We know that the total energy will be:

\[ E = \frac{\hbar^2}{2m} \left[ \left( \frac{n_x}{L_x} \right)^2 + \left( \frac{n_y}{L_y} \right)^2 + \left( \frac{n_z}{L_z} \right)^2 \right] \]

\[ = \frac{\hbar^2 n_r^2}{2mL^2} \left( \frac{n_x^2}{4} + \frac{n_y^2}{4} + \frac{n_z^2}{4} \right) = E_0 \left( \frac{n_x^2}{4} + \frac{n_y^2}{4} + \frac{n_z^2}{4} \right) \]

where \( E_0 = \frac{\hbar^2 n_r^2}{2mL^2} \). We want the six lowest states (or wave functions).

\[
\begin{align*}
& n_x \quad n_y \quad n_z \quad E \text{ (in terms of } E_0) \\
& 1 \quad 1 \quad 1 \quad \frac{3}{2} \\
& 1 \quad 2 \quad 1 \quad \frac{9}{4} \\
& 1 \quad 1 \quad 2 \quad \frac{9}{4} \\
& 1 \quad 2 \quad 2 \quad 3 \\
& 1 \quad 1 \quad 3 \quad \frac{9}{2} \\
& 1 \quad 3 \quad 1 \quad \frac{9}{2}
\end{align*}
\]

\( \Psi_{121} \) and \( \Psi_{112} \) are degenerate w/ energy of \( E = \frac{9}{4} E_0 \)

\( \Psi_{113} \) and \( \Psi_{131} \) are degenerate w/ energy of \( E = \frac{9}{2} E_0 \)

8.5 My energies are: \( E = \frac{\pi^2 \hbar^2}{2mL^2} \left( n_x^2 + n_y^2 + n_z^2 \right) \) since \( L_x = L_y = L_z = L \)

\( a \) For my ground state, \( n_x = n_y = n_z = 1 \) and we have:

\[ E_{111} = \frac{3\pi^2 \hbar^2}{2mL^2} = \frac{3h^2}{8mL^2} = \frac{3}{8} \frac{(hc)^2}{Mpc^2 L^2} = \frac{3}{8} \frac{(1240 \text{ eV}\cdot\text{nm})^2}{(9.38 \text{ MeV})(2 \times 10^{-5} \text{ nm})^2} = 1.54 \text{ MeV} \]

\( b \) For my first excited state, either \( n_x = 2, n_y = 1 = n_z \), \( n_x = n_y = 1 = n_z = 2 \), \( n_x = n_y = n_z = 1 \), \( n_x = n_y = 2, n_z = 1 \), \( n_x = n_y = n_z = 2 \) works. Either way, \( n^2 = 6 \) and we have:

\[ E_{211} = E_{121} - E_{111} = \frac{8 \pi^2 \hbar^2}{2mL^2} = 2E_{111} = 3.08 \text{ MeV} \]

\( c \) For my second excited state, either \( n_x = 2 = n_y = n_z = 1 \), \( n_x = n_y = 2, n_z = 1 \), \( n_x = 1, n_y = n_z = 2 \) works. Then \( n^2 = 9 \) and we have:

\[ E_{311} = E_{212} = E_{122} = \frac{9 \pi^2 \hbar^2}{2mL^2} = 3E_{111} = 4.63 \text{ MeV} \]

\( c \) Both the first excited state and the second excited state are 3-fold degenerate.