Physics 2D Final Exam
Department of Physics, UCSD
Summer Session II - 2010
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Instructions:

1. There are EIGHT questions on the exam — you may attempt ANY SIX.

2. All questions are of EQUAL value.

3. Please write your answers in the blue book and make sure that your three-digit code number is written on all pages of the blue book with indelible ink.

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Some Useful Numbers, Equations, and Identities

Speed of light: \( c = 2.998 \times 10^8 \text{ m/s} \)

Planck’s constant: \( h = 6.626 \times 10^{-34} \text{ J s} \)

\( \hbar = \frac{h}{2\pi}; \quad 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J} \)

Coulomb’s constant: \( k = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2 \)

Electron Charge: \( e = 1.602 \times 10^{-19} \text{ C} \)

Electron Mass: \( m_e = 9.11 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV/c}^2 \)

Rydberg Constant: \( R = 1.097 \times 10^7 \text{ m}^{-1} \)

Atomic Mass Unit: \( u = 1.6606 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV/c}^2 \)

Proton Mass: \( m_p = 1.673 \times 10^{-27} \text{ kg} = 938.3 \text{ MeV/c}^2 = 1.0073 \text{ u} \)

Neutron Mass: \( m_n = 1.675 \times 10^{-27} \text{ kg} = 939.6 \text{ MeV/c}^2 = 1.0087 \text{ u} \)

Compton wavelength for an electron: \( \frac{h}{m_e c} = 0.00243 \text{ nm} \)

Compton scattering formula: \( \lambda' - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta) \)

Photo-electric equation: \( eV_s = hf - \phi = h (f - f_0) \)

Momentum for a relativistic particle: \( p = \gamma m_0 u, \quad \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \)

Energy for a particle: \( E = K + mc^2 = \gamma mc^2 \)

Energy-momentum relation (particle): \( p = \frac{1}{c} \sqrt{E^2 - m_0^2 c^4} = \frac{1}{c} \sqrt{2m_0 c^4 K + K^2} \)

Energy-momentum relation (photon): \( E = pc \)

Relative velocity: \( u' = \frac{u - v}{1 - \frac{uv}{c^2}} \)

Doppler Effect (light source approaching observer): \( f_{\text{obs}} = f_0 \sqrt{\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}}} \)

De Broglie wavelength: \( \lambda = \frac{h}{p} \)
Schrödinger Equation: \(-\frac{h^2}{2m} \frac{d^2 \psi}{dx^2} + U(x) \psi = E \psi\)

1-Dimensional Normalization Condition: \(\int_{-\infty}^{\infty} \psi^* \psi dx = \int_{-\infty}^{\infty} |\psi|^2 dx = 1\)

Harmonic Oscillator Potential: \(U = \frac{1}{2} m \omega^2 x^2\)

For a Hydrogen-like atom:

- Energy: \(E_n = -\frac{ke^2}{2a_0} \frac{n^2}{nt}, \quad n = 1, 2, 3, 4, \ldots\)
- Bohr radius: \(a_0 = \frac{h^2}{4\pi^2 m_e e^2} = 0.529 \times 10^{-10} \text{ m}\)

Volume element in spherical coordinates: \(dV = r^2 \sin \theta dr d\theta d\phi\) or \(4\pi r^2 dr\)

Ground state Wavefunction for Hydrogen: \(\Psi (r, \theta, \phi) = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}\)

Root-Mean-Square deviation: \(\Delta r = \sqrt{r^2 - \bar{r}^2}\)

Expectation Value for an operator \(Q\): \(\bar{Q} = \int_{\text{all space}} dV \Psi^* [Q] \Psi\)

\(\sin^2 \theta = \frac{1}{2} [1 - \cos (2\theta)]\)
\(\cos^2 \theta = \frac{1}{2} [1 + \cos (2\theta)]\)

\(\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{\alpha}}, \quad \alpha > 0\)
\(\int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{\alpha^3}}, \quad \alpha > 0\)
\(\int_a^b x^n e^{-x} dx = -\left( x^n + nx^{n-1} + n(n-1)x^{n-2} + \ldots + n! \right) e^{-x} \bigg|_a^b\)
\(\int_0^\infty x^n e^{-x} dx = n!\)