Uncertainty, Measurement, and Models

Lecture 2
Physics 2CL
Summer Session 2010
Outline

• Brief overview of lab write-ups
• What uncertainty (error) analysis can for you
• Issues with measurement and observation
• What does a model do?
• Brief overview of circuit analysis
Preparing for Lab

• Write-up due at the end of the last meeting
• Prepare by doing Taylor homework and prelab questions
Lab Write-ups

• Begin with lab number & title, date and you and your partners name
• Start with Taylor homework and prelab questions
• State briefly the objective
• Record all data with units and uncertainties
• Brief description of procedure
• Make clear labeled diagrams of setups
• Use graphs to present data, label axes, plot error bars - Origin
Lab Write-up continued

• Include and justify functional fit of data
• Show calculations of final derived quantities, include uncertainty analysis
• State results and comment on the agreement with expectations (or not)
  – Be quantitative (within uncertainty, t-value)
What is uncertainty (error)?

• Uncertainty (or error) in a measurement is not the same as a mistake
• Uncertainty results from:
  – Limits of instruments
    • finite spacing of markings on ruler
  – Design of measurement
    • using stopwatch instead of photogate
  – Less-well defined quantities
    • composition of materials
Understanding uncertainty is important

- for comparing values
- for distinguishing between models
- for designing to specifications/planning

Measurements are less useful (often useless) without a statement of their uncertainty
An example

Batteries

rated for 9 V potential difference across terminals in reality…
Utility of uncertainty analysis

• Evaluating uncertainty in a measurement and calculated quantities
• Propagating errors – ability to extend results to other measurements
• Analyzing a distribution of values
• Quantifying relationships between measured values
Evaluating error in measurements

• To measure height of building, drop rock and measure time to fall:  \( d = \frac{1}{2} gt^2 \)
• Measure times
  2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s
• What is the “best” value
• How certain are we of it?
Calculate “best” value of the time

• Calculate average value (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)

\[ t = \frac{\sum_{i=1}^{n} t_i}{n} \]

\[ t = 2.51666666666666666666666 \text{ s} \]

• Is this reasonable?
Uncertainty in time

• Measured values - (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)

• By inspection can say uncertainty < 0.3 s

• Calculate **standard deviation**

\[ \sigma = \sqrt{\frac{\sum (t_i - \bar{t})^2}{n-1}} \]

\[ \sigma = 0.1949976 \text{ s} \]

\[ \sigma = 0.2 \text{ s} \] (But what does this mean???)
How to quote best value

• Now what is best quote of average value
  – $\bar{t} = 2.51666666666666666666666 \text{ s}$
  – $\bar{t} = 2.52 \text{ s}$

• What is uncertainty
  – Introduce standard deviation of the mean
    $\sigma_t = \sigma/\sqrt{n} = 0.08 \text{ s}$

• Best value is
  – $\bar{t} = 2.52 \pm 0.08 \text{ s}$
Propagation of error

• Same experiment, continued…
• From best estimate of time, get best estimate of distance: 30.6 meters
• Know uncertainty in time, what about uncertainty in distance?
• From error analysis tells us how errors propagate through mathematical function
  \[ \varepsilon(d) = 2 \times \varepsilon(t) \]
  \[ 8\% \text{ uncertainty or } \pm 2.4\text{m} \]
Drawing Conclusions:
The t-value

• Does value agree or not with accepted value of 30.7m?
• How different is it from the accepted? Introduce “t-value” \((t)\)
  \[
t = \frac{|d_{\text{meas}} - d_{\text{accept}}|}{\sigma} = 0.1/2.4 = 0.04
\]
• If value difference \(\leq 1\) then they agree
• Later we will learn what this means in a more quantitative way
Expected uncertainty in a calculated sum $a = b + c$

- Each value has an uncertainty
  - $b = \overline{b} \pm \delta_b$
  - $c = \overline{c} \pm \delta_c$
- Uncertainty for $a$ ($\delta a$) is at most the sum of the uncertainties
  - $\delta a = \delta b + \delta c$
- Better value for $\delta a$ is
  - $\delta a = \sqrt{\delta b^2 + \delta c^2}$
- Best value is
  - $a = \overline{a} \pm \delta a$
Expected uncertainty in a calculated product \( a = b \times c \)

- Each value has an uncertainty
  - \( b = b \pm \delta b \)
  - \( c = c \pm \delta c \)
- Relative uncertainty for \( a \) (\( \epsilon_a \)) is at most the sum of the RELATIVE uncertainties
  \[ \epsilon_a = \frac{\delta a}{a} = \epsilon b + \epsilon c \]
- Better value for \( \delta a \) is
  \[ \epsilon a = \sqrt{\epsilon b^2 + \epsilon c^2} \]
- Best value is
  - \( a = \bar{a} \pm \epsilon a \) (fractional uncertainty)
What about powers in a product

\[ a = b \cdot c^2 \]

- Each value has an uncertainty
  - \( b = b \pm \delta b \)
  - \( c = c \pm \delta c \)
  - \( \varepsilon a = \frac{\delta a}{a} \) (relative uncertainty)
- powers become a prefactor (weighting) in the error propagation
  - \( \varepsilon a = \sqrt{(\varepsilon b^2 + (2 \varepsilon c)^2)} \)
How does uncertainty in $t$ effect the calculated parameter $d$?

\[
d = \frac{1}{2} g t^2
\]

\[
\varepsilon d = \sqrt{(2*\varepsilon t)^2} = 2*\varepsilon t
\]

\[
\varepsilon d = 2*(.09/2.52) = 0.071
\]

\[
\delta d = 0.071*31 \text{ m} = 2.2 \text{ m} = 2 \text{ m}
\]

Statistical error
Relationships

• Know there is a functional relation between $d$ and $t$ 
  $$d = \frac{1}{2} g t^2$$
• $d$ is directly proportional to $t^2$
• Related through a constant $\frac{1}{2} g$
• Can measure time of drop ($t$) at different heights ($d$)
• plot $d$ versus $t$ to obtain constant
Quantifying relationships

\[ d = \frac{1}{2} gt^2 \]

FIT:
\[ g = 8.3 \pm 0.3 \text{ m/s}^2 \]
Different way to plot

\[ d = \frac{1}{2} gt^2 \]

Fit:
- slope = 4.3 ± 0.2 m/s²
- intercept = -10 ± 10 m

slope = \( \frac{1}{2} g \)
Compare analysis of SAME data

• From a fit of the curve $d$ versus $t$ obtained
  – $g = 8.3 \pm 0.3 \text{ m/s}^2$
• From the fit of $d$ versus $t^2$ obtained
  – $g = 8.6 \pm 0.4 \text{ m/s}^2$
• Do the two values agree?
• Which is the better value?
Measurement and Observation

• Measurement: deciding the amount of a given property by observation
  • Empirical
  • Not logical deduction
  • Not all measurements are created equal…
Reproducibility

• Same results under similar circumstances
  – Reliable/precise

• ‘Similar’ - a slippery thing
  – Measure resistance of metal
    • need same sample purity for repeatable measurement
    • need same people in room?
    • same potential difference?
  – Measure outcome of treatment on patients
    • Can’t repeat on same patient
    • Patients not the same
Precision and Accuracy

• Precise - reproducible
• Accurate - close to true value
• Example - temperature measurement
  – thermometer with
    • fine divisions
    • or with coarse divisions
  – and that reads
    • 0 °C in ice water
    • or 5 °C in ice water
Accuracy vs. Precision
Random and Systematic Errors

• Accuracy and precision are related to types of errors
  – random (thermometer with coarse scale)
    • can be reduced with repeated measurements, careful design
  – systematic (calibration error)
    • difficult to detect with error analysis
    • compare to independent measurement
Observations in Practice

• Does a measurement measure what you think it does? Validity
• Are scope of observations appropriate?
  – Incidental circumstances
  – Sample selection bias
• Depends on model
Models

• Model is a construction that represents a subject or imitates a system
• Used to predict other behaviors (extrapolation)
• Provides context for measurements and design of experiments
  – guide to features of significance during observation
Testing model

• Models must be consistent with data
• Decide between competing models
  – elaboration: extend model to region of disagreement
  – precision: prefer model that is more precise
  – simplicity: Ockham’s razor
Oscilloscope – screen
Oscilloscope – voltage scale
Series and Parallel

Simple circuit

Series: \( R \) and \( \varepsilon \)

Parallel: \( R_1 \) and \( R_2 \)
Uncertainties - Series and Parallel

\[ R_{\text{TOTAL}} = R_1 + R_2 \]
\[ \delta R_{\text{TOTAL}} = \sqrt{\delta R_1^2 + \delta R_2^2} \]

\[ \frac{1}{R_{\text{TOTAL}}} = \frac{1}{R_1} + \frac{1}{R_2} \]
\[ R_{\text{TOTAL}} = \frac{R_2 \times R_1}{R_1 + R_2} \]
\[ \varepsilon R_{\text{TOTAL}} = \sqrt{\varepsilon(R_2 \times R_1)^2 + \varepsilon(R_2 + R_1)^2} \]
Circuit Analysis

**Loop rule**

$$\sum_{k} \Delta V_{k} = \sum_{j} E_{j}$$

**Junction rule**

$$\sum_{k} i_{k} = 0$$
Kirchoff's Rules

<table>
<thead>
<tr>
<th>Component</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>$\Delta V = V_b - V_a = -IR$</td>
<td>Current flows through a resistor.</td>
</tr>
<tr>
<td>EMF Source</td>
<td>$\Delta V = V_b - V_a = +\mathcal{E}$</td>
<td>Voltage change across the source.</td>
</tr>
<tr>
<td>Capacitor</td>
<td>$\Delta V = V_b - V_a = +\frac{q}{C}$</td>
<td>Charge stored in the capacitor.</td>
</tr>
</tbody>
</table>
Circuit Exp. 1

**Charge**

\[ V_c = \frac{q(t)}{C} = V_{\max} \left( 1 - e^{-\frac{t}{RC}} \right) \]

**Discharge**

\[ V_c(t) = V_{\max} e^{-\frac{t}{RC}} \]
Reminder

- perform lab #0
- Read and Prepare for lab # 1 on Tuesday/Wednesday
- Read Taylor through chapter 3
- Do assigned homework – Taylor problems 3.7, 3.36, 3.41