Uncertainty, Measurement, and Models
Overview Exp #1

Lab Session 2
Physics 2BL
Summer Session 2010
# Lab TAs

<table>
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<td>694439 A50</td>
<td>TuTh</td>
<td>11:00a-1:50p</td>
<td>Paul Hemphill, Stefan Progovac</td>
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<tr>
<td>694440 A51</td>
<td>TuTh</td>
<td>2:00p-4:50p</td>
<td>Paul Hemphill, Stefan Progovac</td>
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Present in labs to assist
Outline

• What uncertainty (error) analysis can for you
• Issues with measurement and observation
• What does a model do?
What is uncertainty (error)?

• Uncertainty (or error) in a measurement is not the same as a mistake

• Uncertainty results from:
  – Limits of instruments
    • finite spacing of markings on ruler
  – Design of measurement
    • using stopwatch instead of photogate
  – Less-well defined quantities
    • composition of materials
Understanding uncertainty is important

- for comparing values
- for distinguishing between models
- for designing to specifications/planning

Measurements are less useful (often useless) without a statement of their uncertainty
An example

Batteries
  rated for 1.5 V potential difference across terminals
  in reality…
Utility of uncertainty analysis

- Evaluating uncertainty in a measurement
- Propagating errors – ability to extend results through calculations or to other measurements
- Analyzing a distribution of values
- Quantifying relationships between measured values
Evaluating error in measurements

- To measure height of building, drop rock and measure time to fall:  \[ d = \frac{1}{2} gt^2 \]
- Measure times
  2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s
- What is the “best” value
- How certain are we of it?
Calculate “best” value of the time

- Calculate average value (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)
  \[ t = \frac{\sum_{i=1}^{n} t_i}{n} \]
  \[ t = 2.51666666666666666666667 \text{ s} \]

- Is this reasonable?
  Significant figures
Uncertainty in time

- Measured values - (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)

- By inspection can say uncertainty < 0.4 s

- Calculate standard deviation

  \[ \sigma = \sqrt{\frac{\sum (t_i - \bar{t})^2}{n-1}} \]
  \[ \sigma = 0.2137288 \text{ s} \]
  \[ \sigma = 0.2 \text{ s} \quad \text{(But what does this mean???)} \]
How to quote best value

• What is uncertainty in average value?
  – Introduce standard deviation of the mean
    \[ \sigma_t = \frac{\sigma}{\sqrt{n}} = 0.08725 \text{ s} = 0.09 \text{ s} \]

• Now what is best quote of average value
  – \( \bar{t} = 2.51666666666666666666 \text{ s} \)
  – \( \bar{t} = 2.52 \text{ s} \)

• Best value is
  – \( \bar{t} = 2.52 \pm 0.09 \text{ s} \)
Propagation of error

• Same experiment, continued…
• From best estimate of time, get best estimate of distance: 31 meters
• Know uncertainty in time, what about uncertainty in distance?
• From error analysis tells us how errors propagate through mathematical functions
  (2 meters)
Expected uncertainty in a calculated sum $a = b + c$

- Each value has an uncertainty
  - $b = \bar{b} \pm \delta b$
  - $c = \bar{c} \pm \delta c$

- Uncertainty for $a$ ($\delta a$) is at most the sum of the uncertainties
  $$\delta a = \delta b + \delta c$$

- Better value for $\delta a$ is
  $$\delta a = \sqrt{(\delta b^2 + \delta c^2)}$$

- Best value is
  - $a = \bar{a} \pm \delta a$
Expected uncertainty in a calculated product $a = b \times c$

- Each value has an uncertainty
  - $b = b \pm \delta b$
  - $c = c \pm \delta c$

- Relative uncertainty for $a$ ($\varepsilon_a$) is at most the sum of the RELATIVE uncertainties
  $$\varepsilon_a = \frac{\delta a}{a} = \varepsilon b + \varepsilon c$$

- Better value for $\delta a$ is
  $$\varepsilon a = \sqrt{(\varepsilon b^2 + \varepsilon c^2)}$$

- Best value is
  - $a = a \pm \varepsilon a$ (fractional uncertainty)
What about powers in a product

\[ a = b \cdot c^2 \]

– Each value has an uncertainty
  - \( b = b \pm \delta b \)
  - \( c = c \pm \delta c \)
  - \( \varepsilon a = \delta a/a \quad (\text{relative uncertainty}) \)

– powers become a prefactor (weighting) in the error propagation
  - \( \varepsilon a^2 = (\varepsilon b^2 + (2 \varepsilon c)^2) \)
How does uncertainty in $t$ effect the calculated parameter $d$?

$$d = \frac{1}{2} g t^2$$

$$\varepsilon_d = (2 \varepsilon t)^2 = 2 \varepsilon t$$

$$\varepsilon_d = 2 \times \frac{.09}{2.52} = 0.071$$

$$\delta d = .071 \times 31 \text{ m} = 2.2 \text{ m} = 2 \text{ m}$$

Statistical error
Relationships

• Know there is a functional relation between d and t  \[ d = \frac{1}{2} g t^2 \]
• d is directly proportional to \( t^2 \)
• Related through a constant \( \frac{1}{2} g \)
• Can measure time of drop (t) at different heights (d)
• plot d versus t to obtain constant
Quantifying relationships

\[ d = \frac{1}{2} gt^2 \]

FIT:
\[ g = 8.3 \pm 0.3 \text{ m/s}^2 \]
Different way to plot

\[ d = \frac{1}{2} g t^2 \]

Fit:
- slope = 4.3 ± 0.2 m/s²
- intercept = -10 ± 10 m

slope = \( \frac{1}{2} g \)
Compare analysis of SAME data

- From a fit of the curve $d$ versus $t$ obtained
  - $g = 8.3 \pm 0.3 \text{ m/s}^2$
- From the fit of $d$ versus $t^2$ obtained
  - $g = 8.6 \pm 0.4 \text{ m/s}^2$
- Do the two values agree?
- Which is the better value?
Measurement and Observation

• Measurement: deciding the amount of a given property by observation
• Empirical
• Not logical deduction
• Not all measurements are created equal…
Reproducibility

- Same results under similar circumstances
  - Reliable/precise
- ‘Similar’ - a slippery thing
  - Measure resistance of metal
    - need same sample purity for repeatable measurement
    - need same people in room?
    - same potential difference?
  - Measure outcome of treatment on patients
    - Can’t repeat on same patient
    - Patients not the same
Precision and Accuracy

- Precise - reproducible
- Accurate - close to true value
- Example - temperature measurement
  - thermometer with
    - fine divisions
    - or with coarse divisions
  - and that reads
    - 0 °C in ice water
    - or 5 °C in ice water
Accuracy vs. Precision

Precision

Accuracy
Random and Systematic Errors

• Accuracy and precision are related to types of errors
  – random (thermometer with coarse scale)
    • can be reduced with repeated measurements, careful design
  – systematic (calibration error)
    • difficult to detect with error analysis
    • compare to independent measurement
Observations in Practice

• Does a measurement measure what you think it does? Validity

• Are scope of observations appropriate?
  – Incidental circumstances
  – Sample selection bias

• Depends on model
Models

- Model is a construction that represents a subject or imitates a system
- Used to predict other behaviors (extrapolation)
- Provides context for measurements and design of experiments
  - guide to features of significance during observation
Testing model

• Models must be consistent with data
• Decide between competing models
  – elaboration: extend model to region of disagreement
  – precision: prefer model that is more precise
  – simplicity: Ockham’s razor
Experiment 1 Overview: Density of Earth

\[ \rho = \frac{M_E}{\frac{4}{3} \pi R_E^3} = \frac{3g}{4\pi GR_E} = \frac{GM_E m}{R_E^2} = mg \]

- Measure \( \Delta t \) between sunset on cliff and at sea level
Experiment 1: Height of Cliff

rangepfinder to get $L$

Sextant to get $\theta$

Wear comfortable shoes

Make sure you use $\theta$ and not $(90 - \theta)$
Experiment 1:
Determine $g$

F = $-mg \sin(\phi) = -mg \phi$

F = $ml \ddot{\phi}$

$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{l}{g}}$

period
Experiment 1: Pendulum

- For release angle $\theta_i$, you should have a set of time data $(t_{i1}^P, t_{i2}^P, t_{i3}^P, ..., t_{iN}^P)$.

- Calculate the average, $\bar{t}^P$, and the the standard deviation, $\sigma_{tP}$, of this data.

- Divide $\bar{t}^P$ and $\sigma_{tP}$ by $p$ to get average time of a single period, $\bar{T}$ and standard deviation of a single period $\sigma_T$.

- Calculate SDOM, $\sigma_T = \frac{\sigma_T}{\sqrt{N}}$.

- Now you should have $\bar{T} \pm \sigma_T$ for your data at $\theta_i$.

- Repeat these calculations for data at each release angle.

Grading rubric uploaded on website
Reminder

- Prepare for lab
- Read Taylor chapter 4
- Homework Taylor 4.6, 4.14, 4.18, 4.26
  (separate sheet)