Temperature coefficient of resistivity

\[ \rho = \rho_0 [1 + \alpha (T - T_o)] \]

\[ R = R_0 [1 + \alpha (T - T_o)] \]

\( T_0 = \) reference temperature

\( \alpha = \) temperature coefficient of resistivity, units of \((^\circ C)^{-1}\)

For Ag, Cu, Au, Al, W, Fe, Pt, Pb: values of \(\alpha\) are \(\sim 3-5 \times 10^{-3} \ (^\circ C)^{-1}\)
Typical tungsten filament: ~1 m long, but 0.05mm in radius.

Calculate typical R.

\[ A = \pi (5 \times 10^{-5} \text{m})^2 = 7.9 \times 10^{-9} \text{ m}^2 \]

\[ \rho = 5.6 \times 10^{-8} \Omega \text{m} \text{ (Table 17.1)} \]

\[ R = \rho \frac{L}{A} = \left( 5.6 \times 10^{-8} \Omega \text{m} \right) \frac{1 \text{m}}{7.9 \times 10^{-9} \text{ m}^2} = 7.1 \Omega \]

Note: As per section 17.6, the resistivity value used above is valid only at a temperature of 20°C, so this derived value of R holds only for T=20°C.
Calculate \( \rho \) at \( T=4000^\circ\text{C} \), assuming a linear \( \rho\)-\( T \) relation:

For tungsten, \( \alpha = 4.5 \times 10^{-3}/^\circ\text{C} \)

\[
\rho = \rho_0[1+\alpha(T-T_0)] = 8.3 \times 10^{-7} \ \Omega\text{m}
\]

\[
R = \rho L/A = 106 \ \Omega.
\]

(note-- this is still less than the estimate of \( >200 \ \Omega \) we’ll derive in class in a few minutes... I suspect the \( \rho\)-\( T \) relation in reality may not be strictly linear over such a wide range of temperature; my guess would be that the above value of \( \alpha \) may only be valid for temperatures of tens to hundreds of \( ^\circ\text{C} \))
Superconductors

For some materials, as temperature drops, resistance suddenly plummets to 0 below some $T_c$.

Once a current is set up, it can persist without any applied voltage because $R \to 0$!
Superconductors

Applications:

• Energy storage at power plants
• Superconducting magnets with much stronger magnetic fields than normal electromagnets
• Superconducting distribution power lines could eliminate resistive losses
More recently: As the field has advanced, materials with higher values of $T_c$ get discovered.

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_c$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zn</td>
<td>0.88</td>
</tr>
<tr>
<td>Al</td>
<td>1.19</td>
</tr>
<tr>
<td>Sn</td>
<td>3.72</td>
</tr>
<tr>
<td>Hg</td>
<td>4.15</td>
</tr>
<tr>
<td>Pb</td>
<td>7.18</td>
</tr>
<tr>
<td>Nb</td>
<td>9.46</td>
</tr>
<tr>
<td>Nb$_3$Sn</td>
<td>18.05</td>
</tr>
<tr>
<td>Nb$_3$Ge</td>
<td>23.2</td>
</tr>
<tr>
<td>YBa$_2$Cu$_3$O$_7$</td>
<td>90</td>
</tr>
<tr>
<td>Bi–Sr–Ca–Cu–O</td>
<td>105</td>
</tr>
<tr>
<td>Tl–Ba–Ca–Cu–O</td>
<td>125</td>
</tr>
<tr>
<td>HgBa$_2$Ca$_2$Cu$_3$O$_8$</td>
<td>134</td>
</tr>
</tbody>
</table>
Electrical Energy and Power

Power dissipated in a R is due to collisions of charge carriers with the lattice. Electrical potential energy is converted to thermal energy in the resistor--a light bulb filament thus glows or toaster filaments give off heat (and turn orange)
Power dissipated in a resistor

Power = work / time = \( q \Delta V / \Delta t \)

\[ P = I \times \Delta V \]

\[ P = I^2 \times R \]

\[ P = \Delta V^2 / R \]

UNITs:

\[ P = I \times V = \text{Amp} \times \text{Volt} = \text{C/s} \times \text{J/C} = \text{J/s} = \text{WATT} \]
Example: A typical household incandescent lightbulb is connected to a 120V outlet. The power output is 60 Watts. What's the current through the bulb? What’s R of the filament?
Example: A typical household incandescent lightbulb is connected to a 120V outlet. The power output is 60 Watts. What's the current through the bulb? What’s R of the filament?

\[ \Delta V = 120 \text{ V (rel. to ground)} \]
\[ P = I \Delta V \rightarrow I = \frac{P}{\Delta V} = \frac{60\text{W}}{120\text{V}} = 0.5 \text{ A} \]

\[ P = \frac{\Delta V^2}{R} \rightarrow \quad R = \frac{\Delta V^2}{P} = \frac{(120\text{V})^2}{60 \text{ W}} = 240 \, \Omega \]

Note -- a few slides earlier, we’d estimated the typical resistance of a tungsten light bulb filament at 4000°C -- that estimate of ~106 Ω assumed for simplicity a constant coefficient of resistivity \( \alpha \) from 20°C to 4000°C, which might not be the case in reality. If the actual value of \( \alpha \) increases as \( T \) increases, then the dependence of \( \rho \) on \( T \) will also be non-linear.
A heating element in an electric range is rated at 2000 W. Find the current required if the voltage is 240 V. Find the resistance of the heating element.

\[ P = I\Delta V \rightarrow I = \frac{P}{\Delta V} = \frac{2000\text{W}}{240\text{V}} = 8.3 \text{ A} \]

\[ R = \frac{\Delta V^2}{P} = \frac{(240\text{V})^2}{2000\text{W}} = 28.8 \Omega \]
Cost of electrical power

1 kilowatt-hour = 1000 W * 1 hour = 1000 J/s (3600s) = 3.6e6 J.

1kWh costs about $0.13, typically

How much does it cost to keep a single 100W light bulb on for 24 hours?
(100W)*24hrs = 2400 W-hr = 2.4kWh
2.4kWh*$0.13 = $0.31

So how much does it cost per week to keep the ~40 fluorescent lights in this classroom on for 40 hours per week? (assume P=20W, since fluor. bulbs are ~4x as efficient as producing visible light as incandescent light bulbs).
40x20W*40hr = 32000 W-hr = 32kWh
32kWh*$0.13 = $4.16
How many rooms are there on campus?
Power Transmission

Transmitting electrical power is done much more efficiently at higher voltages due to the desire to minimize $(I^2R)$ losses.

Consider power transmission to a small community which is 100 mi from the power plant and which consumes power at a rate of 10 MW.

In other words, the generating station needs to supply whatever power it takes such that $P_{\text{req}} = 10$ MW arrives at the end user (compensating for $I^2R$ losses): $P_{\text{generated}} = P_{\text{loss}} + P_{\text{req}}$

Consider three cases:
A: $V=2000$ V; $I=5000$ A ($P_{\text{req}} = IV = 10^7$ W)
B: $V=20000$ V; $I=500$ A ($P_{\text{req}} = IV = 10^7$ W)
C: $V=200000$ V; $I=50$ A ($P_{\text{req}} = IV = 10^7$ W)
Power Transmission

Resistance/length = 0.0001 $\Omega$ / foot.
Length of transmission line = 100 mile = 528000 feet.
Total $R = 52.8 \, \Omega$.

**A:** $P_{\text{loss}} = I^2R = (5000A)^2(52.8\Omega) = 1.33 \times 10^3$ MW

$P_{\text{generated}} = P_{\text{loss}} + P_{\text{req}} = 1.33 \times 10^3$ MW + 10 MW = $1.34 \times 10^3$ MW

**Efficiency of transmission** = $P_{\text{req}} / P_{\text{generated}} = 0.75\%$

**B:** $P_{\text{loss}} = I^2R = (500A)^2(52.8\Omega) = 13.3$ MW

$P_{\text{generated}} = P_{\text{loss}} + P_{\text{req}} = 13.3$ MW + 10 MW = $23.3$ MW

**Efficiency of transmission** = $P_{\text{req}} / P_{\text{generated}} = 43\%$

**C:** $P_{\text{loss}} = I^2R = (50A)^2(52.8\Omega) = 0.133$ MW

$P_{\text{generated}} = P_{\text{loss}} + P_{\text{req}} = 0.133$ MW + 10 MW = $10.133$ MW

**Efficiency of transmission** = $P_{\text{req}} / P_{\text{generated}} = 98.7\%$ (most reasonable)

*Lower current during transmission yields a reduction in $P_{\text{loss}}$.
You can do the same exercise for local distribution lines (assume $P_{\text{req}} = 0.1 \text{ MW}$), which are usually a few miles long (so the value of $R$ is $\sim$ a few) and need to distribute power from substations to local neighborhoods at a voltage of at least a few thousand volts (keeping currents under $\sim 30\text{A}$, roughly) to have a transmission efficiency above $\sim 90\%$. 
Ch 18: Direct-Current Circuits

EMF

Resistors in Series & in Parallel

Kirchhoff’s Junction & Loop Rules for complex circuits

RC Circuits

Household circuits & Electrical Safety
Sources of EMF

In a closed circuit, the source of EMF is what drives and sustains the current.

EMF = work done per charge: Joule / Coulomb = Volt
Sources of EMF

In a closed circuit, the source of EMF is what drives and sustains the current.

EMF = work done per charge: Joule / Coulomb = Volt

Assume internal resistance \( r \) of battery is negligible.

Here, \( \mathcal{E} = IR \)
From A to B: Potential increases by $\Delta V = +\varepsilon$

From B to A: Potential decreases by $\Delta V = -\varepsilon$.

From C to D: Potential decreases by $\Delta V = -IR = -\varepsilon$
If circuit is grounded: V at points A & D will be zero.

From A to B: Potential increases by \( \Delta V = +\varepsilon \)

From B to A: Potential decreases by \( \Delta V = -\varepsilon \).

From C to D: Potential decreases by \( \Delta V = -IR = -\varepsilon \)
Why is this useful?

The middle voltage can be 'tailored' to any voltage we desire (between 0 and \( \varepsilon \)) by adjusting \( R_1 \) and \( R_2 \)!
Resistors connected in series

What’s $R_{eq}$ in terms of $R_1$ and $R_2$?

$\Delta V = I R_{eq}$
Resistors connected in series

Note: Current is the same in $R_1$ and $R_2$.

$\Delta V_1 = IR_1$

$\Delta V_2 = IR_2$

$\Delta V = \Delta V_1 + \Delta V_2$

$\Delta V = IR_1 + IR_2 = I(R_1+R_2)$

$\Delta V = IR_{eq}$

$R_{eq} = R_1 + R_2$

For $N$ resistors in series:

$R_{eq} = R_1 + R_2 + \ldots + R_N$

Note that $R_{eq}$ is larger than any one individual $R$ value.
Resistors connected in series

Find $R_{eq}$:

$$R_{eq} = 4\Omega + 7\Omega + 1\Omega + 2\Omega = 14\Omega$$
Understanding the Series Law

\[ R = \rho \frac{L}{A} \]

means \( R \) is prop.to \( L \)

Total \( R \) is prop. to \( (L_1 + L_2) \)