18.5 RC Circuits

Introduction to time-dependent currents and voltages.

Applications: timing circuits, clocks, computers, charging + discharging capacitors
RC circuit: charging

At time $t=0$, close Switch
### RC circuit: charging

#### Graphs:
- **Q(t):**
  - Initial charge: $0.632C\varepsilon$
  - Final charge: $0.368\varepsilon/R$
  - Time constant: $\tau = RC$

- **I(t):**
  - Initial current: $\varepsilon/R$
  - Final current: $0$

#### Time Table:

<table>
<thead>
<tr>
<th></th>
<th>$t=0$</th>
<th>$t \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta V_C$</td>
<td>0</td>
<td>$\varepsilon(1-e^{-(t/\tau)})$</td>
</tr>
<tr>
<td>$Q$</td>
<td>0</td>
<td>$C\varepsilon(1-e^{-(t/\tau)})$</td>
</tr>
<tr>
<td>$\Delta V_R$</td>
<td>$\varepsilon$</td>
<td>$\varepsilon(e^{-(t/\tau)})$</td>
</tr>
<tr>
<td>$I$</td>
<td>$\varepsilon/R$</td>
<td>$(\varepsilon/R)(e^{-(t/\tau)})$</td>
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Time constant $\tau = RC$

RC is called the time constant: it's a measure of how fast the capacitor is charged up.

It has units of time:
$RC = \frac{(V/I)(q/V)}{q/I} = q/I = q/(q/t) = t$

At $t = RC$, $Q(t)$ and $\Delta V_C(t)$ go to $1 - 1/e = 0.63$ of the final values

At $t = RC$, $I(t)$ and $\Delta V_R(t)$ go to $1/e$ of the initial values
Time constant $\tau = RC$

Think about why increasing R and/or C would increase the time to charge up the capacitor:

When charging up: $\tau$ will increase with C because the capacitor can store more charge. Increases with R because the flow of current is lower.
Time constant $\tau = RC$

$$V_c = \frac{q(t)}{C} = V_{\text{max}} \left(1 - e^{-t/RC}\right)$$

$V_{\text{max}}$

$V_c(t)$

$i(t) = \frac{V_{\text{max}}}{R} e^{-t/RC}$

$\tau_1 < \tau_2 < \tau_3$
You want to make a so-called flasher circuit that charges a capacitor through a resistor up to a voltage at which a neon bulb discharges once every 5.0 sec. If you have a 10 microfarad capacitor what resistor do you need?

Solution: Have the flash point be equal to $0.63 \Delta V_{C,\text{max}}$ (i.e., let $t=\tau$)

$$\tau = RC \rightarrow R = \frac{\tau}{C} = \frac{5\text{s}}{10^{-6}\text{F}} = 5 \times 10^5 \text{ Ohms}$$

This is a very big resistance, but 5 seconds is pretty long in "circuit" time
Discharging an RC circuit:

First, disconnect from EMF source.

Q is at maximum value, \( Q_{\text{max}} = C\varepsilon \)

\( \Delta V_C \) is at maximum value of \( \Delta V_{C,\text{max}} = \varepsilon \)
Discharging an RC circuit:

Then close switch at time $t=0$. 
Discharging an RC circuit:

Circuit now has only R and C.

From loop rule: $-\Delta V_C - \Delta V_R = 0$
Discharging an RC circuit:

Circuit now has only R and C.

From loop rule: \(- \Delta V_C - \Delta V_R = 0\)

As capacitor discharges, Q and \(\Delta V_C\) decrease with time.

\(\Delta V_R\) will track \(\Delta V_C\)

So \(I = \frac{\Delta V_R}{R}\) will jump from 0 to \(\varepsilon/R\) at \(t=0\), then exponentially decay
\[ I(t) = \frac{\varepsilon}{R} \left( e^{-\frac{t}{\tau}} \right) \]

\[ \tau = RC \]

\[ Q \]

\[ Q_{\text{max}} = C\varepsilon \]

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</tr>
<tr>
<td>( \Delta V_R )</td>
<td>( \varepsilon )</td>
</tr>
<tr>
<td>( I )</td>
<td>( \frac{\varepsilon}{R} )</td>
</tr>
</tbody>
</table>
Think about why increasing R and/or C would increase the time to discharge the capacitor:

$\tau$ will increase with C because there is more stored charge in the capacitor to unload. $\tau$ increases with R because the flow of current is lower.
Given a 12 μF capacitor being discharged through a 2000 Ω resistor. How long does it take for the voltage drop across the resistor to reach 5% of the initial voltage?

Solution:

First, calculate τ: \( \tau = 2000 \Omega \times 12 \times 10^{-6} \text{ F} = 24 \text{ ms} \)

Then: \( V = V_0 \exp(-t/\tau) \)

\( V / V_0 = \exp(-t/\tau) \)

Take ln of both sides: \( \ln(V/V_0) = -t/\tau \)

Solve for t: \( t = -\tau \ln(V/V_0) = -0.024 \text{ s ( ln(0.05) )} = 0.072 \text{ sec} \)
Household circuits

Circuits are in parallel. All devices have same potential. If one device fails, others will continue to work at required potential.

$\Delta V$ is 120 V above ground potential

Heavy-duty appliances (electric ranges, clothes dryers) require 240 V. Power co. supplies a line which is 120V BELOW ground potential so TOTAL potential drop is 240 V
Circuit breakers or fuses are connected in series.

Fuses: melt when I gets too high, opening the circuit

Circuit breakers: opens circuit without melting. So they can be reset.

Many circuit breakers use electromagnets, to be discussed in future chapters
Example: Consider a microwave oven, a toaster, and a space heater, all operating at 120 V:

Toaster: 1000 W
Microwave: 800 W
Heater: 1300 W

How much current does each draw? \( I = \frac{P}{\Delta V} \)
Toaster: \( I = \frac{1000W}{120V} = 8.33A \)
Microwave: \( I = \frac{800W}{120V} = 6.67A \)
Heater: \( I = \frac{1300W}{120V} = 10.8A \)

Total current (if all operated simultaneously) = 25.8 A
(So the breaker should be able to handle this level of current, otherwise it'll trip)
Electrical Safety
\( R_{\text{skin(dry)}} \sim 10^5 \Omega \)

<table>
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<tr>
<th>CURRENT RANGE</th>
<th>EFFECT</th>
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<tr>
<td>0.5–2 mA</td>
<td>Threshold of sensation</td>
</tr>
<tr>
<td>10–15 mA</td>
<td>Involuntary muscle contractions; can’t let go</td>
</tr>
<tr>
<td>15–100 mA</td>
<td>Severe shock; muscle control lost; breathing difficult</td>
</tr>
<tr>
<td>100–200 mA</td>
<td>Fibrillation of heart; death within minutes</td>
</tr>
<tr>
<td>&gt;200 mA</td>
<td>Cardiac arrest; breathing stops; severe burns</td>
</tr>
</tbody>
</table>

So for \( \Delta V = 10,000V \):

\[ I = \frac{\Delta V}{R} = \frac{10,000V}{10^5 \Omega} = 0.1 \text{ A} = \text{dangerous.} \]

But \( R_{\text{skin(wet)}} \) is much, much lower, \( \sim 10^3 \Omega \):

So in this case, when \( \Delta V = 120V \), \( I \) is also \( \sim 0.1 \text{ A} = \text{dangerous} \)
18.8: Conduction of electrical signals by neutrons
Ch. 19: Magnetism

Bar magnets.
Planetary magnetic fields.
Forces on moving charges.
Electrical motors (electrical energy → mechanical energy)
Generators (mechanical → electrical energy)
Magnetic data storage: magnetic tapes, computer drives
MRI (magnetic resonance imaging)
A magnet has two poles (magnetic dipole)
North–South

Opposite poles attract

Like poles repel
stable

stable

unstable

unstable
No magnetic monopoles

No *magnetic monopoles* are found (i.e. there is no magnetic equivalent of charge).

If you cut a magnetic bar in two....

You get two smaller magnets:

You do not get separate N & S magnetic charges -- no matter how small the magnet. This continues down to the scale of a single atom!
Magnetic Field Lines

B-field lines flow from N to S pole.

Can be traced out using a compass

When placed in external magnetic fields, magnetic dipoles (e.g., compass needle) orient themselves parallel to B-field lines.
Magnetic Field Lines

Can also be traced out with Fe filings
Soft/Hard Magnetic materials

Soft magnetic materials (e.g., Fe): Easily magnetized, but can lose magnetization easily

Hard materials: more difficult to magnetize, but retain magnetism for a long time (“permanent magnet”)  
Ex.: metal alloys such as Alnico (Aluminum, Nickel, Cobalt)

Ferromagnetic materials: materials which can become magnetized and can be attracted to other magnets
Magna-doodle
Ferromagnetic fluids

“Leaping ferrofluid demonstration:”
http://www.youtube.com/watch?v=Rg9xSLdXKXk

Sachiko Kodama, Yasushi Miyajima "Morpho Towers -- Two Stand”:
http://www.youtube.com/watch?v=me5Zzm2TXh4
(see also sachikokodama.com)

Ferrofluid: how it works:
http://www.youtube.com/watch?v=PvtUt02zVAs

Ferrofluid on the track of a Magnetized Meatgrinder
http://www.youtube.com/watch?v=OE2pB1pyZN0