8-11. $\frac{n_2}{n_1} = \frac{g_2 e^{-E_2/kT}}{g_1 e^{-E_1/kT}} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT}$

$$e^{(E_2-E_1)/kT} = \frac{g_2}{g_1} \times \frac{n_1}{n_2} = (E_2 - E_1)/kT = \ln\left(\frac{g_2}{g_1} \times \frac{n_1}{n_2}\right)$$

$$T = \frac{E_2 - E_1}{k \ln\left[(g_2/g_1)(n_1/n_2)\right]} = \frac{10.2eV}{(8.67 \times 10^{-5} eV/K)\ln(4 \times 10^6)} = 7790K$$

8-12. $\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT} = \frac{3}{1} e^{-\left[\frac{4\times10^{-3}eV}{(8.67\times10^{-5}eV/K)(300K)}\right]} = 0.155$

8-14. $c_v = 3R/M$

(a) Al: $c_v = \frac{3(1.99cal/mole\cdot K)}{27.0g/mole} = 0.221cal/g\cdot K \quad \{0.215cal/g\cdot K\}$

(b) Cu: $c_v = \frac{3(1.99cal/mole\cdot K)}{62.5g/mole} = 0.0955cal/g\cdot K \quad \{0.0920cal/g\cdot K\}$

(c) Pb: $c_v = \frac{3(1.99cal/mole\cdot K)}{207g/mole} = 0.0288cal/g\cdot K \quad \{0.0305cal/g\cdot K\}$

The values for each element shown in brackets are taken from the *Handbook of Chemistry and Physics* and apply at 25° C.

8-17. For hydrogen: $E_n = -\frac{mk^2e^4}{2\hbar^2}\frac{1}{n^2} = -\frac{13.605687}{n^2}eV$ using values of the constants accurate to six decimal places.

$E_1 = -13.605687eV$

$E_2 = -3.401422eV \qquad E_2 - E_1 = 10.204265eV$

$E_3 = -1.511743eV \qquad E_3 - E_1 = 12.093944eV$

(a) $\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT} = \frac{8}{2} e^{-10.20427/0.02586} = 4e^{-395} = 4 \times 10^{-172} \approx 0$

$$\frac{n_3}{n_1} = \frac{g_3}{g_1} e^{-(E_3 - E_1)/kT} = \frac{18}{2} e^{-12.09394/0.02586} = 9e^{-468} = 9 \times 10^{-203} \approx 0$$

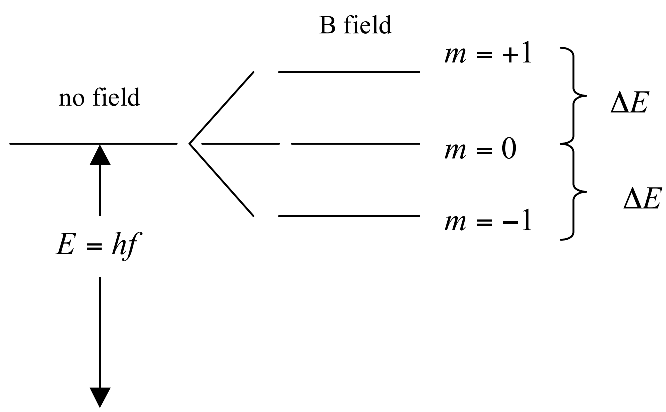
(b) $$\frac{n_2}{n_1} = 0.01 = 4e^{-10.20427/kT} \quad \rightarrow \quad e^{-10.20427/kT} = 0.0025$$

$$-10.20427/kT = \ln(0.0025) = -5.99146$$

$$T = \frac{10.20427eV}{(5.99146)(8.61734 \times 10^{-5} eV/K)} = 19{,}760K$$

(c) $$\frac{n_3}{n_1} = 9e^{-12.09394/(8.61734\times10^{-5})(19{,}760)} = 0.00742 = 0.7\%$$

8-18.



Neglecting the spin, the $3p$ state is doubly degenerate: $\ell = 0,1$ hence, there are two $m = 0$ levels equally populated.

$$E = hf = hc/\lambda = 1.8509eV \quad (\lambda = 670.79nm)$$

$$\Delta E = \frac{e\hbar B}{2m_e} = 2.315 \times 10^{-4} eV$$

(a) The fraction of atoms in each m-state relative to the ground state is: (Example 8-2)

$$\frac{n_{+1}}{n} = e^{-1.8511/0.02586} = e^{-71.58} = 10^{-31.09} = 8.18 \times 10^{-32}$$

$$\frac{n_0}{n} = 2 \times e^{-1.8509/0.02586} = 2e^{-71.57} = 2 \times 10^{-31.08} = 1.64 \times 10^{-31}$$

$$\frac{n_0}{n_{-1}} = e^{-1.8507/0.02586} = e^{-71.56} = 10^{-31.08} = 8.30 \times 10^{-32}$$

(b) The brightest line with the B-field "on" will be the transition from the $m = 0$ level, the center line of the Zeeman spectrum. With that as the "standard", the relative intensities will be: $8.30/16.4/8.18 \quad \rightarrow \quad 0.51/1.00/0.50$

8-21. Assuming the gases are ideal gases, the pressure is given by: $P = \frac{2}{3}\frac{N\langle E\rangle}{V}$ for classical, FD, and BE particles. P_{FD} will be highest due to the exclusion principle, which, in effect, limits the volume available to each particle so that each strikes the walls more frequently than the classical particles. On the other hand, P_{BE} will be lowest, because the particles tend to be in the same state, which in effect, is like classical particles with a mutual attraction, so they strike the walls less frequently.

8-23. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m\langle E\rangle}} = \frac{h}{\sqrt{2m(3kT/2)}} = \frac{h}{(3mkT)^{1/2}}$

The distance between molecules in an ideal gas $(V/N)^{1/3}$ is found from

$$PV = nRT = nRT\left(N_A/N_A\right) = NkT \rightarrow (V/N)^{1/3} = (kT/P)^{1/3}$$

and equating this to λ above, $(kT/P)^{1/3} = \frac{h}{(3mkT)^{1/2}}$

$\frac{kT}{P} = \frac{h^3}{(3mkT)^{3/2}}$ and solving for T, yields: $T^{5/2} = \frac{P}{k}\frac{h^3}{(3mk)^{3/2}}$

$$T = \left[\frac{Ph^3}{k(3mk)^{3/2}}\right]^{5/2} = \left[\frac{(101kPa)(6.63\times10^{-34}J\cdot s)^3}{3(2\times1.67\times10^{-27}kg)(1.38\times10^{-23}J/K)^{5/2}}\right]^{2/5} = 4.4K$$

8-26. $T_C = \frac{h^2}{2mk}\left[\frac{N}{2\pi(2.315)V}\right]^{2/3}$ (Equation 8-48)

The density of liquid *Ne* is 1.207 g/cm^3, so

$$\frac{N}{V} = \frac{(1.207g/cm^3)(6.022\times10^{23}molecules/mol)(10^6cm^2/m^3)}{20.18g/mol} = 3.601\times10^{28}/m^3$$

$$T = \frac{(6.626\times10^{-34}J\cdot s)^2}{2(20u\times1.66\times10^{-27}kg/u)(.381\times10^{-23}J/K)}\left[\frac{3.601\times10^{28}m^3}{2\pi(2.315)}\right]^{2/3} = 0.895K$$

Thus, T_C at which ^{20}Ne would become a superfluid is much lower than its freezing temperature of 24.5K.

8-28. $\langle E\rangle = \dfrac{hf}{e^{hf/kT}-1}$ (Equation 8-60)

(a) For $T = 10hf/k$; $hf = kT/10 \rightarrow \langle E\rangle = \dfrac{hf}{e^{1/10}-1} = \dfrac{kT/10}{0.1051} = 0.951kT$

(b) For $T = hf/k$; $hf = kT \rightarrow \langle E\rangle = \dfrac{hf}{e^{1}-1} = \dfrac{kT}{1.718} = 0.582kT$

(c) For $T = 0.1hf/k$; $hf = 10kT \rightarrow \langle E\rangle = \dfrac{hf}{e^{10}-1} = \dfrac{10kT}{2.20\times10^{4}} = 4.54\times10^{-4}kT$

According to equipartition $\langle E\rangle = kT$ in each case.

8-29. $C_V = 3N_A k\left(\dfrac{hf}{kT}\right)^2 \dfrac{e^{hf/kT}}{\left(e^{hf/kT}-1\right)^2}$ As $T\rightarrow\infty$, hf/kT gets small and

$e^{hf/kT} \approx 1 + hf/kT + \cdots$

$$C_V = 3N_A k\left(\frac{hf}{kT}\right)^2 \frac{\left(1+hf/kT+\cdots\right)}{\left(hf/kT\right)^2} \approx 3N_A k = 3N_A\left(R/N_A\right) = 3R$$

The rule of Dulong and Petit.

8-31. $C_V = 3R\left(\dfrac{hf}{kT}\right)^2 \dfrac{e^{hf/kT}}{\left(e^{hf/kT}-1\right)^2}$ (Equation 8-62)

At the Einstein temperature $T_E = hf/k$,

$$C_V = 3R(1)^2 \frac{e^1}{\left(e^1-1\right)^2} = 3R(0.9207) = 3(8.31J/K\cdot mol)(0.9207)$$

$$= 22.95K/K\cdot mol = 5.48cal/K\cdot mol$$

8-35. Approximating the nuclear potential with an infinite square well and ignoring the Coulomb repulsion of the protons, the energy levels for both protons and neutrons are given by $E_n = \left(n^2h^2\right)/\left(8mL^2\right)$ and six levels will be occupied in ^{22}Ne, five levels with 10 protons and six levels with 12 neutrons.

$$E_F\,(\text{protons}) = \frac{(5)^2\,(1240 MeV \cdot fm)^2}{8\,(1.0078u \times 931.5 MeV/u)(3.15 fm)^2} = 516 MeV$$

$$E_F\,(\text{neutrons}) = \frac{(6)^2\,(1240 MeV \cdot fm)^2}{8\,(1.0087u \times 931.5 MeV/u)(3.15 fm)^2} = 742 MeV$$

$$\langle E\rangle(\text{protons}) = (3/5)E_F = 310 MeV$$

$$\langle E\rangle(\text{neutrons}) = (3/5)E_F = 445 MeV$$

As we will discover in Chapter 11, these estimates are nearly an order of magnitude too

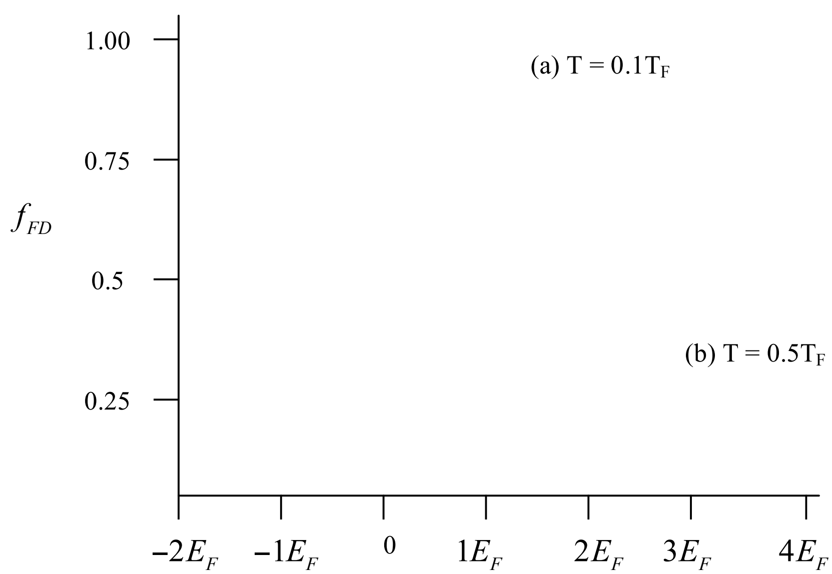
large. The number of particles is not a large sample.

8-36. $E_1 = h^2/8mL^2$. All 10 bosons can be in this level, so $E_1\,(\text{total}) = 10h^2/8mL^2$.

8-37. (a) $f_{FD}(E) = \dfrac{1}{e^{(E-E_F/kT)}+1}$ (Equation 8-68)

$$= \frac{1}{e^{(E-E_F)/0.1E_F}+1} = \frac{1}{e^{10(E-E_F)/E_F}+1}$$

(b) $f_{FD}(E) = \dfrac{1}{e^{(E-E_F)/0.5E_F}+1} = \dfrac{1}{e^{2(E-E_F)/E_F}+1}$



8-38. $\frac{N_O}{N} \approx 1 - \left(\frac{T}{T_c}\right)^{3/2}$ (Equation 8-52)

(a) $\frac{N_O}{N} \approx 1 - \left(\frac{T_c / 2}{T_c}\right)^{3/2} = 1 - \left(\frac{1}{2}\right)^{3/2} = 0.646$

(b) $\frac{N_O}{N} \approx 1 - \left(\frac{T_c / 4}{T_c}\right)^{3/2} = 1 - \left(\frac{1}{4}\right)^{3/2} = 0.875$