7-9. (a) For $n = 3,\ \ell = 0, 1, 2$

(b) For $\ell = 0, m = 0$. For $\ell = 1, m = -1, 0, +1$. For $\ell = 2, m = -2, -1, 0, +1, +2$.

(c) There are nine different m-states, each with two spin states, for a total of 18 states for

$n = 3$.

7-10. (a) For $\ell = 4$

$$L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{4(5)}\hbar = \sqrt{20}\,\hbar$$

$$m_\ell = 4\hbar$$

$$\theta_{\min} = \cos^{-1}\frac{4}{\sqrt{20}} \quad \rightarrow \quad \theta_{\min} = 26.6°$$

(b) For $\ell = 2$

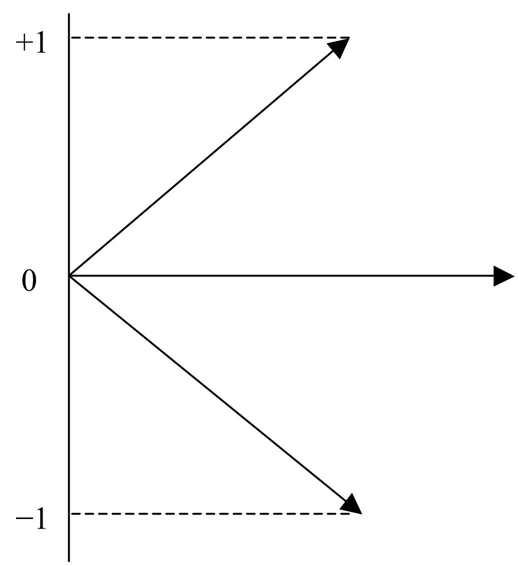
$$L = \sqrt{6}\,\hbar \qquad m_\ell = 2\hbar$$

$$\theta_{\min} = \cos^{-1}\frac{2}{\sqrt{6}} \quad \rightarrow \quad \theta_{\min} = 35.3°$$

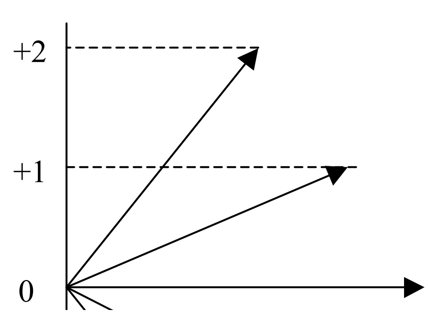
7-12. (a)

$\ell = 1$

$|\mathbf{L}| = \sqrt{2}\hbar$



(b)



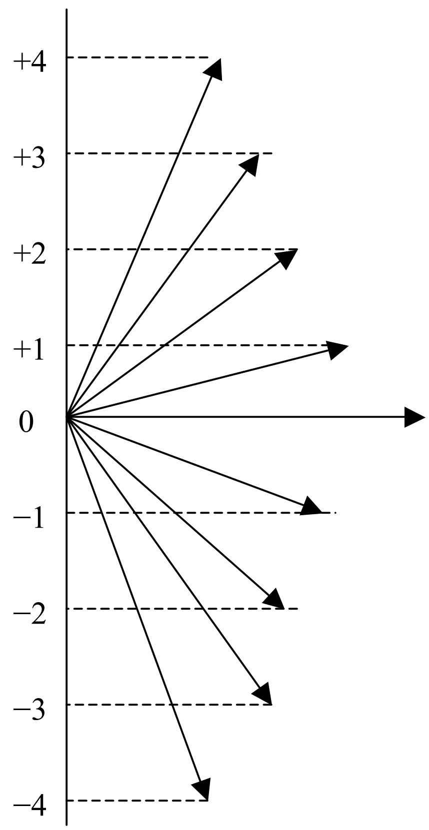
$\ell = 2$

$|\mathbf{L}| = \sqrt{6}\hbar$

(c)

$\ell = 4$

$|\mathbf{L}| = \sqrt{20}\hbar$



(d) $|\boldsymbol{L}| = \sqrt{\ell(\ell+1)}\hbar$ (See diagrams above.)

7-13. $L^2 = L_x^2 + L_y^2 + L_z^2 \rightarrow L_x^2 + L_y^2 = L^2 - L_z^2 = \ell(\ell+1)\hbar^2 - (m\hbar)^2 = (6 - m^2)\hbar^2$

(a) $\left(L_x^2 + L_y^2\right)_{\text{min}} = \left(6 - 2^2\right)\hbar^2 = 2\hbar^2$

(b) $\left(L_x^2 + L_y^2\right)_{\text{max}} = \left(6 - 0^2\right)\hbar^2 = 6\hbar^2$

(c) $L_x^2 + L_y^2 = (6-1)\hbar^2 = 5\hbar^2$ L_x and L_y cannot be determined separately.

(d) $n = 3$

7-15. $\boldsymbol{L} = \boldsymbol{r}$? $\boldsymbol{p}$ $\frac{d\boldsymbol{L}}{dt} = \frac{d\boldsymbol{r}}{dt} \times \boldsymbol{p} + \boldsymbol{r} \times \frac{d\boldsymbol{p}}{dt}$

$\frac{d\boldsymbol{r}}{dt} \times \boldsymbol{p} = \boldsymbol{v} \times m\boldsymbol{v} = m\boldsymbol{v} \times \boldsymbol{v} = 0$ and $\boldsymbol{r} \times \frac{d\boldsymbol{p}}{dt} = \boldsymbol{r} \times \boldsymbol{F}$. Since for $V = V(r)$, i.e., central forces,

$\boldsymbol{F}$ is parallel to $\boldsymbol{r}$, then $\boldsymbol{r} \times \boldsymbol{F} = 0$ and $\frac{d\boldsymbol{L}}{dt} = 0$

7-16. (a) For $\ell = 3$, $n = 4, 5, 6, \ldots$ and $m = -3, -2, -1, 0, 1, 2, 3$

(b) For $\ell = 4$, $n = 5, 6, 7, \ldots$ and $m = -4, -3, -2, -1, 0, 1, 2, ,3 ,4$

(c) For $\ell = 0$, $n = 1$ and $m = 0$

(d) The energy depends only on n. The minimum in each case is:

$$E_4 = -13.6eV/n^2 = -13.6eV/4^2 = -0.85eV$$

$$E_5 = -13.6eV/5^2 = -0.54eV$$

$$E_1 = -13.6eV$$

7-17. (a) $6f$ state: $n = 6$, $\ell = 3$

(b) $E_6 = -13.6eV/n^2 = -13.6eV/6^2 = -0.38eV$

(c) $L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{3(3+1)}\hbar = \sqrt{12}\,\hbar = 3.65\times10^{-34}\,J$□$s$

(d) $L_z = m\hbar$ $L_z = -3\hbar, -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar, 3\hbar$

7-20. (a) For the ground state, $P(r)\Delta r = \psi^2(4\pi r^2)\Delta r = \frac{4r^2}{a_0^3}e^{-2r/a_0}\Delta r$

For $\Delta r = 0.03a_0$, at $r = a_0$ we have $P(r)\Delta r = \frac{4a_0^2}{a_0^3}e^{-2}(0.03a_0) = 0.0162$

(b) For

$\Delta r = 0.03a_0$, at $r = 2a_0$ we have $P(r)\Delta r = \frac{4(2a_0)^2}{a_0^3}e^{-4}(0.03a_0) = 0.0088$

7-21. $P(r) = Cr^2 e^{-2Zr/a_0}$ For $P(r)$ to be a maximum,

$$\frac{dP}{dt} = C\left[r^2\left(-\frac{2Z}{a_0}\right)e^{-2Zr/a_0} + 2re^{-2Zr/a_0}\right] = 0 \rightarrow C \times \frac{2Zr}{a_0}\left(\frac{a_0}{Z} - r\right)e^{-2Zr/a_0} = 0$$

This condition is satisfied with $r = 0$ or $r = a_0/Z$. For $r = 0$, $P(r) = 0$ so the maximum

$P(r)$ occurs for $r = a_0/Z$.

7-22. $$\int \psi^2 d\tau = \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi^2 r^2 \sin\theta \, dr \, d\theta \, d\phi = 1$$

$$= 4\pi \int_0^\infty \psi^2 r^2 dr = 4\pi C_{210}^2 \int_0^\infty \left(\frac{Zr}{a_0}\right)^2 r^2 e^{-Zr/a_0} dr = 1$$

$$= 4\pi C_{210}^2 \int_0^\infty \left(\frac{Z^2 r^4}{a_0^2}\right) e^{-Zr/a_0} dr = 1$$

Letting $x = Zr/a_0$, we have that $r = a_0 x/Z$ and $dr = a_0 dx/Z$ and substituting

these above,

$$\int \psi^2 d\tau = \frac{4\pi a_0^3 C_{210}^2}{Z^3} \int_0^\infty x^4 e^{-x} dx$$

Integrating on the right side

$$\int_0^\infty x^4 e^{-x} dx = 6$$

Solving for C_{210}^2 yields: $C_{210}^2 = \dfrac{Z^3}{24\pi a_0^3} \rightarrow C_{210} = \left(\dfrac{Z^3}{24\pi a_0^3}\right)^{1/2}$

7-26. For the most likely value of r, $P(r)$ is a maximum, which requires that (see Problem 7-24)

$$\frac{dP}{dr} = A\cos^2\theta\left[r^4\left(-\frac{Z}{a_0}\right)e^{-Zr/a_0} + 4r^3 e^{-Zr/a_0}\right] = 0$$

For hydrogen $Z = 1$ and $A\cos^2\theta\left(r^3/a_0\right)\left(4a_0 - r\right)e^{-r/a_0} = 0$. This is satisfied for

$r = 0$

and $r = 4a_0$. For $r = 0$, $P(r) = 0$ so the maximum $P(r)$ occurs for $r = 4a_0$.

7-33. (a) There should be four lines corresponding to the four m_J values −3/2, −1/2,

+1/2, +3/2.

(b) There should be three lines corresponding to the three m_ℓ values −1, 0, +1.

7-68. $P(r) = \dfrac{4Z^3}{a_0^3} r^2 e^{-2Zr/a_0}$ (See Problem 7-63)

For hydrogen, $Z = 1$ and at the edge of the proton $r = R_0 = 10^{-15}m$. At that point,

the

exponential factor in P(r) has decreased to:

$$e^{-2R_0/a_0} = e^{-2\left(10^{-15}\right)/\left(0.529\times10^{-10}m\right)} = e^{-\left(3.78\times10^{-5}\right)} \approx 1 - 3.78\times10^{-5} \approx 1$$

Thus, the probability of the electron in the hydrogen ground state being inside the

nucleus,

to better than four figures, is:

$$P(r) = \frac{4r^2}{a_0^3} \qquad P = \int_0^{r_0} P(r)dr = \int_0^{R_0} \frac{4r^2}{a_0^3} = \frac{4}{a_0^3}\int_0^{R_0} r^2 dr = \frac{4}{a_0^3}\frac{r^3}{3}\bigg|_0^{R_0}$$

$$= \frac{4}{a_0^3}\left(\frac{R_0^3}{3}\right) = \frac{4\left(10^{-15}m\right)^3}{3\left(0.529\times10^{-10}m\right)^3} = 9.0\times10^{-15}$$

7-70. (a) Substituting $\psi(r,\theta)$ into Equation 7-9 and carrying out the indicated

operations

yields (eventually):

$$-\frac{\hbar^2}{2\mu}\psi(r,\theta)\left[2/r^2 - 1/4a_0^2\right] - \frac{\hbar^2}{2\mu}\psi(r,\theta)\left(-2/r^2\right) + V\psi(r,\theta) = E\psi(r,\theta)$$

Canceling $\psi(r,\theta)$ and recalling that $r^2 = 4a_0^2$ (because ψ given is for $n = 2$)

we

have $-\frac{\hbar^2}{2\mu}\left(-1/4a_0^2\right)+v=E$

The circumference of the $n = 2$ orbit is:

$C=2\pi\left(4a_0\right)=2\lambda \rightarrow a_0=\lambda/4\pi=1/2k.$

Thus, $-\frac{\hbar^2}{2\mu}\left(-\frac{1}{4/4k^2}\right)+V=E \rightarrow \frac{\hbar^2k^2}{2\mu}+V=E$

(b) or $\frac{p^2}{2m}+v=E$ and Equation 7-9 is satisfied.

$$\int_0^\infty \psi^2 dx=\int A^2\left(\frac{r}{a_0}\right)^2 e^{-r/a_0}\cos^2\theta r^2\sin\theta dr d\theta d\phi=1$$

$$A^2\int_0^\infty\left(\frac{r}{a_0}\right)^2 e^{-r/a_0}r^2 dr\int_0^\pi\cos^2\theta\sin\theta d\theta\int_0^{2\pi}d\phi=1$$

Integrating (see Problem 7-22),

$$A^2\left(6a_0^3\right)\left(2/3\right)\left(2\pi\right)=1$$

$$A^2=1/8a_0^3\pi \rightarrow A=\sqrt{1/8a_0^3\pi}$$