5-3. $E_k = eV_o = \dfrac{p^2}{2m\lambda^2} = \dfrac{(hc)^2}{2mc^2\lambda^2}$ $\quad V_o = \dfrac{1}{e} \cdot \dfrac{(1240eV \cdot nm)^2}{2(5.11\times10^5 eV)(0.04nm)^2} = 940V$

5-4. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE_k}} = \dfrac{hc}{\sqrt{2mc^2E_k}}$ (from Equation 5-2)

(a) For an electron: $\lambda = \dfrac{1240eV \cdot nm}{\left[(2)(0.511\times10^6 eV)(4.5\times10^3 eV)\right]^{1/2}} = 0.0183nm$

(b) For a proton: $\lambda = \dfrac{1240eV \cdot nm}{\left[(2)(983.3\times10^6 eV)(4.5\times10^3 eV)\right]^{1/2}} = 4.27\times10^{-4}nm$

(c) For an alpha particle: $\lambda = \dfrac{1240eV \cdot nm}{\left[(2)(3.728\times10^9 eV)(4.5\times10^3 eV)\right]^{1/2}} = 2.14\times10^{-4}nm$

5-11. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2m_pE_k}} = 0.25nm$

Squaring and rearranging,

$$E_k = \frac{h^2}{2m_p\lambda^2} = \frac{(hc)^2}{2(m_pc^2)\lambda^2} = \frac{(1240eV \cdot nm)^2}{2(938\times10^6 eV)(0.25nm)^2} = 0.013eV$$

$n\lambda = D\sin\phi \quad \rightarrow \quad \sin\phi = n\lambda / D = (1)(0.25nm)/(0.304nm)$

$\sin\phi = 0.822 \quad \rightarrow \quad \phi = 55°$

5-12. (a) $n\lambda = D\sin\phi \quad \therefore \quad D = \dfrac{n\lambda}{\sin\phi} = \dfrac{nhc}{\sin\phi\sqrt{2mc^2E_k}}$

$$= \frac{(1)(1240eV \cdot nm)}{(\sin 55.6°)\left[2(5.11\times10^5 eV)(50eV)\right]^{1/2}} = 0.210nm$$

(b) $\sin\phi = \frac{n\lambda}{D} = \frac{(1)(1240 eV\cdot nm)}{(0.210nm)\left[2(5.11\times10^5 eV)(100eV)\right]^{1/2}} = 0.584$

$\phi = \sin^{-1}(0.584) = 35.7°$

5-17. (a) $y = y_1 + y_2$

$$= 0.002m\cos(8.0x/m - 400t/s) + 0.002m\cos(7.6x/m - 380t/s)$$
$$= 2(0.002m)\cos\left[\frac{1}{2}(8.0x/m - 7.6x/m) - \frac{1}{2}(400t/s - 380t/s)\right]$$
$$\times\cos\left[\frac{1}{2}(8.0x/m + 7.6x/m) - \frac{1}{2}(400t/s + 380t/s)\right]$$
$$= 0.004m\cos(0.2x/m - 10t/s)\times\cos(7.8x/m - 390t/s)$$

(b) $v = \frac{\bar{\omega}}{\bar{k}} = \frac{390/s}{7.8/m} = 50m/s$

(c) $v_s = \frac{\Delta\omega}{\Delta k} = \frac{20/s}{0.4/m} = 50m/s$

(d) Successive zeros of the envelope requires that $0.2\Delta x/m = \pi$, thus

$\Delta x = \frac{\pi}{0.2} = 5\pi m$ with $\Delta k = k_1 - k_2 = 0.4m^{-1}$ and $\Delta x = \frac{2\pi}{\Delta k} = 5\pi m.$

5-23. (a) The particle is found with equal probability in any interval in a force-free region. Therefore, the probability of finding the particle in any interval Δx is proportional to Δx. Thus, the probability of finding the sphere *exactly* in the middle, i.e., with $\Delta x = 0$ is zero.

(b) The probability of finding the sphere somewhere within 24.9*cm* to 25.1*cm* is proportional to Δx =0.2*cm*. Because there is a force free length L = 48*cm* available to the sphere and the probability of finding it somewhere in L is unity, then the probability that it will be found in Δx = 0.2*cm* between 24.9*cm* and 25.1*cm* (or any interval of equal size) is: $P\Delta x = (1/48)(0.2cm) = 0.00417.$

5-24. Because the particle must be in the box $\int_0^L \psi^* \psi \, dx = 1 = \int_0^L A^2 \sin^2(\pi x / L) \, dx = 1$

Let $u = \pi x / L; \quad x = 0 \to u = 0; \quad x = L \to u = \pi$ and $dx = (L/\pi) du$, so we have

$$\int_0^\pi A^2 (L/\pi) \sin^2 u \, du = A^2 (L/\pi) \int_0^\pi \sin^2 u \, du = 1$$

$$(L/\pi) A^2 \int_0^\pi \sin^2 u \, du = (L/\pi) A^2 \left[\frac{u}{2} - \frac{\sin 2u}{4} \right]_0^\pi = (L/\pi) A^2 (\pi/2) = (LA^2)/2 = 1$$

$$\therefore \quad A^2 = 2/L \to A = (2/L)^{1/2}$$

5-25. (a) At $x = 0$: $\quad Pdx = |\psi(0,0)|^2 \, dx = |Ae^0|^2 \, dx = A^2 dx$

(b) At $x = \sigma$: $\quad Pdx = \left| Ae^{-\sigma^2/4\sigma^2} \right|^2 dx = \left| Ae^{-1/4} \right|^2 dx = 0.61 A^2 dx$

(c) At $x = 2\sigma$: $\quad Pdx = \left| Ae^{-4\sigma^2/4\sigma^2} \right|^2 dx = \left| Ae^{-1} \right|^2 dx = 0.14 A^2 dx$

(d) The electron will most likely be found at $x = 0$, where Pdx is largest.

5-27. $\Delta E \Delta t \approx \hbar \;\to\; \Delta E \approx \hbar / \Delta t = \dfrac{1.055 \times 10^{-34} \, J \cdot s}{10^{-7} s \left(1.609 \times 10^{-19} \, J/eV\right)} \approx 6.6 \times 10^{-9} eV$

5-34. $\Delta \omega \Delta t \approx 1 \quad \to \quad 2\pi \Delta f \, \Delta t \approx 1$

For the visible spectrum the range of frequencies is $\Delta f = (7.5 - 4.0) \times 10^{14} = 3.5 \times 10^{14} Hz$

The time duration of a pulse with a frequency uncertainty of Δf is then:

$$\Delta t = \frac{1}{2\pi \Delta f} = \frac{1}{2\pi \times 3.5 \times 10^{14} Hz} = 4.5 \times 10^{-16} s = 0.45 \, fs$$

5-35. The size of the object needs to be of the order of the wavelength of the $10 MeV$ neutron.

$\lambda = h/p = h/\gamma m u$. γ and u are found from:

$E_k = m_n c^2 (\gamma - 1)$ or $\gamma - 1 = 10 MeV / 939 MeV$

$\gamma = 1 + 10/939 = 1.0106 = 1/\left(1 - u^2/c^2\right)^{1/2}$ or $u = 0.14c$

Then, $\lambda = \frac{h}{\gamma mu} = \frac{hc}{\left[\gamma mc^2 (u/c)\right]} = \frac{1240 eV \cdot nm}{\left[(1.0106)(939\times 10^6 eV)(0.14)\right]} = 9.33\, fm$

Nuclei are of this order of size and could be used to show the wave character of $10 MeV$ neutrons.

5-36. (a) $\Delta E = 135 MeV$, the rest energy of the pion.

(b) $\Delta E \Delta t \approx \frac{\hbar}{2}$

$$\Delta t = \frac{\hbar}{2\Delta E} = \frac{6.58\times 10^{-16} eV \cdot s}{2\times 135\times 10^6 eV} = 2.44\times 10^{-24} s$$

5-40. (a) For a proton or neutron:

$\Delta x \Delta p \approx \frac{\hbar}{2}$ and $\Delta p = m\Delta v$ assuming the particle speed to be non-relativistic.

$$\Delta v = \frac{\hbar}{2m\Delta x} = \frac{1.055\times 10^{-34} J\cdot s}{2(1.67\times 10^{-27} kg)(10^{-15} m)} = 3.16\times 10^7 m/s \approx 0.1c \text{ (non-relativistic)}$$

(b) $E_k \approx \frac{1}{2}mv^2 = \frac{(1.67\times 10^{-27} kg)(3.16\times 10^7 m/s)^2}{2} = 8.34\times 10^{-13} J = 5.21 MeV$

(c) Given the proton or neutron velocity in (a), we expect the electron to be relativistic, in which case, $E_k = mc^2(\gamma - 1)$ and

$$\Delta p = \frac{\hbar}{2\Delta x} \approx \gamma mv \quad \rightarrow \quad \gamma v \approx \frac{\hbar}{2m\Delta x}$$

For the relativistic electron we assume $v \approx c$

$$\gamma \approx \frac{\hbar}{2mc\Delta x} = \frac{1.055\times 10^{-34} J\cdot s}{2(9.11\times 10^{-31} kg)(3.00\times 10^8 m/s)(10^{-15} m)} = 193$$

$$E_k = mc^2(\gamma - 1) = (9.11\times 10^{-31} kg)(3.00\times 10^8 m/s)^2 (192) = 1.58\times 10^{-11} J = 98 MeV$$

5-41. (a) $E^2 = p^2c^2 + m^2c^4 \qquad E = hf = \hbar\omega \qquad p = h/\lambda = \hbar/k \qquad \hbar^2\omega^2 = \hbar^2k^2c^2 + m^2c^4$

$$v = \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} = \frac{\sqrt{\hbar^2 k^2 c^2 + m^2 c^4}}{\hbar k} = c\sqrt{1 + m^2 c^2 / \hbar^2 k^2} > c$$

(b) $$v_s = \frac{d\omega}{dk} = \frac{d}{dk}\sqrt{\frac{k^2 c^2 + m^2 c^4}{\hbar k}} = \frac{c^2 k}{\sqrt{\frac{k^2 c^2 + m^2 c^4}{\hbar^2}}}$$

$$= \frac{c^2 k}{\omega} = \frac{c^2 \hbar k}{\hbar\omega} = \frac{c^2 p}{E} = u \quad \text{(by Equation 2-41)}$$

5-46. (a) $E \geq \hbar^2 / 2mL^2$ (Equation 5-28) and $E = \hbar^2 / 2mA^2$

(b) For electron with $A = 10^{-10} m$:

$$E = \frac{(\hbar c)^2}{2mc^2 A^2} = \frac{(197.3eV \cdot nm)^2}{2(0.511 \times 10^6 eV)(10^{-1} nm)^2} = 3.81eV$$

For electron with $A = 1cm$ or $A = 10^{-2} m$:

$$E = 3.81eV\left(10^{-1}\right)^2 / \left(10^7 nm\right)^2 = 3.81 \times 10^{-16} eV$$

(c) $$E = \frac{\hbar^2}{2mL^2} = \frac{\left(1.055 \times 10^{-34} J \cdot s\right)^2}{2\left(100 \times 10^{-3} g \times 10^{-3} kg / g\right)\left(2 \times 10^{-2}\right)^2} = 1.39 \times 10^{-61} J = 8.7 \times 10^{-43} eV$$