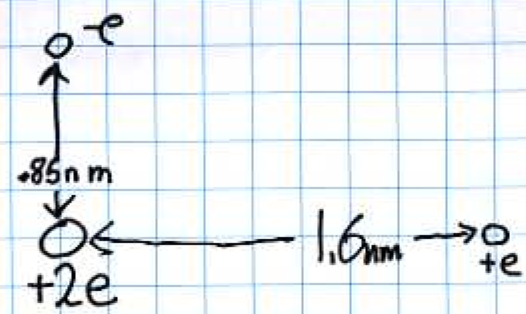
23.11



note $\underset{\sim}{A}$ or $\vec{A}$ indicates a vector (or $\mathbb{A}$)

$\hat{r}$ means in the direction of r

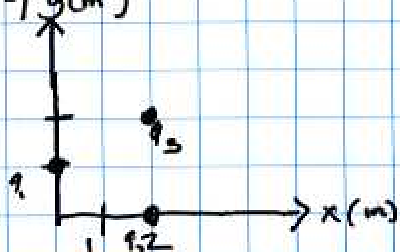
$$F_{ij} = \sum_i \frac{K q_i q_j}{r_{ij}^2}\hat{r} = \frac{-2Ke^2}{(.85\times10^{-9})^2}(-\hat{y}) + \frac{Ke^2}{(1.6\times10^{-9})^2}(-\hat{x})$$

minus comes from $\hat{r}$ going from the source of Force, the electron and proton

$e = 1.6\times10^{-19}C$

$K = 8.99\times10^{9}\ Nm^2/C^2$

23.19



$q_1 = 68\mu C \quad q_2 = -34\mu C \quad q_3 = 15\mu C$

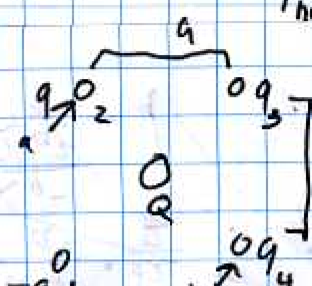
$r_{13} = \sqrt{5}\ m \quad r_{23} = 2m$

$$F_{13} = K\frac{q_1 q_3}{r_{13}^2}\left(\frac{2}{\sqrt{5}}\hat{x} + \frac{1}{\sqrt{5}}\hat{y}\right)$$

$$F_{23} = \frac{K q_2 q_3}{r_{23}^2}(\hat{y})$$

$$F_{net} = F_{13} + F_{23}$$

23.22

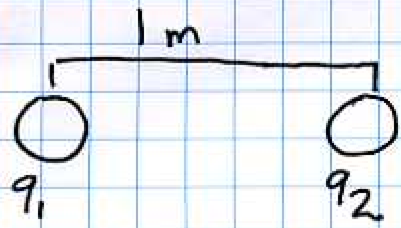


a,b will cancel

$$F_1 = \frac{-KqQ}{a^2/2}\frac{1}{\sqrt{2}}\hat{x} - \frac{KqQ}{a^2/2}\frac{1}{\sqrt{2}}\hat{y}$$

$$F_3 = \frac{+KqQ}{a^2/2}\frac{1}{\sqrt{2}}(-\hat{x}) + \frac{KqQ}{a^2/2}\frac{1}{\sqrt{2}}(-\hat{y})$$

23.24



$F = -2.5\,N$

after touching (charges redistribute) $F = +2.5\,N$

$q_1' = q_2'$

$q_1' + q_2' = q_1 + q_2$ (charge is conserved)

$$\frac{\cancel{K} q_1 q_2}{\cancel{r^2}} = -\frac{\cancel{K} q_1' q_2'}{\cancel{r^2}}$$

$$q_1 = q_1' + q_2' - q_2$$

$$(q_1' + q_2' - q_2)\, q_2 = -q_1' q_2'$$

$$-q_2^2 + (q_1' + q_2')\, q_2 + q_1' q_2' = 0$$

$$q_2 = \frac{-(q_1' + q_2') \pm \sqrt{(q_1' + q_2')^2 + 4 q_1' q_2'}}{-2}$$

(q_1 is the other sign on $\pm$ from q_2)

23.30

65µC

$$\underset{\sim}{E} = K \frac{q}{r^2} \hat{r}$$

$q = 65\,\mu C$

a) $\underset{\sim}{E} = \dfrac{K\, 65\,\mu C}{.5^2} \hat{x}$

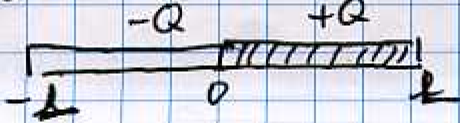
b) $\underset{\sim}{E} = \dfrac{K q}{.5} \left(\dfrac{\sqrt{2}}{2} \hat{x} + \dfrac{\sqrt{2}}{2} \hat{y} \right)$

c) $\underset{\sim}{E} = \dfrac{K q}{(.25^2 + .75^2)^{3/2}} (-.25\hat{x} + .75\hat{y})$

23.39 $P = qd$ $P = 6.2\times10^{-30}\,C\,m$ $q = 1.6\times10^{-19}\,C$

so $d = \dfrac{6.2\times10^{-30}}{1.6\times10^{-19}} \cong 4\times10^{-11}\,m$, about the size of a hydrogen atom in the ground state ($.5\times10^{-10}\,m$)

23.46



$\lambda = \frac{Q}{L}$

a) $dQ = \lambda\, dl \quad dE = \frac{K\lambda\, dl}{(l-x)^2}\Big|_0^L$ and $\frac{-K\lambda\, dl}{(l-x)^2}\Big|_{-L}^0$

$E = K\lambda\int_0^L \frac{dl}{(l-x)^2} - K\lambda\int_{-L}^0 \frac{dl}{(l-x)^2} = K\lambda\left[\frac{+1}{3}\frac{1}{(l-x)^3}\Big|_0^L + \frac{1}{3(l-x)^3}\Big|_{-L}^0\right]$

$= K\lambda\frac{1}{3}\left[\frac{1}{(x-L)^3} - \frac{1}{x^3} - \frac{1}{x^3} + \frac{1}{(x+L)^3}\right]$

b) for $x \gg L$ $\quad \frac{1}{(x-L)^3} - \frac{1}{x^3} = \frac{x^3-(x-L)^3}{x^3(x-L)^3} = \frac{\cancel{x^3} - \cancel{x^3} + 3x^2L - 3xL^2 + L^3}{x^3(x-L)^3}$ small

$\cong \frac{3(x-L)x^2L}{x^3(x-L)^3} \approx \frac{3L}{x(x-L)^2}$

similar for $\frac{1}{(x+L)^3} - \frac{1}{x^3} \cong \frac{3L}{x^2(x-L)^2}\frac{3L}{x^2}\frac{(x-L)^2+(x+L)^2}{(x-L)^2(x+L)^2} = \frac{2x^2+2L^2}{(x^2-L^2)^2}\left(\frac{3L}{x^2}\right.$

~~$E \approx K\lambda\frac{1}{3}\left(\frac{3}{x^3} + \frac{3}{x^3}\right) = ?$~~

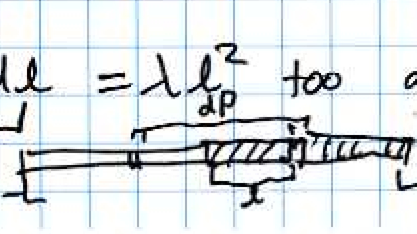
actually going about it this way leads to madness

so we note that the integral in part a can be written as $K\lambda\int_0^L\left[\frac{1}{(x-l)^2} - \frac{1}{(x+l)^2}\right]dl = K\lambda\int_0^L\frac{(x+l)^2-(x-l)^2}{(x^2-l^2)^2}\,dl$

$= K\lambda\int_0^L\frac{4xl}{(x^2-l^2)^2}\,dl \approx K\lambda\int_0^L\frac{4l}{x^3}\,dl = \frac{2K\lambda L^2}{x^3}$

c) $E = \frac{2Kp}{x^3}$ so $p = \lambda L^2$

note $\int_0^L \underbrace{2\lambda l\, dl}_{dp} = \lambda L^2$ too as expected



23.48 a) you can work through this if you want but you honestly should just quote the result or note that $a = \pi r^2$ so $da = 2\pi r\,dr$

b) $dq = \sigma 2\pi r\,dr = \sigma da$

c) 

exploit symmetry

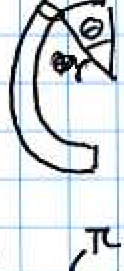
only $\hat{x}$ direction will matter (rest will cancel

$$dE = \underbrace{\sigma 2\pi r\,dr}_{\text{all charge at } r} \frac{K\,x}{(r^2+x^2)^{3/2}}$$

d) $$\underset{\sim}{E} = \int_0^R \frac{\sigma 2\pi r K x}{(r^2+x^2)^{3/2}} = 2\pi\sigma K\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

note as $R \to \infty$ $E \to 2\pi\sigma K$ (no x dependence)

23.50 exploit symmetry to argue that the y component of $\underset{\sim}{E}$ will cancel



$$dQ = \underbrace{\frac{Q}{\alpha\pi}}_{\lambda} \underbrace{\alpha\,d\theta}_{d\ell}$$

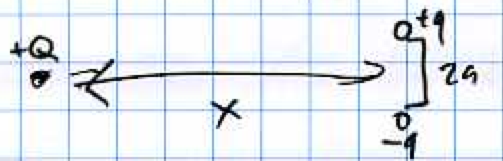
for shits and giggles

$$d\underset{\sim}{E} = \frac{Q}{\pi a^2} \underbrace{\sin\theta\,d\theta}_{x \text{ component}}\,\hat{x} + \frac{Q}{\pi a^2} \cos\theta\,d\theta\,\hat{y}$$

$$\frac{Q}{\pi a^2}\int_0^{\pi} \sin\theta\,\hat{x} + \cos\theta\,\hat{y}\,d\theta = \frac{Q}{\pi a^2}\left[2\hat{x} + 0\hat{y}\right]$$

$$= \frac{2Q}{\pi a^2}\hat{x}$$

23.68 a)



$x >> a$

that $x >> a$ means that we can tak $x^2 + a^2 \to x^2$ and in the $\hat{x}$ direction (unless they cancel, more on that in c)

$$\tau_{Q-q} = \frac{-KqQa}{x^2} \quad \text{(note counterclockwise is +)}$$

$$\tau_{Q+q} = \frac{-KqQa}{x^2} \qquad \tau_{net} = \frac{-2KqQa}{x^2}$$

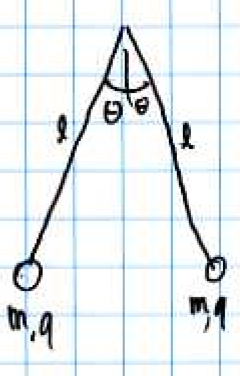
b) note 23.7a $E = -\frac{2Kp}{y^3}$ so F on Q is

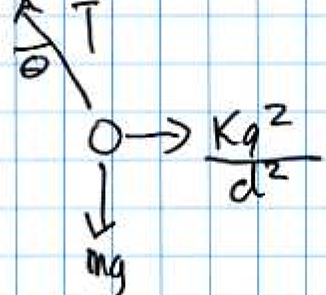
$p = 2aq$ $\qquad = -\frac{2KaqQ}{x^3}$

by newton's second law, $F_{12} = -F_{21}$ so the Force on the dipole would be $\frac{2KaqQ}{x^3}\hat{y}$

c) and up

23.78





$d = 2l\sin\theta$

$T\cos\theta = mg \qquad \tan\theta = \frac{Kq^2}{d^2 mg} = \frac{Kq^2}{4l^2\sin^2\theta\, mg}$

$T\sin\theta = \frac{Kq^2}{d^2}$

so $q = \pm 2l\sin\theta\sqrt{mg\tan\theta/k}$

24.9 (I had a problem on a final that used said messy integral)

$$\Phi_E = \underset{\sim}{E}\cdot\underset{\sim}{A} = E\pi R^2$$

all line go through both

24.10 $\underset{\sim}{E} = E_0 \frac{y}{a}\hat{k}$ $\quad \Phi_E = \oiint \underbrace{E\cdot dA}_{k\,dx\,dy} = E_0\int_0^a y\,dy = E_0\frac{a^2}{2}$

24.15 a) $\underbrace{\oint E\cdot da}_{\text{net flux}} = \frac{Q_{enc}}{\varepsilon_0}$

a) $\frac{6.1\mu C}{\varepsilon_0}$

b) $-\frac{6.1\mu C}{\varepsilon_0}$

c) 0

24.17 E goes like $1/r^2$ for spherical symmetric distributions

so doubling r, decreases E by $1/4$

24.20 a) the charge is on the balloon's surface so inside there is no fields

$Q_{enc}=0$

b) $\cancel{E=\frac{KQ}{.190^2}\hat{r}}$ $\quad E_1 = \frac{KQ}{.70^2}$ $\quad E_2 = \frac{KQ}{1.90^2}$

$$E_2 = E_1 \cdot \frac{.70^2}{1.90^2}$$

c) $Q = \frac{E_1 \cdot .70^2}{K}$

24.23 a) $Q_{enc} = -2Q$ so $E = \frac{-2KQ}{(R/2)^2}\hat{r}$

b) $Q_{enc} = -Q$ so $\underset{\sim}{E} = \frac{-KQ}{(2R)^2}\hat{r}$

c) $Q_{enc} = 0$ so $E = 0$

24.26 $E = \frac{K\,Q_{enc}}{r^2}$ $Q_{enc} = \int_a^r 4\pi r^2 \rho\, dr = \frac{4}{3}\pi(r^3 - a^3)\rho$

$E = \frac{K\,\frac{4}{3}\pi(r^3-a^3)\rho}{r^2}$ $a \to 0$ $E \to \frac{4}{3}K\pi r\rho$

24.31

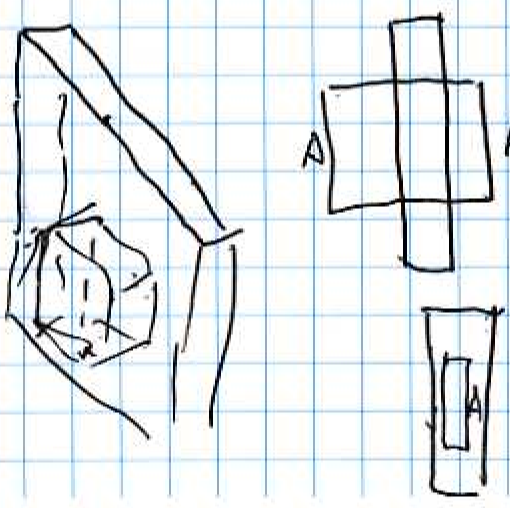
$2\pi r_2 l E_{out} = \frac{\pi R^2 l \rho}{\varepsilon_0}$

$E_{out} = \frac{\pi R^2 \rho l}{2\pi r l \varepsilon_0} = \cancel{2K\pi R^2}$

$2\pi r l E_{in} = \frac{\pi r^2 l \rho}{\varepsilon_0}$

$E_{in} = \frac{\pi r^2 l \rho}{\varepsilon_0 2\pi r l} = \frac{r\rho}{2\varepsilon_0}$

24.36



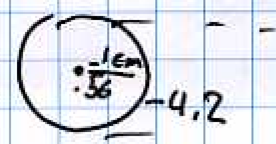
$2E_{out}A = \frac{dA\rho}{\varepsilon_0}$

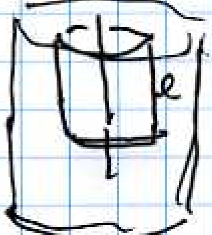
$E_{out} = \frac{dA\rho}{2A\varepsilon_0}$

$2E_{in}A = \frac{xA\rho}{\varepsilon_0}$

$E_{in} = \frac{xA\rho}{2A\varepsilon_0}$

24.28





a) $2\pi r l E_1 = \dfrac{5.6\,nC/m \cdot l}{\varepsilon_0}$

$$E_1 = \frac{5.6\,nC/m}{2\pi r \varepsilon_0} \qquad r = .50\text{ cm}$$

b) $2\pi r l E_2 = \dfrac{(5.6 - 4.2)\,nC/m\ l}{\varepsilon_0}$

$$E_2 = \frac{1.4\,nC/m}{2\pi r \varepsilon_0} \qquad r = 1.5\text{ cm}$$

24.38 a) at 4mm the rod is basically infinant 50cm >> 4mm

so $E = \dfrac{\frac{2\,\mu C}{50cm}}{2\pi(1.04cm)\cdot\frac{1m}{100cm}\ \varepsilon_0}$

b) 23 m away the rod is basically a point 23m >> 50cm

$$E = \frac{2\,\mu C}{4\pi\varepsilon_0 (23m)^2}$$

24.39 at 1cm from the plate, it is effectively an infinite sheet

$E = \dfrac{\sigma}{2\varepsilon_0}$ thus $\sigma = 2\varepsilon_0 E$

$Q_{net} = \sigma \cdot A = \sigma \cdot .75^2$

15m >> .75m so $E = \dfrac{K Q_{net}}{(15m)^2}$