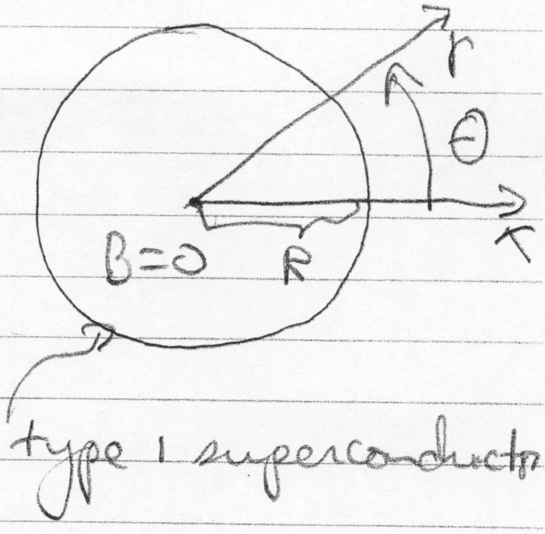
(11.) (a)



$\underset{\sim}{B} = B_0 \hat{x}$

as $r \to \infty$

Outside of cylinder

$$\underset{\sim}{\nabla} \times \underset{\sim}{B} = 0 \Rightarrow \underset{\sim}{B} = -\underset{\sim}{\nabla}\phi$$

$$0 = \underset{\sim}{\nabla} \cdot \underset{\sim}{B} = -\nabla^2 \phi$$

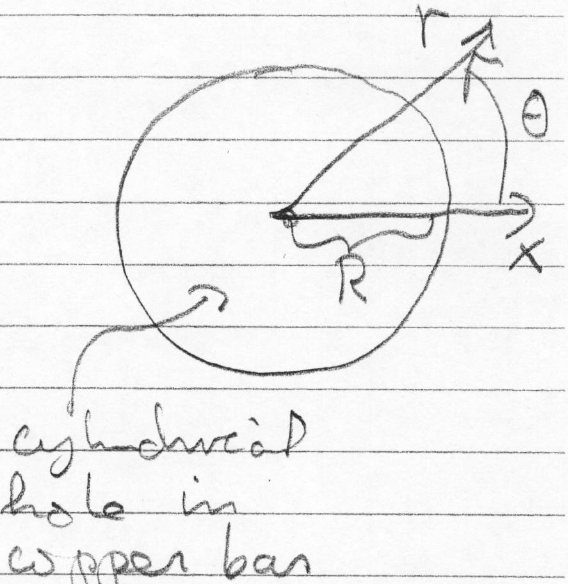
b.c. at cylinder is $0 = \underset{\sim}{B} \cdot \hat{r} = -\left.\frac{\partial \phi}{\partial r}\right|_{r=R}$

$$\phi = -B_0 r \cos\theta + \frac{A}{r} \cos\theta$$

$$0 = \left.\frac{\partial \phi}{\partial r}\right|_{R=a} = \left(-B_0 - \frac{A}{R^2}\right) \cos\theta$$

$$\therefore \quad A = -B_0 R^2$$

(11.) (b)



$\underset{\sim}{J} = J_0 \hat{x}$

as $\underset{-}{r} \to \infty$

In copper

$\underset{\sim}{\nabla} \times \underset{\sim}{E} = 0 \implies \underset{\sim}{E} = -\underset{\sim}{\nabla}\varphi$

$\underset{\sim}{J} = -\sigma \underset{-}{\nabla}\varphi \quad , \quad 0 = \frac{\partial \rho}{\partial t} = -\nabla \cdot J = \sigma \nabla^2 \varphi$

at surface of hole

$0 = J \cdot \hat{r} = -\sigma \left.\frac{\partial \varphi}{\partial r}\right|_{r=R}$

as $r \to \infty$

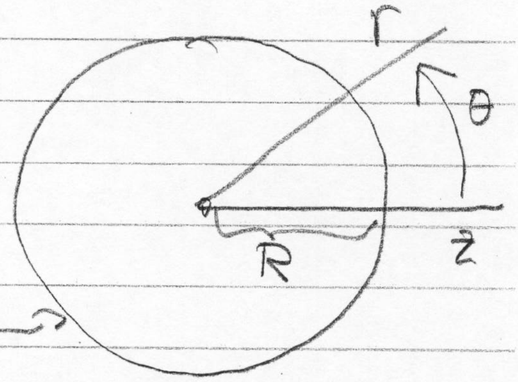
$\varphi = -\frac{J_0}{\sigma} x = -\frac{J_0}{\sigma} r\cos\theta$

$$\therefore \quad \Phi = -\frac{J_o}{\sigma} r\cos\theta + \frac{A}{r}\cos\theta$$

$$0 = \left.\frac{\partial\Phi}{\partial r}\right|_{r=R} = \left(-\frac{J_o}{\sigma} - \frac{A}{R^2}\right)\cos\theta$$

$$\therefore \quad A = -\frac{J_o R^2}{\sigma}$$

(12)



$\underset{\sim}{B} = \hat{z} B_o$
as $r \to \infty$

$\underset{\sim}{\nabla} \times \underset{\sim}{B} = 0$ outside sphere

$\therefore \quad \underset{\sim}{B} = -\underset{\sim}{\nabla}\Phi \qquad 0 = \underset{\sim}{\nabla}\cdot\underset{\sim}{B} = -\nabla^2\Phi$

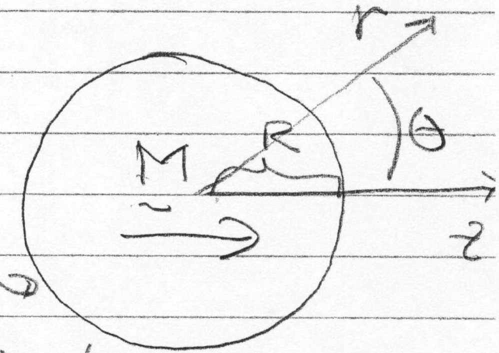
at surface of sphere $0 = \hat{r}\cdot\underset{\sim}{B} = -\left.\frac{\partial\Phi}{\partial r}\right|_R$

$$\varphi = -B_0 r \cos\theta + \frac{A}{r^2}\cos\theta$$

$$0 = \left.\frac{\partial\varphi}{\partial r}\right|_{r=R} = \left(-B_0 - \frac{A2}{R^3}\right)\cos\theta$$

$$\therefore \quad A = -\frac{B_0 R^3}{2}$$

(13)



uniformly magnetized sphere

$$\underline{\nabla} \times \underline{H} = \frac{4\pi}{c}\underline{J}_f = 0$$

$$\underline{H} = -\underline{\nabla}\varphi$$

$$\underline{B} = -\underline{\nabla}\varphi + 4\pi\underline{M}$$

$$0 = \nabla\cdot B = -\nabla^2\varphi + 4\pi\,\underline{\nabla}\cdot\underline{M}$$

non-zero at surface only

$$\varphi(r,\theta) = \begin{cases} \varphi_1(r,\theta) & r < R \\ \varphi_2(r,\theta) & r > R \end{cases}$$

$$\nabla^2\varphi_1 = \nabla^2\varphi_2 = 0$$

at $r = R$ B_n, H_t are continuous

$$\left. -\frac{\partial\varphi_1}{\partial r}\right|_{r=R} + 4\pi \underbrace{\underset{\sim}{M}\cdot\hat{r}}_{M\cos\theta} = \left. -\frac{\partial\varphi_2}{\partial r}\right|_{r=R}, \quad \varphi_1(R,\theta) = \varphi_2(R,\theta)$$

$$\varphi_2(r,\theta) \rightarrow \underset{\text{const}}{C} \quad \text{as } r \rightarrow \infty$$

Try

$$\varphi_1(r,\theta) = Ar\cos\theta + D$$

$$\varphi_2(r,\theta) = \frac{E}{r^2}\cos\theta + C$$

$$AR\cos\theta + D = \frac{E}{R^2}\cos\theta + C$$

$$\therefore \quad AR = \frac{E}{R^2}, \qquad D = C$$

$$-A\cos\theta + 4\pi M\cos\theta = +\frac{2E}{R^2}\cos\theta$$

$$-A + 4\pi M = \frac{2E}{R^3}$$

$$\therefore \quad \frac{4\pi M}{3} = \cancel{3}A$$

$$E = R^3 \frac{4\pi M}{3}$$

(14)

$$E(x,y,z,t) \simeq \left[E_0(x,y)(\hat{x} \pm i\hat{y}) + \psi(x,y)\hat{z}\right] e^{ikz - i\omega t}$$

$$B(x,y,z,t) = \mp i E(x,y,z,t) \quad , \quad k = \frac{\omega}{c}$$

Real part is implied

$\psi(x,y)$ is of order $\frac{1}{kE_0}|\nabla E_0| \ll 1$

find $\psi(x,y)$ such that

$$\nabla \times E = +\frac{i\omega}{c} B \, , \quad \nabla \times B = -\frac{i\omega}{c} E$$

$$\nabla \cdot E = \nabla \cdot B = 0$$

to first order in $\frac{1}{kE_0}|\nabla E_0| \ll 1$

$$0 = \nabla \cdot E \simeq \frac{\partial E_0}{\partial x} \pm i \frac{\partial E_0}{\partial y} + ik\psi$$

$$\therefore \psi(x,y) = -\frac{1}{ik}\left(\frac{\partial E_0}{\partial x} \pm i \frac{\partial E_0}{\partial y}\right)$$

$$\nabla \cdot B = \mp i \nabla \cdot E \simeq 0$$

$$\nabla \times E \simeq \left\{ ik E_0 \, \hat{z} \otimes (\hat{x} \pm i\hat{y}) + \frac{\partial E_0}{\partial x}(\pm i \, \hat{x} \otimes \hat{y}) + \frac{\partial E_0}{\partial y}(\hat{y} \otimes \hat{x}) \right\} e^{ikz - i\omega t}$$

$$\{ \quad \} \simeq \left\{ ik E_0 (\hat{y} \mp i\hat{x}) + \hat{z}\left(\pm i \frac{\partial E_0}{\partial x} - \frac{\partial E_0}{\partial y}\right) \right\}$$

$$\simeq ik \left\{ E_0 (\hat{y} \mp i\hat{x}) + \frac{\hat{z}}{ik}\left(\pm i \frac{\partial E_0}{\partial x} - \frac{\partial E_0}{\partial y}\right) \right\}$$

$$\simeq \frac{i\omega}{c}(\mp i) \left\{ E_0(x,y)(\hat{x} \pm i\hat{y}) + 4(x,y)\hat{z} \right\}$$

$$\therefore \nabla \times E \simeq \frac{i\omega}{c} \underbrace{(\mp i) E}_{B}$$

Likewise

$$\nabla \times B = \nabla \times (\mp i) E = (\mp i) \nabla \times E = (\mp i)^2 \frac{i\omega}{c} E$$

$$\nabla \times B = -\frac{i\omega}{c} E$$

(15)

$$U = \int dx\,dy\, \frac{\underset{\sim}{E} \cdot \underset{\sim}{E}^*}{8\pi} \simeq \int dx\,dy\, \frac{E_0^2(x,y)}{4\pi}$$

$$L_z = \int dx\,dy\, \hat{z} \cdot \frac{\underset{\sim}{r} \times (\underset{\sim}{E} \times \underset{\sim}{B}^*)}{8\pi c}$$

$$\text{''} = \int dx\,dy\, (\hat{z} \times \underset{\sim}{r}) \cdot \frac{(\underset{\sim}{E} \times \underset{\sim}{B}^*)}{8\pi c}$$

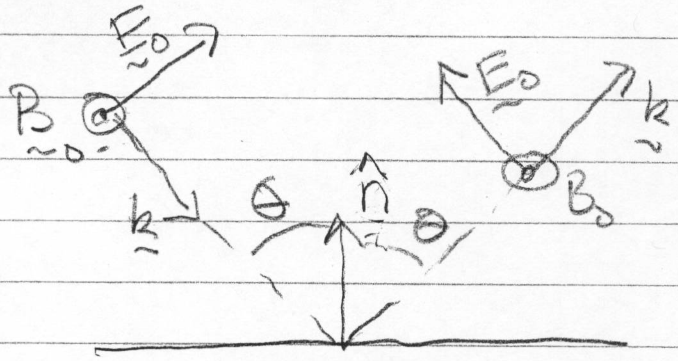
$$\text{''} = \int dx\,dy\, (-x\hat{y} + y\hat{x}) \cdot \frac{\underset{\sim}{E} \times \underset{\sim}{E}^*}{8\pi c}(\pm i)$$

$$\text{''} \simeq \frac{(\pm i)}{8\pi c}\int dx\,dy \left(-x\hat{y} + y\hat{x}\right) \cdot \left[2E_0 \underbrace{\psi^*}_{\frac{1}{ik}\left(\frac{\partial E_0}{\partial x} \mp i \frac{\partial E_0}{\partial y}\right)} (-\hat{y} \pm i\hat{x})\right]$$

$$\text{''} \simeq \frac{\pm 1}{8\pi c k}\int dx\,dy\, (\mp x \pm i y)\left[\frac{\partial}{\partial x} E_0^2 \mp i \frac{\partial E_0^2}{\partial y}\right]$$

$$\text{''} \simeq \frac{\pm 1}{\omega}\,\frac{1}{8\pi}\int dx\,dy\, [E_0^2 + E_0^2] = \frac{\pm 1}{\omega}\, U$$

(16)



(a) time average stress

write out better

$$\langle \hat{n} \cdot \underset{\sim}{T} \hat{n} \rangle = \frac{\hat{n} \cdot}{8\pi}\left[\underset{\sim}{E}\underset{\sim}{E}^* + \underset{\sim}{B}\underset{\sim}{B}^* - \frac{1}{2}\underset{\approx}{\mathbb{I}}\left(|E|^2 + |B|^2\right)\right] \cdot \hat{n}$$

$$= \hat{n} \frac{|2E_0|^2 \sin^2\theta}{16\pi} - \frac{|2B_0|^2}{16\pi}$$

$$= \underbrace{-\hat{n}\frac{E_0^{\,2}\cos^2\theta}{4\pi}}_{\text{pressure}}$$

(b) time average momentum density

$$\underset{\sim}{g} = \hat{k}|E_0|^2/8\pi c$$

$$\frac{\text{Force}}{\text{area}} = -\hat{n}\,\hat{n}\cdot\hat{k}\,\hat{n}\cdot\mathbf{g}\,2c$$

$$= -\hat{n}\frac{|E_0|^2}{4\pi}\cos^2\theta$$

(17)

in mirror frame

$$\mathbf{E}_0' = \gamma\left[\mathbf{E}_0 + \frac{\mathbf{V}}{c}\times\mathbf{B}_0\right] = \gamma\,\mathbf{E}_0\left(1-\frac{V}{c}\right)$$

$$\mathbf{B}_0' = \gamma\left[\mathbf{B}_0 - \frac{\mathbf{V}}{c}\times\mathbf{E}_0\right] = \gamma\,\mathbf{B}_0\left(1-\frac{V}{c}\right)$$

pressure measured by observer on the mirror

$$\frac{\text{Force}}{\text{area}} = \frac{|E_0'|^2}{4\pi} = \frac{E_0^2}{4\pi}\frac{\left(1-\frac{V}{c}\right)^2}{1-V^2/c^2}$$

$$" = \frac{E_0^2}{4\pi}\frac{(1-V/c)}{(1+V/c)}$$

(18)

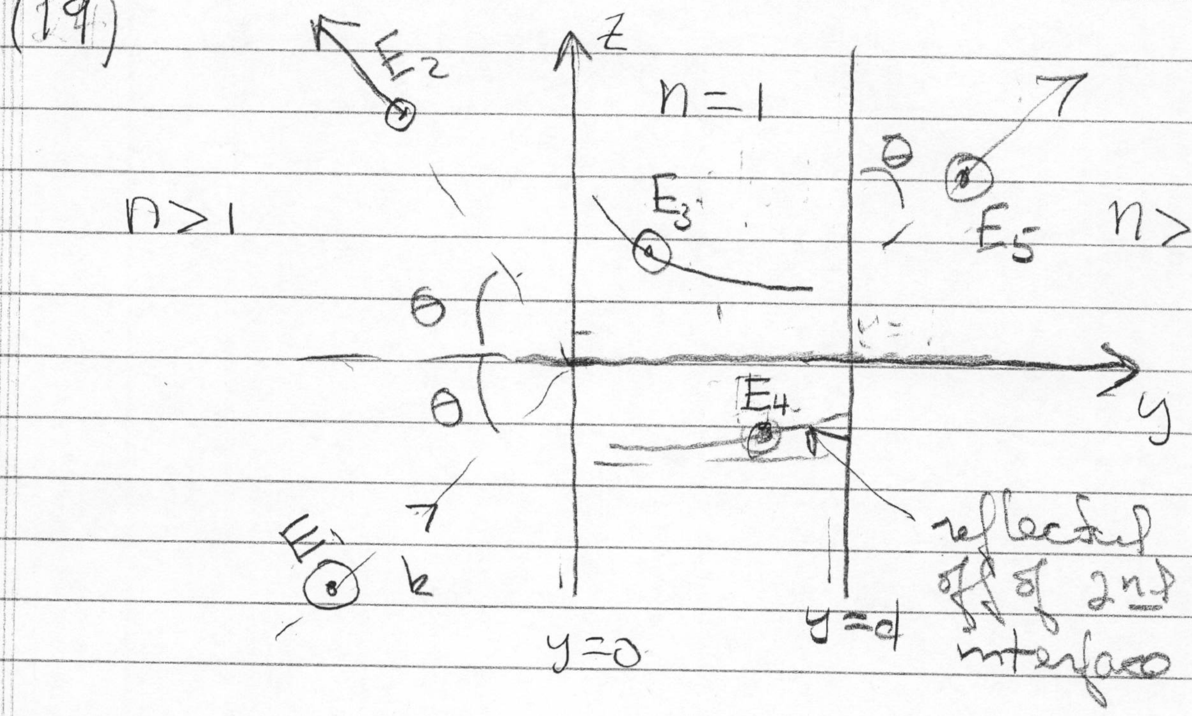
$$R_{\parallel} = \frac{|E''|^2}{|E|^2} = \frac{|n_2^2\cos\theta - n_1 n_2\cos\theta'|^2}{|n_2^2\cos\theta + n_1 n_2 \cos\theta'|^2}$$

$$R_{\parallel} = \frac{\left|\cos\theta - \frac{n_1}{n_2}\cos\theta'\right|^2}{\left|\cos\theta + \frac{n_1}{n_2}\cos\theta'\right|^2} = \frac{|\sin\theta\cos\theta - \sin\theta'\cos\theta'|^2}{|\sin\theta\cos\theta + \sin\theta'\cos\theta'|^2}$$

$$R_{\parallel} = \left|\frac{\sin 2\theta - \sin 2\theta'}{\sin 2\theta + \sin 2\theta'}\right|^2 = \left|\frac{\sin(\theta-\theta')\cos(\theta+\theta')}{\sin(\theta+\theta')\cos(\theta-\theta')}\right|^2$$

$$R_{\parallel} = \left|\frac{\tan(\theta-\theta')}{\tan(\theta+\theta')}\right|^2$$

(19)



Given that $n\sin\theta > 1$, find transmitted power to lowest order in small parameter $\exp[-|k_y'|d]$. Prime refers to wave number in gap.

At first interface, we can neglect reflected wave from 2nd interface (i.e., E_4) since this wave is of order $\exp[-2|k_y'|d]$ at location of 1st interface.

b.c. at first interface

(1) continuity of $E_t = E_x$

$$E_1 + E_2 = E_3$$

(2) continuity of $H_t = H_z$

in medium use

$$\frac{i\omega}{c} \underset{\sim}{H} \simeq \frac{i\omega}{c} \underset{\sim}{B} = \underset{\sim}{\nabla} \times \underset{\sim}{E} = i\underset{\sim}{k} \times \underset{\sim}{E}$$

$$\underset{\sim}{H} = \frac{c\underset{\sim}{k} \times \underset{\sim}{E}}{\omega}, \quad H_z = -\frac{ck_y}{\omega} E_x$$

in gap $\sim e^{-|k_y'|y + ik_z'z}$

$$\frac{i\omega}{c} \underset{\sim}{H} \cdot \hat{z} = \hat{z} \cdot \underset{\sim}{\nabla} \times \underset{\sim}{E} = |k_y'| E_x$$

$$\therefore \frac{ck\cos\theta}{\omega}(E_2 - E_1) = \frac{c}{i\omega} |k_y'| E_3$$

also

$$k_z'^2 + k_y'^2 = \frac{\omega'^2}{c^2} = \frac{\omega^2}{c^2} = \frac{k^2}{n^2}$$

$$\overset{\parallel}{k_z^2}$$

$$k_y'^2 = \frac{k^2}{n^2} - k_z^2 = \frac{k^2}{n^2}\underbrace{(1 - n^2\sin^2\theta)}_{\text{negative}}$$

$$|k_y'| = \frac{k}{n}\sqrt{n^2\sin^2\theta - 1}$$

$$\therefore \cancel{\cos}\theta\,(E_2 - E_1) = -i\,\frac{\overset{|k_y|}{\sqrt{n^2\sin^2\theta - 1}}}{n\cos\theta}\,E_3$$

$$\therefore 2E_1 = E_3\left[1 + i\,\frac{\sqrt{n^2\sin^2\theta - 1}}{n\cos\theta}\right]$$

at 2nd interface

$$E_3 e^{-|k_y'|d} + E_4 = E_5$$

$$\frac{c|k_y'|}{i\omega} E_3 e^{-|k_y|d} - \frac{c|k_y'|}{i\omega} E_4 = -\frac{ck\cos\theta}{\omega} E_5$$

from different sign in ~~exponential~~

$E_4 \sim e^{+|k_y'|y}$

$$\therefore E_3 e^{-|k_y'|d} - E_4 = \frac{-ik\cos\theta}{|k_y'|} E_5$$

$$2|E_3| e^{-|k_y'|d} = \left[\frac{-in\cos\theta}{\sqrt{n^2\sin^2\theta - 1}} + 1\right] E_5$$

Transmission coefficient

$$T = \frac{|E_5|^2}{|E_1|^2} = \left|\frac{E_5}{E_3}\right|^2 \left|\frac{E_3}{E_1}\right|^2 \sim e^{-2|k_y'|d}$$

given above

(20) Jackson 7.12

$$\epsilon(\omega) = \epsilon_0 - \underbrace{\frac{4\pi n e^2 f_1}{m[\omega^2 + i\omega\gamma]}}_{\frac{4\pi\sigma}{-i\omega}}$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega/\gamma}, \quad \text{where } \sigma_0 = \frac{n e^2 f_1}{m\omega\gamma}$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{H}(\underset{\sim}{r}, \omega) = -\frac{i\omega}{c}\epsilon(\omega)\underset{\sim}{D}(\underset{\sim}{r}, \omega)$$

$$0 = \underset{\sim}{\nabla} \cdot \underset{\sim}{\nabla} \times \underset{\sim}{H} = -\frac{i\omega}{c}\epsilon(\omega)\underbrace{\nabla \cdot D(\underset{\sim}{r}, \omega)}_{4\pi\rho(\underset{\sim}{r}, \omega)}$$

$$0 = [-i\omega\epsilon_0 + 4\pi\sigma(\omega)]\,\rho(\underset{\sim}{r}, \omega)$$

solve for ω, assuming $\rho(\underline{r},\omega) \neq 0$

$$i\omega\epsilon_0 = \frac{4\pi\sigma_0}{1 - i\omega/\gamma}$$

$$\frac{\omega^2\epsilon_0}{\gamma} + i\omega\epsilon_0 - 4\pi\sigma_0 = 0$$

$$\omega^2 + i\omega\gamma - \frac{4\pi\sigma_0\gamma}{\epsilon_0} = 0$$

$$\omega = \frac{-i\gamma \pm \sqrt{\frac{16\pi\sigma_0\gamma}{\epsilon_0} - \gamma^2}}{2}$$

$$\therefore\ -i\omega = -\frac{\gamma}{2} \pm \underbrace{\sqrt{\frac{4\pi\sigma_0\gamma}{\epsilon_0} - \frac{\gamma^2}{4}}}$$

$$\simeq \sqrt{\frac{4\pi\sigma_0\gamma}{\epsilon_0}} = \omega_p$$

$$\rho(\underline{r},t) = \mathrm{Re}\,\rho(\underline{r},0)\,e^{-i\omega t}$$

$$\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}D(\omega)e^{-i\omega t} \simeq \int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}e^{-i\omega t}\Big\{\epsilon(0)E(\omega)$$

$$+\frac{d\epsilon}{d\omega}\Big|_{\omega=0}\omega E(\omega) \quad +\frac{1}{2}\frac{d^2\epsilon}{d\omega^2}\Big|_{\omega=0}\omega^2 E(\omega)$$

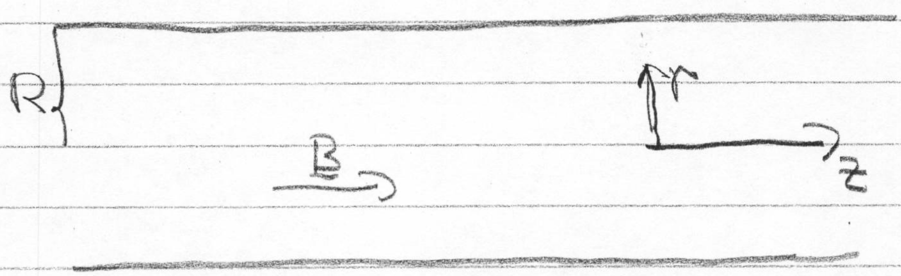
skip

$$D(t) \simeq \epsilon(0)E(t) - \frac{1}{i}\frac{d\epsilon}{d\omega}\Big|_{\omega=0}\frac{dE}{dt}$$

$$-\frac{1}{2}\frac{d^2\epsilon}{d\omega^2}\Big|_{\omega=0}\frac{d^2E}{dt^2} + \cdots$$

∴ same result

(21)



$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial B_z}{\partial r} = \frac{4\pi\sigma}{c^2}\frac{\partial B_z}{\partial t}$$

$$B_z(R,t)=0 \quad , \quad B_z(r,0)=B_0$$

Look for soln of form

$$B_z(r,t) = \sum_{n=1}^{\infty} A_n J_0\left(\frac{r}{R}x_{0n}\right) e^{-\nu_n t}$$

$$0 = \left[\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\right] B_z = \sum_{n=1}^{\infty}\left[-\left(\frac{x_{0n}}{R}\right)^2 + \frac{4\pi\sigma}{c^2}\nu_n\right] A J_0\left(\frac{r}{R}x_{0n}\right) e^{-\nu_n t}$$

satisfied if $\nu_n = \dfrac{c^2 x_{0n}^2}{4\pi\sigma R^2}$

b.c. at $r=R$ is satisfied

initial condition is satisfied
if A_n's are chosen so that

$$B_0 = B_z(r, t=0) = \sum_n A_n J_n\left(\frac{r}{R}x_{0n}\right)$$

$$\therefore \; B_0 \underbrace{\int_0^R r\,dr\, J_0\left(\frac{r}{R}x_{0n}\right)}_{(R^2/x_{0n})\, J_1(x_{0n})} = A_n \underbrace{\int_0^R r\,dr\, J_n^2\left(\frac{r}{R}x_{0n}\right)}_{\frac{R^2}{2} J_1^2(x_{0n})}$$

(22)

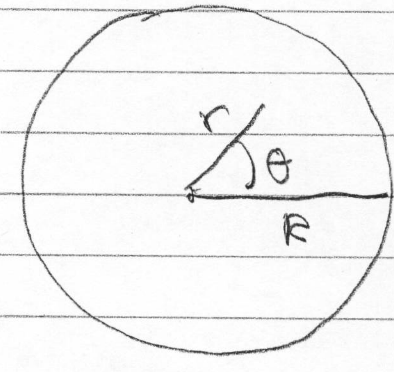
$$\langle S_x \rangle = \frac{c}{8\pi} \mathrm{Re}\left(\underset{\sim}{E}_0 \times \underset{\sim}{H}_0^* \cdot \hat{x}\right)$$

$$'' \quad = \frac{c}{8\pi} \mathrm{Re}\,(i-1) \frac{c}{4\pi\sigma\delta} |\underset{\sim}{H}_0|^2 \underbrace{\hat{z} \times \hat{y} \cdot \hat{x}}_{-1}$$

$$'' \quad = \frac{c^2}{32\pi^2\sigma\delta} |\underset{\sim}{H}_0|^2$$ same answer as in lecture

(23) See Jackson pages 357–359

(24)



TM modes

$$\underbrace{\left[\nabla_t^2 + \frac{\omega}{c^2} - k^2\right]}_{\gamma^2} E_z(r, \theta) = 0$$

$$E_z(R, \theta) = 0$$

Try $E_z(r,\theta) = J_\ell\left(\frac{r}{R} x_{\ell n}\right) e^{i\ell\theta}$, where

$$J_\ell(x_{\ell n}) = 0$$

$$\nabla_t^2 = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{r^2 \partial\theta^2}$$

$$0 = [\nabla_t^2 + \gamma^2] J_l(\tfrac{r}{R} x_{ln}) e^{il\theta} = \left[-\frac{x_{ln}^2}{R^2} + \gamma^2\right] J_l(\tfrac{r}{R} x_{ln}) e^{il\theta}$$

$$\therefore \gamma^2 = \frac{x_{ln}^2}{R^2}$$

TE modes

$$[\nabla_t^2 + \gamma^2] B_z(r,\theta) = 0$$

$$\left.\frac{\partial B_z}{\partial r}\right|_{r=R} = 0$$

Try $B_z(r,R) = J_l(\frac{r}{R} x'_{ln}) e^{il\theta}$

where $J_l'(x'_{ln}) = 0$

$$\therefore \gamma^2 = \frac{x'^2_{ln}}{R^2}$$

(25) for TE modes

$$\langle S_z \rangle = \frac{c}{8\pi} \mathrm{Re}\left[E_x B_y^* - E_y B_x^*\right]$$

$$'' = \frac{c}{8\pi} \frac{1}{\gamma^4} \frac{\omega}{c} k \left[\left|\frac{\partial B_z}{\partial y}\right|^2 + \left|\frac{\partial B_z}{\partial x}\right|^2\right]$$

$$\langle P \rangle = \iint \langle S_z \rangle dA = \frac{\omega k}{8\pi \gamma^4} \iint \nabla_t B_z \cdot \nabla_t B_z^* dA$$

$$\nabla_t \cdot (B_z^* \nabla_t B_z) = \nabla_t B_z \cdot \nabla_t B_z^* + B_z^* \nabla_t^2 B_z$$

$$\langle P \rangle = \frac{\omega k}{8\pi \gamma^4} \left\{ \underbrace{\int \hat{n} \cdot B_z^* \nabla_t B_z \, dl}_{= 0} - \iint B_z^* \underbrace{\nabla_t^2 B_z}_{-\gamma^2 B_z} dA \right.$$

$$\langle P \rangle = \frac{\omega k}{\gamma^2} \iint \frac{|B_z|^2}{8\pi} dA$$

(26) For TM modes the time average energy per unit length is given by

$$\langle U\rangle = \frac{1}{16\pi}\iint\left[|E_z|^2 + |\underset{\sim}{E_t}|^2 + |\underset{\sim}{B_t}|^2\right] dA$$

$$|\underset{\sim}{E_t}|^2 = \frac{k^2}{\gamma^4}|\underset{\sim}{\nabla_t} E_z|^2 \quad , \quad |B_t|^2 = \frac{\omega^2}{c^2\gamma^4}|\underset{\sim}{\nabla_t} E_z|^2$$

$$\iint|\underset{\sim}{\nabla_t} E_z|^2 dA = -\iint E_z^* \underbrace{\underset{\sim}{\nabla_t^2} E_z}_{-\gamma^2 E_z} = \gamma^2\iint|E_z|^2 dA$$

see previous problem

$$\therefore \langle U\rangle = \frac{1}{16\pi}\iint\left[|E_z|^2 + \frac{1}{\gamma^2}\Big(k^2 + \underbrace{\frac{\omega^2}{c^2}}\Big)|E_z|^2\right] dA$$

$$= \frac{\omega^2}{c^2} - k^2$$

$$\langle U\rangle = \frac{1}{8\pi}\frac{\omega^2}{c^2\gamma^2}\iint|E_z|^2 dA$$

$$\langle P\rangle / \langle U\rangle = \frac{\omega k}{\omega^2/c^2} = \frac{c^2}{(\omega/k)} = v_g$$

(27) TE cavity modes for cylindrical cavity of radius R and height L

$$B_z = \psi(r,\theta)\sin kz\, e^{-i\omega t}$$

$$E_z = 0$$

$$\underset{\sim}{E}_t = \frac{i\omega/c\ \hat{z}\times\underset{\sim}{\nabla}_t B_z}{\frac{\omega^2}{c^2}-k^2}$$

$$\left[\nabla_t^2 + \frac{\omega^2}{c^2} - k^2\right]\psi(r,\theta) = 0$$

$$\left.\frac{\partial\psi}{\partial r}\right|_R = \frac{\partial\psi}{\partial n} = 0 \quad \text{on walls}$$

$$B_z(z=0,L) = 0$$

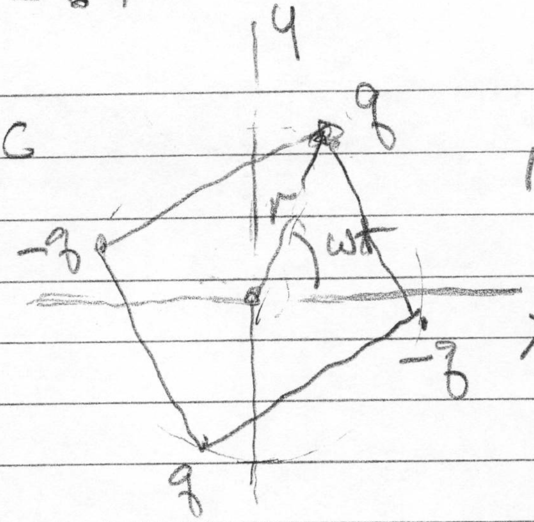
$$\underset{\sim}{E}_t(z=0,L) = 0$$

$$k = \pi m/L$$

$$\psi = J_l\left(\frac{r}{R}x'_{ln}\right)e^{il\theta}$$

$$\therefore\ \omega^2/c^2 = (\pi m/L)^2 + (x'_{ln}/R)^2$$

(28) $a\omega \ll c$



$r = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$

$$Q_{\alpha\beta} = \int d^3\underline{r}\left(3x_\alpha x_\beta - \delta_{\alpha\beta} r^2\right)\rho(r,t)$$

$$= \int d^3\underline{r}\; 3x_\alpha x_\beta\, \rho(\underline{r},t) \quad \text{for this case}$$

$$Q_{xx} = 3q\left(\frac{a^2}{2}\cos^2\omega t - \frac{a^2}{2}\sin^2\omega t\right)2$$

$$Q_{yy} = 3q\left(\frac{a^2}{2}\sin^2\omega t - \frac{a^2}{2}\cos^2\omega t\right)2$$

$$Q_{xx} = -Q_{yy} = 3qa^2\cos 2\omega t$$

$$Q_{xy} = Q_{yx} = 3q\frac{a^2}{2}(\cos\omega t\sin\omega t)4$$

$$= 3qa^2\sin 2\omega t$$

$$Q_{xx} = -Q_{yy} = \mathrm{Re}\, 3q\, a^2 e^{2i\omega t}$$

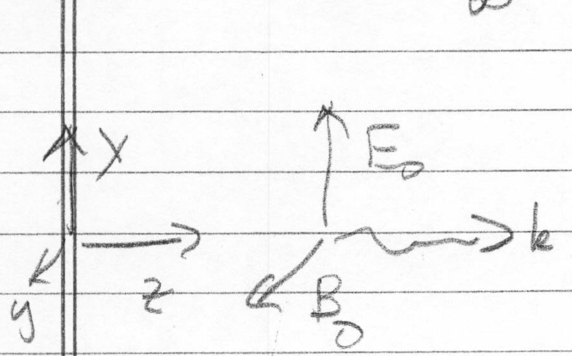
$$Q_{xy} = +Q_{yx} = \mathrm{Re}\, -i\, 3q\, a^2 e^{2i\omega t}$$

$$\langle P \rangle = \frac{1}{360} \frac{(2\omega)^6}{c^5} \underbrace{\sum_{\alpha,\beta} |Q_{\alpha\beta}|^2}_{4(3q^2a^2)}$$

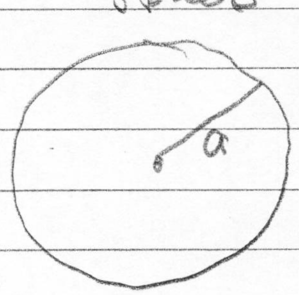
(29)

$$\underset{\sim}{E}(\underset{\sim}{r},t) = \mathrm{Re}\, \underset{\sim}{E}_0 e^{i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t}$$

$$a \ll \frac{c}{\omega}, \quad \delta \ll a$$



conducting sphere



(a) use static approximation to determine electric and magnetic dipoles induced in sphere

electric dipole

$\underset{\sim}{p} = \underset{\sim}{E}_0 a^3$ (see page 56 notes)

magnetic dipole

$\underset{\sim}{m} = -\underset{\sim}{B}_0 \frac{a^3}{2}$ (see homework # 17)

(b) In radiation zone

$$\underset{\sim}{B} = \left[k^2 \hat{r} \times \underset{\sim}{p} - k^2 \hat{r} \times (\hat{r} \times \underset{\sim}{m})\right] \frac{e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}}{r}$$

$$\underset{\sim}{B} = k^2 a^3 \left[\hat{r} \times \underset{\sim}{E}_0 + \hat{r} \times \frac{(\hat{r} \times \underset{\sim}{B}_0)}{2}\right] \frac{e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}}{r}$$

let $\underset{\sim}{E}_0 = \hat{x} E_0$, $\underset{\sim}{B}_0 = \hat{y} E_0$

$$\underset{\sim}{B} = \frac{k^2 a^3 e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}}{r} E_0 \left[\hat{r} \otimes \hat{x} + \frac{1}{2} \hat{r} \otimes (\hat{r} \otimes \hat{y})\right]$$

$$\langle \underset{\sim}{S} \rangle = \frac{c}{8\pi} |B|^2 \hat{r}$$

$$\frac{d\langle P \rangle}{d\Omega} = \hat{r} r^2 \cdot \langle \underset{\sim}{S} \rangle$$

$$\frac{d\sigma}{d\Omega} = \frac{\frac{d\langle P \rangle}{d\Omega}}{c|E_0|^2/8\pi} = (k^4 a^4) a^2 \cdot \left[\hat{r} \otimes \hat{x} + \frac{1}{2} \hat{r} \otimes (\hat{r} \otimes \hat{y})\right]^2$$