# Chapter 28
# Atomic Physics

## Quick Quizzes

**1.** (b). The allowed energy levels in a one-electron atom may be expressed as $E_n = -Z^2(13.6 \text{ eV})/n^2$, where Z is the atomic number. Thus, the ground state $(n = 1 \text{ level})$ in helium, with $Z = 2$, is lower than the ground state in hydrogen, with $Z = 1$.

**2.** (a). The energy of the photon emitted when the electron in a one-electron atom makes a transition from a state having principal quantum number $n_i$ to one having principal quantum number $n_f$ is

$$E_\gamma = Z^2(13.6 \text{ eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Thus, for given values of $n_i$ and $n_f$, the energy of the photon emitted by a helium atom, with $Z = 2$, is four times that of the photon emitted when an electron makes the corresponding transition in a hydrogen atom, with $Z = 1$.

**3.** **(a)** For $n = 5$, there are 5 allowed values of $\ell$, namely $\ell$ = 0, 1, 2, 3, and 4.

**(b)** Since $m_\ell$ ranges from $-\ell$ to $+\ell$ in integer steps, the largest allowed value of $\ell$ ($\ell$ = 4 in this case) permits the greatest range of values for $m_\ell$. For $n = 5$, there are 9 possible values for $m_\ell$: -4, -3, -2, –1, 0, +1, +2, +3, and +4.

**(c)** For each value of $\ell$, there are $2\ell + 1$ possible values of $m_\ell$. Thus, there is 1 distinct pair with $\ell = 0$; 3 distinct pairs possible with $\ell = 1$; 5 distinct pairs with $\ell = 2$; 7 distinct pairs with $\ell = 3$; and 9 distinct pairs with $\ell = 4$. This yields a total of 25 distinct pairs of $\ell$ and $m_\ell$ that are possible when $n = 5$.

**4.** (d). Krypton has a closed configuration consisting of filled $n$=1, $n$=2, and $n$=3 shells as well as filled 4*s* and 4*p* subshells. The filled $n = 3$ shell (the next to outer shell in Krypton) has a total of 18 electrons, 2 in the 3*s* subshell, 6 in the 3*p* subshell and 10 in the 3*d* subshell.

## Answers to Even Numbered Conceptual Questions

**2.** Neon signs do not emit a continuous spectrum. They emit many discrete wavelengths as could be determined by observing the light from the sign through a spectrometer. However, they do not emit all wavelengths. The specific wavelengths and intensities account for the color of the sign.

**4.** An atom does not have to be ionized to emit light. For example, hydrogen emits light when a transition carries an electron from a higher state to the $n = 2$ state.

**6.** Classically, the electron can occupy any energy state. That is, all energies would be allowed. Therefore, if the electron obeyed classical mechanics, its spectrum, which originates from transitions between states, would be continuous rather than discrete.

**8.** The de Broglie wavelength of macroscopic objects such as a baseball moving with a typical speed such as 30 m/s is very small and impossible to measure. That is, $\lambda = h/mv$, is a very small number for macroscopic objects. We are not able to observe diffraction effects because the wavelength is much smaller than any aperture through which the object could pass.

**10.** In both cases the answer is yes. Recall that the ionization energy of hydrogen is 13.6 eV. The electron can absorb a photon of energy less than 13.6 eV by making a transition to some intermediate state such as one with $n = 2$. It can also absorb a photon of energy greater than 13.6 eV, but in doing so, the electron would be separated from the proton and have some residual kinetic energy.

**12.** It replaced the simple circular orbits in the Bohr theory with electron clouds. More important, quantum mechanics is consistent with Heisenberg's uncertainty principle, which tells us about the limits of accuracy in making measurements. In quantum mechanics, we talk about the probabilistic nature of the outcome of a measurement of a system, a concept which is incompatible with the Bohr theory. Finally, the Bohr theory of the atom contains only one quantum number $n$, while quantum mechanics provides the basis for additional quantum numbers to explain the finer details of atomic structure.

**14.** Each of the given atoms has a single electron in an $\ell = 0$ (or $s$) state outside a fully closed-shell core, shielded from all but one unit of the nuclear charge. Since they reside in very similar environments, one would expect these outer electrons to have nearly the same electrical potential energies and hence nearly the same ionization energies. This is in agreement with the given data values. Also, since the distance of the outer electron from the nuclear charge should tend to increase with $Z$ (to allow for greater numbers of electrons in the core), one would expect the ionization energy to decrease somewhat as atomic number increases. This is also in agreement with the given data.

**16.** One assumption is natural from the standpoint of classical physics: The electron feels an electric force of attraction which supplies the centripetal acceleration and holds it in orbit. The other assumptions are in sharp contrast to the behavior of ordinary-size objects: The electron's angular momentum must be one of a set of certain special allowed values. During the time when it is in one of these quantized orbits, the electron emits no electromagnetic radiation. The atom radiates a photon when the electron makes a quantum jump from one orbit to a lower one.

**18.** **(a)** $n$, $\ell$, and $m_\ell$ are integers; $m_s$ is fractional

**(b)** $n$ and $\ell$ are always positive; $m_\ell$ and $m_s$ can be negative

**(c)** $\ell_{\text{max}} = n - 1 = 1$

**(d)** $m_\ell$ can have values of $-1$, 0, or 1

## Answers to Even Numbered Problems

**4.** **(a)** $2.3\times10^{2}$ N **(b)** 1.4 MeV

**6.** 45 fm

**8.** **(a)** $2.19\times10^{6}$ m/s **(b)** 13.6 eV **(c)** –27.2 eV

**10.** **(a)** 3.03 eV **(b)** 410 nm **(c)** $7.32\times10^{14}$ Hz

**12.** **(a)** transition II **(b)** transition I **(c)** transitions II and III

**14.** **(a)** 12.1 eV **(b)** 12.1 eV, 10.2 eV, and 1.89 eV

**20.** **(a)** 6 **(b)** $1.88\times10^{3}$ nm (in the Paschen series)

**22.** **(a)** $1.52\times10^{-16}$ s **(b)** $8.23\times10^{9}$ revolutions
**(c)** Yes, for $8.23\times10^{9}$ ªelectron yearsº
**(d)** The electron moves so quickly that it can never meaningfully be said to be on any particular side of the nucleus.

**24.** $4.43\times10^{4}$ m/s

**26.** **(a)** $2.89\times10^{34}$ $\text{kg}\cdot\text{m}^2/\text{s}$ **(b)** $2.74\times10^{68}$ **(c)** $7.30\times10^{-69}$

**28.** **(a)** $E_n = -54.4\text{ eV}/n^2$ **(b)** 54.4 eV

**30.** **(a)** $4.42\times10^{7}$ m$^{-1}$ **(b)** 30.1 nm **(c)** ultraviolet

**34.** **(a)** 4 **(b)** 7

**36.** **(a)** $1s^2 2s^2 2p^4$
**(b)** $(n=1,\ \ell=0,\ m_\ell=0,\ m_s=\pm\frac{1}{2})$; $(n=2,\ \ell=0,\ m_\ell=0,\ m_s=\pm\frac{1}{2})$
$(n=2,\ \ell=1,\ m_\ell=0,\ m_s=\pm\frac{1}{2})$; $(n=2,\ \ell=1,\ m_\ell=1,\ m_s=\pm\frac{1}{2})$

**38.** **(a)** 2 **(b)** 8 **(c)** 18 **(d)** 32 **(e)** 50

**40.** 0.155 nm, 8.03 kV

**42.** $Z=32$, germanium

**44.** 137

**46.** **(a)** 4.20 mm **(b)** $1.05\times10^{19}$ photons
**(c)** $8.82\times10^{16}$ photons/mm$^3$

**48.** **(a)** 137 **(b)** $1/2\pi\alpha$ **(c)** $4\pi/\alpha$

**50.** The simplest diagram has 4 states with energies of -4.100 eV, -1.000 eV, -0.1000 eV, and 0.

**52.** **(a)** 135 eV
**(b)** $\approx 10$ times the magnitude of the ground state energy of hydrogen.

**54.** when $n\to\infty$, $f\to f_{classical}=4\pi^2 m_e k_e^2 e^4/h^3 n^3$

**56.** **(a)** $2.56\times10^{-4}$ nm **(b)** $-2.82\times10^{3}$ eV, $-704$ eV, $-313$ eV

**58.** **(a)** $n_f=1$ **(b)** $n_i=3$

## Problem Solutions

**28.1** The Balmer equation is $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, or $\lambda = \frac{4}{R_H}\left(\frac{n^2}{n^2-4}\right)$

When $n = 3$,

$$\lambda = \frac{4}{1.097\,37\times10^7\ \text{m}^{-1}}\left(\frac{9}{9-4}\right) = 6.56\times10^{-7}\ \text{m} = \boxed{656\ \text{nm}}$$

When $n = 4$,

$$\lambda = \frac{4}{1.097\,37\times10^7\ \text{m}^{-1}}\left(\frac{16}{16-4}\right) = 4.86\times10^{-7}\ \text{m} = \boxed{486\ \text{nm}}$$

When $n = 5$,

$$\lambda = \frac{4}{1.097\,37\times10^7\ \text{m}^{-1}}\left(\frac{25}{25-4}\right) = 4.34\times10^{-7}\ \text{m} = \boxed{434\ \text{nm}}$$

**28.2** Start with Balmer's equation, $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = R_H\left(\frac{n^2-4}{4n^2}\right)$

or $$\lambda = \frac{4}{R_H}\left(\frac{n^2}{n^2-4}\right)$$

Substituting $R_H$=1.097 3732 $\times 10^7\ \text{m}^{-1}$, we obtain

$$\lambda = \frac{(3.645\times10^{-7}\ \text{m})n^2}{n^2-4} = \boxed{\frac{364.5n^2}{n^2-4}\ \text{nm}} \quad \text{where } n = 3, 4, 5, \ldots.$$

**28.3** (a) From Coulomb's law,

$$|F| = \frac{k_e|q_1q_2|}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.0\times10^{-10}\ \text{m})^2} = \boxed{2.3\times10^{-8}\ \text{N}}$$

(b) The electrical potential energy is

$$PE = \frac{k_e q_1 q_2}{r} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-1.60\times10^{-19}\ \text{C}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{1.0\times10^{-10}\ \text{m}}$$

$$= -2.3\times10^{-18}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{-14\ \text{eV}}$$

**28.4** (a) From Coulomb's law,

$$F = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(1.0\times10^{-15}\ \text{m}\right)^2} = \boxed{2.3\times10^{2}\ \text{N}}$$

(b) The electrical potential energy is

$$PE = \frac{k_e q_1 q_2}{r} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{1.0\times10^{-15}\ \text{m}}$$

$$= 2.3\times10^{-13}\ \text{J}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{+1.4\ \text{MeV}}$$

**28.5** (a) The electrical force supplies the centripetal acceleration of the electron, so $m\frac{v^2}{r} = \frac{k_e e^2}{r^2}$ or $v = \sqrt{\frac{k_e e^2}{mr}}$

$$v = \sqrt{\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.0\times10^{-10}\ \text{m}\right)}} = \boxed{1.6\times10^{6}\ \text{m/s}}$$

(b) $\boxed{\text{No.}}$ $\frac{v}{c} = \frac{1.6\times10^{6}\ \text{m/s}}{3.00\times10^{8}\ \text{m/s}} = 5.3\times10^{-3} << 1$, so the electron is not relativistic.

(c) The de Broglie wavelength for the electron is $\lambda = \frac{h}{p} = \frac{h}{mv}$, or

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.6\times10^{6}\ \text{m/s}\right)} = 4.6\times10^{-10}\ \text{m} = \boxed{0.46\ \text{nm}}$$

(d) $\boxed{\text{Yes.}}$ The wavelength and the atom are roughly the same size.

**28.6** Assuming a head-on collision, the $\alpha$-particle comes to rest momentarily at the point of closest approach. From conservation of energy,

$$KE_f + PE_f = KE_i + PE_i \text{, or } 0 + \frac{k_e(2e)(79e)}{r_f} = KE_i + \frac{k_e(2e)(79e)}{r_i}$$

With $r_i \to \infty$, this gives the distance of closest approach as

$$r_f = \frac{158 k_e e^2}{KE_i} = \frac{158\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{5.0\ \text{MeV}\left(1.60\times10^{-13}\ \text{J/MeV}\right)}$$

$$= 4.5\times10^{-14}\ \text{m} = \boxed{45\ \text{fm}}$$

**28.7** (a) $r_n = n^2 a_0$ yields $r_2 = 4(0.052\,9\ \text{nm}) = \boxed{0.212\ \text{nm}}$

(b) With the electrical force supplying the centripetal acceleration,

$$\frac{m_e v_n^2}{r_n} = \frac{k_e e^2}{r_n^2}\text{, giving } v_n = \sqrt{\frac{k_e e^2}{m_e r_n}} \text{ and } p_n = m_e v_n = \sqrt{\frac{m_e k_e e^2}{r_n}}$$

Thus,

$$p_2 = \sqrt{\frac{m_e k_e e^2}{r_2}} = \sqrt{\frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{0.212\times10^{-9}\ \text{m}}}$$

$$= \boxed{9.95\times10^{-25}\ \text{kg}\cdot\text{m/s}}$$

(c) $L_n = n\left(\dfrac{h}{2\pi}\right) \to L_2 = 2\left(\dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi}\right) = \boxed{2.11\times10^{-34}\ \text{J}\cdot\text{s}}$

(d) $KE_2 = \dfrac{1}{2}m_e v_2^2 = \dfrac{p_2^2}{2m_e} = \dfrac{\left(9.95\times10^{-25}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)} = 5.44\times10^{-19}\ \text{J} = \boxed{3.40\ \text{eV}}$

(e) $PE_2 = \dfrac{k_e(-e)e}{r_2} = -\dfrac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(0.212\times10^{-9}\ \text{m}\right)}$

$$= -1.09\times10^{-18}\ \text{J} = \boxed{-6.80\ \text{eV}}$$

(f) $E_2 = KE_2 + PE_2 = 3.40\ \text{eV} - 6.80\ \text{eV} = \boxed{-3.40\ \text{eV}}$

**28.8** (a) With the electrical force supplying the centripetal acceleration,

$$\frac{m_e v_n^2}{r_n} = \frac{k_e e^2}{r_n^2}\text{, giving } v_n = \sqrt{\frac{k_e e^2}{m_e r_n}}$$

where $r_n = n^2 a_0 = n^2 (0.052\,9 \text{ nm})$

Thus,

$$v_1 = \sqrt{\frac{k_e e^2}{m_e r_1}} = \sqrt{\frac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19} \text{ C})^2}{(9.11\times 10^{-31} \text{ kg})(0.052\,9\times 10^{-9} \text{ m})}} = \boxed{2.19\times 10^6 \text{ m/s}}$$

(b) $$KE_1 = \frac{1}{2} m_e v_1^2 = \frac{k_e e^2}{2r_1} = \frac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19} \text{ C})^2}{2(0.052\,9\times 10^{-9} \text{ m})}$$

$$= 2.18\times 10^{-18} \text{ J} = \boxed{13.6 \text{ eV}}$$

(c) $$PE_1 = \frac{k_e(-e)e}{r_1} = -\frac{(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19} \text{ C})^2}{(0.052\,9\times 10^{-9} \text{ m})}$$

$$= -4.35\times 10^{-18} \text{ J} = \boxed{-27.2 \text{ eV}}$$

**28.9** Since the electrical force supplies the centripetal acceleration,

$$\frac{m_e v_n^2}{r_n} = \frac{k_e e^2}{r_n^2} \text{ or } v_n^2 = \frac{k_e e^2}{m_e r_n}$$

From $L_n = m_e r_n v_n = n\hbar$, we have $r_n = \dfrac{n\hbar}{m_e v_n}$, so

$$v_n^2 = \frac{k_e e^2}{m_e}\left(\frac{m_e v_n}{n\hbar}\right) \text{ which reduces to } \boxed{v_n = \frac{k_e e^2}{n\hbar}}$$

**28.10** (b) From $\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

or $\lambda = \frac{1}{R_H}\left(\frac{n_i^2 n_f^2}{n_i^2 - n_f^2}\right)$ with $n_i = 6$ and $n_f = 2$

$$\lambda = \frac{1}{1.097\ 37 \times 10^7\ \text{m}^{-1}}\left[\frac{(36)(4)}{36-4}\right] = 4.10 \times 10^{-7}\ \text{m} = \boxed{410\ \text{nm}}$$

(a) $E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})}{410 \times 10^{-9}\ \text{m}} = 4.85 \times 10^{-19}\ \text{J} = \boxed{3.03\ \text{eV}}$

(c) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{410 \times 10^{-9}\ \text{m}} = \boxed{7.32 \times 10^{14}\ \text{Hz}}$

**28.11** The energy of the emitted photon is

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})}{656\ \text{nm}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right)\left(\frac{1\ \text{ev}}{1.602 \times 10^{-19}\ \text{J}}\right) = 1.89\ \text{eV}$$

This photon energy is also the difference in the electron's energy in its initial and final orbits. The energies of the electron in the various allowed orbits within the hydrogen atom are

$$E_n = -\frac{13.6}{n^2}\ \text{eV} \qquad \text{where} \qquad n = 1, 2, ,3, \ldots$$

giving $E_1 = -13.6$ eV, $E_2 = -3.40$ eV, $E_3 = -1.51$ eV, $E_4 = -0.850$ eV, …

Observe that $E_\gamma = E_3 - E_2$. Thus, the transition was from

$\boxed{\text{the } n = 3 \text{ orbit to the } n = 2 \text{ orbit}}$

**28.12** The change in the energy of the electron is

$$\Delta E = E_f - E_i = 13.6\text{ eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

Transition I: $\Delta E = 13.6\text{ eV}\left(\frac{1}{4} - \frac{1}{25}\right) = 2.86\text{ eV}$ (absorption)

Transition II: $\Delta E = 13.6\text{ eV}\left(\frac{1}{25} - \frac{1}{9}\right) = -0.967\text{ eV}$ (emission)

Transition III: $\Delta E = 13.6\text{ eV}\left(\frac{1}{49} - \frac{1}{16}\right) = -0.572\text{ eV}$ (emission)

Transition IV: $\Delta E = 13.6\text{ eV}\left(\frac{1}{16} - \frac{1}{49}\right) = 0.572\text{ eV}$ (absorption)

(a) Since $\lambda = \frac{hc}{E_\gamma} = \frac{hc}{-\Delta E}$, $\boxed{\text{transition II}}$ emits the shortest wavelength photon.

(b) The atom gains the most energy in $\boxed{\text{transition I}}$

(c) The atom loses energy in $\boxed{\text{transitions II and III}}$

**28.13** The energy absorbed by the atom is

$$E_\gamma = E_f - E_i = 13.6\text{ eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

(a) $E_\gamma = 13.6\text{ eV}\left(\frac{1}{9} - \frac{1}{25}\right) = \boxed{0.967\text{ eV}}$

(b) $E_\gamma = 13.6\text{ eV}\left(\frac{1}{25} - \frac{1}{49}\right) = \boxed{0.266\text{ eV}}$

**28.14** (a) The energy absorbed is

$$\Delta E = E_f - E_i = 13.6\text{ eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = 13.6\text{ eV}\left(\frac{1}{1} - \frac{1}{9}\right) = \boxed{12.1\text{ eV}}$$

(b) Three transitions are possible as the electron returns to the ground state. These transitions and the emitted photon energies are

$n_i = 3 \to n_f = 1$: $|\Delta E| = 13.6\text{ eV}\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \boxed{12.1\text{ eV}}$

$n_i = 3 \to n_f = 2$: $|\Delta E| = 13.6\text{ eV}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \boxed{1.89\text{ eV}}$

$n_i = 2 \to n_f = 1$: $|\Delta E| = 13.6\text{ eV}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \boxed{10.2\text{ eV}}$

**28.15** From $\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, it is seen that (for a fixed value of $n_f$) $\lambda_{max}$ occurs when $n_i = n_f + 1$ and $\lambda_{min}$ occurs when $n_i \to \infty$.

(a) For the Lyman series $(n_f = 1)$,

$$\frac{1}{\lambda_{max}} = \left(1.097\,37 \times 10^7\text{ m}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \to \lambda_{max} = 1.22 \times 10^{-7}\text{ m} = \boxed{122\text{ nm}}$$

and

$$\frac{1}{\lambda_{min}} = \left(1.097\,37 \times 10^7\text{ m}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{\infty}\right) \to \lambda_{min} = 9.11 \times 10^{-8}\text{ m} = \boxed{91.1\text{ nm}}$$

(b) For the Paschen series $(n_f = 3)$,

$$\frac{1}{\lambda_{max}} = \left(1.097\,37 \times 10^7\text{ m}^{-1}\right)\left(\frac{1}{3^2} - \frac{1}{4^2}\right) \to \lambda_{max} = 1.87 \times 10^{-6}\text{ m} = \boxed{1.87 \times 10^3\text{ nm}}$$

and

$$\frac{1}{\lambda_{min}} = \left(1.097\,37 \times 10^7\text{ m}^{-1}\right)\left(\frac{1}{3^2} - \frac{1}{\infty}\right) \to \lambda_{min} = 8.20 \times 10^{-7}\text{ m} = \boxed{820\text{ nm}}$$

**28.16** The electron is held in orbit by the electrical force the proton exerts on it. Thus,

$$m_e \frac{v^2}{r} = \frac{k_e e^2}{r^2} \qquad \text{or} \qquad v = \sqrt{\frac{k_e e^2}{m_e r}}$$

If we divide by the speed of light, and recognize that in the first Bohr orbit $r = a_0$ where $a_0 = 0.052\,9 \text{ nm}$, this becomes

$$\frac{v_1}{c} = \sqrt{\frac{k_e e^2}{m_e c^2 r_1}} = \sqrt{\frac{k_e e^2}{m_e c^2 a_0}} = \sqrt{\frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(9.11 \times 10^{-31} \text{ kg}\right)\left(3.00 \times 10^8 \text{ m/s}\right)^2 \left(0.052\,9 \times 10^{-9} \text{ m}\right)}}$$

or $\frac{v_1}{c} = 7.28 \times 10^{-3} = \frac{1}{137}$ Thus, $\boxed{v_1 = (1/137)c}$

**28.17** The batch of excited atoms must make these six transitions to get back to the ground state: $n_i = 2 \to n_f = 1$, also $n_i = 3 \to n_f = 2$ and $n_i = 3 \to n_f = 1$, and also $n_i = 4 \to n_f = 3$ and $n_i = 4 \to n_f = 2$ and $n_i = 4 \to n_f = 1$. Thus, the incoming light must have just enough energy to produce the $n_i = 1 \to n_f = 4$ transition. It must be the third line of the Lyman series in the absorption spectrum of hydrogen. The incoming photons must have wavelength given by

$$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = \frac{15 R_{\text{H}}}{16} \text{ or } \lambda = \frac{16}{15 R_{\text{H}}} = \frac{16}{15\left(1.097\,37 \times 10^7 \text{ m}^{-1}\right)} = \boxed{97.2 \text{ nm}}$$

**28.18** The magnetic force supplies the centripetal acceleration, so

$$\frac{mv^2}{r} = qvB\text{, or } r = \frac{mv}{qB}$$

If angular momentum is quantized according to

$$L_n = mv_n r_n = 2n\hbar\text{, then } mv_n = \frac{2n\hbar}{r_n}$$

and the allowed radii of the path are given by

$$r_n = \frac{1}{qB}\left(\frac{2n\hbar}{r_n}\right) \quad \text{or} \quad \boxed{r_n = \sqrt{\frac{2n\hbar}{qB}}}$$

**28.19** (a) The energy emitted by the atom is

$$\Delta E = E_4 - E_2 = -13.6\text{ eV}\left(\frac{1}{4^2} - \frac{1}{2^2}\right) = 2.55\text{ eV}$$

The wavelength of the photon produced is then

$$\lambda = \frac{hc}{E_\gamma} = \frac{hc}{\Delta E} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(2.55\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}$$

$$= 4.88\times10^{-7}\text{ m} = \boxed{488\text{ nm}}$$

(b) Since momentum must be conserved, the photon and the atom go in opposite directions with equal magnitude momenta. Thus, $p = m_{atom}v = h/\lambda$ or

$$v = \frac{h}{m_{atom}\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{\left(1.67\times10^{-27}\text{ kg}\right)\left(4.88\times10^{-7}\text{ m}\right)} = \boxed{0.814\text{ m/s}}$$

**28.20** (a) Starting from the $n=4$ state, there are 6 possible transitions as the electron returns to the ground $(n=1)$ state. These transitions are: $n=4\to n=1$, $n=4\to n=2$, $n=4\to n=3$, $n=3\to n=1$, $n=3\to n=2$, and $n=2\to n=1$. Since there is a different change in energy associated with each of these transitions there will be $\boxed{\text{6 different wavelengths}}$ observed in the emission spectrum of these atoms.

(b) The longest observed wavelength is produced by the transition involving the smallest change in energy. This is the $n=4\to n=3$ transition, and the wavelength is

$$\lambda_{\max} = \frac{hc}{E_4 - E_3} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.998\times10^{8}\text{ m/s}\right)}{-13.6\text{ eV}\left(\frac{1}{4^2} - \frac{1}{3^2}\right)}\left(\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)$$

or $\lambda_{\max} = \boxed{1.88\times10^{3}\text{ nm}}$

Since this transition terminates on the $n=3$ level, this is part of the $\boxed{\text{Paschen series}}$

**28.21** When the centripetal acceleration is supplied by the gravitational force,

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \text{ or } v^2 = \frac{GM}{r}$$

(a) With $PE = -GMm/r$, the total energy is

$$E = KE + PE = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{m}{2}\left(\frac{GM}{r}\right) - \frac{GMm}{r} = \boxed{-\frac{GMm}{2r}}$$

(b) Using the Bohr quantization rule, $L_n = mv_n r_n = n\hbar$, so $v_n = \frac{n\hbar}{mr_n}$ and

$$v^2 = \frac{GM}{r} \text{ becomes } \left(\frac{n\hbar}{mr_n}\right)^2 = \frac{GM}{r_n}$$

which reduces to $r_n = \frac{n^2\hbar^2}{GMm^2} = \boxed{n^2 r_0}$ with

$$r_0 = \frac{\hbar^2}{GMm^2}$$

$$= \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{4\pi^2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)^2}$$

$$r_0 = \boxed{2.32\times10^{-138}\ \text{m}}$$

(c) The energy in the $n^{\text{th}}$ orbit is $E_n = -\frac{GMm}{2r_n} = -\frac{GMm}{2}\left(\frac{GMm^2}{n^2\hbar^2}\right) = -\frac{E_0}{n^2}$, where

$$E_0 = \frac{G^2M^2m^3}{2\hbar^2}$$

$$= \frac{4\pi^2\left(6.67\times10^{-11}\right)^2\left(1.99\times10^{30}\right)^2\left(5.98\times10^{24}\right)^3}{2\left(6.63\times10^{-34}\right)^2} = \boxed{1.71\times10^{182}\ \text{J}}$$

(d) $r_n = n^2 r_0$, so $n^2 = \frac{r_n}{r_0} = \frac{1.49\times10^{11}\ \text{m}}{2.32\times10^{-138}\ \text{m}} = 6.42\times10^{148}$

or $n = \boxed{2.53\times10^{74}}$

(e) No, the quantum numbers are too large, and the allowed energies are essentially continuous in this region.

**28.22** (a) The time for one complete orbit is $T = \dfrac{2\pi r}{v}$

From Bohr's quantization postulate, $L = m_e v r = n\hbar$

we see that $v = \dfrac{n\hbar}{m_e r}$ Thus, the orbital period becomes:

$$T = \frac{2\pi m_e r^2}{n\hbar} = \frac{2\pi m_e \left(a_0 n^2\right)^2}{n\hbar} = \frac{2\pi m_e a_0^2}{\hbar} n^3$$

or $T = t_0 n^3$ where

$$t_0 = \frac{2\pi m_e a_0^2}{\hbar} = \frac{2\pi\left(9.11\times10^{-31}\text{ kg}\right)\left(0.052\,9\times10^{-9}\text{ m}\right)^2}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{1.52\times10^{-16}\text{ s}}$$

(b) With $n = 2$, we have $T = 8t_0 = 8\left(1.52\times10^{-16}\text{ s}\right) = 1.21\times10^{-15}\text{ s}$

Thus, if the electron stays in the $n = 2$ state for $10\ \mu\text{s}$, it will make

$$\frac{10.0\times10^{-6}\text{ s}}{1.21\times10^{-15}\text{ s/rev}} = \boxed{8.23\times10^{9}\text{ revolutions}}$$ of the nucleus

(c) $\boxed{\text{Yes, for } 8.23\times10^{9} \text{ “electron years”}}$

(d) The electron moves so quickly that it can never meaningfully be said to be on any particular side of the nucleus.

**28.23** (a) The wavelength emitted in the $n_i = 2 \rightarrow n_f = 1$ transition is

$$\lambda = \frac{1}{R_{\text{H}}}\left(\frac{n_i^2 n_f^2}{n_i^2 - n_f^2}\right) = \frac{1}{\left(1.097\,37\times10^{7}\text{ m}^{-1}\right)}\left(\frac{(4)(1)}{4-1}\right) = 1.22\times10^{-7}\text{ m}$$

and the frequency is $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\text{ m/s}}{1.22\times10^{-7}\text{ m}} = \boxed{2.47\times10^{15}\text{ Hz}}$

From $L_n = m_e v_n r_n = n\hbar$, the speed of the electron is $v_n = n\hbar/m_e r_n$

Therefore, with $r_n = n^2 a_0$, the orbital frequency is

$$f_{orb} = \frac{1}{T} = \frac{v_n}{r_n} = \frac{n\hbar}{2\pi m_e r_n^2} = \left(\frac{h}{4\pi^2 m_e a_0^2}\right)\frac{1}{n^3} = \frac{6.59\times10^{15}\ \text{Hz}}{n^3}$$

For the $n = 2$ orbit, $f_{orb} = \dfrac{6.59\times10^{15}\ \text{Hz}}{(2)^3} = \boxed{8.23\times10^{14}\ \text{Hz}}$

(b) For the $n_i = 10\,000 \rightarrow n_f = 9\,999$ transition,

$$\lambda = \frac{1}{\left(1.0977\,37\times10^7\ \text{m}^{-1}\right)}\left[\frac{(10\,000)^2(9\,999)^2}{(10\,000)^2-(9\,999)^2}\right] = 4.56\times10^4\ \text{m}$$

and $$f = \frac{c}{\lambda} = \frac{3.00\times10^8\ \text{m/s}}{4.56\times10^4\ \text{m}} = \boxed{6.59\times10^3\ \text{Hz}}$$

For the $n = 10\,000$ orbit, $f_{orb} = \dfrac{6.59\times10^{15}\ \text{Hz}}{(10\,000)^3} = \boxed{6.59\times10^3\ \text{Hz}}$

For small *n*, significant differences between classical and quantum results appear. However, as *n* becomes large, classical theory and quantum theory approach one another in their results. This is in agreement with the correspondence principle.

**28.24** Each atom gives up its kinetic energy in emitting a photon, so

$$KE = \frac{1}{2}mv^2 = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{121.6\times10^{-9}\ \text{m}} = 1.64\times10^{-18}\ \text{J}$$

$$v = \sqrt{\frac{2(KE)}{m_{atom}}} = \sqrt{\frac{2\left(1.64\times10^{-18}\ \text{J}\right)}{1.67\times10^{-27}\ \text{kg}}} = \boxed{4.43\times10^4\ \text{m/s}}$$

**28.25** For minimum initial kinetic energy, $KE_{total} = 0$ after collision. Hence, the two atoms must have equal and opposite momenta before impact. The atoms then have the same initial kinetic energy, and that energy is converted into excitation energy of the atom during the collision. Therefore,

$$KE_{atom} = \frac{1}{2} m_{atom} v^2 = E_2 - E_1 = 10.2 \text{ eV}$$

or $$v = \sqrt{\frac{2(10.2 \text{ eV})}{m_{atom}}} = \sqrt{\frac{2(10.2 \text{ eV})\left(1.60\times10^{-19} \text{ J/eV}\right)}{1.67\times10^{-27} \text{ kg}}} = \boxed{4.42\times10^{4} \text{ m/s}}$$

**28.26** (a) $L = mvr = m\left(\frac{2\pi r}{T}\right) r$

$$= \frac{2\pi\left(7.36\times10^{22} \text{ kg}\right)\left(3.84\times10^{8} \text{ m}\right)^2}{2.36\times10^{6} \text{ s}} = \boxed{2.89\times10^{34} \text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) $n = \frac{L}{\hbar} = \frac{2\pi L}{h} = \frac{2\pi\left(2.89\times10^{34} \text{ kg}\cdot\text{m}^2/\text{s}\right)}{6.63\times10^{-34} \text{ J}\cdot\text{s}} = \boxed{2.74\times10^{68}}$

(c) The gravitational force supplies the centripetal acceleration so

$$\frac{mv^2}{r} = \frac{GM_E m}{r^2}, \text{ or } rv^2 = GM_E$$

Then, from $L_n = mv_n r_n = n\hbar$ or $v_n = \frac{n\hbar}{mr_n}$,

we have $r_n\left(\frac{n\hbar}{mr_n}\right)^2 = GM_E$ which gives $r_n = n^2\left(\frac{\hbar^2}{GM_E m^2}\right) = n^2 r_1$

Therefore, when $n$ increases by 1, the fractional change in the radius is

$$\frac{\Delta r}{r} = \frac{r_{n+1} - r_n}{r_n} = \frac{(n+1)^2 r_1 - n^2 r_1}{n^2 r_1} = \frac{2n+1}{n^2} \approx \frac{2}{n}$$

$$\frac{\Delta r}{r} \approx \frac{2}{2.74\times10^{68}} = \boxed{7.30\times10^{-69}}$$

**28.27** (a) From $E_n = -\frac{Z^2(13.6 \text{ eV})}{n^2}$, $E_1 = -\frac{(3)^2(13.6 \text{ eV})}{(1)^2} = \boxed{-122 \text{ eV}}$

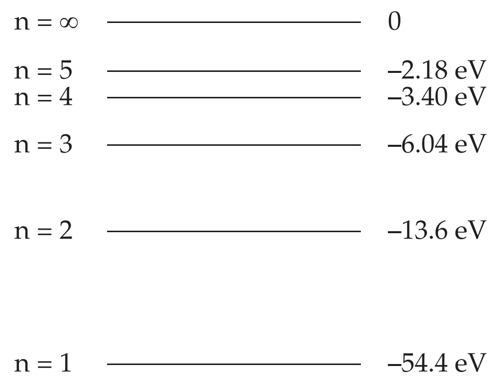
(b) Using $r_n = \dfrac{n^2 a_0}{Z}$ gives $r_1 = \dfrac{(1)^2 a_0}{3} = \dfrac{0.052\,9 \times 10^{-9}\text{ m}}{3} = \boxed{1.76 \times 10^{-11}\text{ m}}$

**28.28** (a) The energy levels of a hydrogen-like ion whose charge number is $Z$ are given by

$$E_n = (-13.6\text{ eV})\frac{Z^2}{n^2}$$

For Helium, $Z = 2$ and the energy levels are

$$\boxed{E_n = -\frac{54.4\text{ eV}}{n^2} \quad n = 1, 2, 3, \ldots}$$




(b) For $He^+$, $Z = 2$, so we see that the ionization energy (the energy required to take the electron from the $n = 1$ to the $n = \infty$ state) is

$$E = E_\infty - E_1 = 0 - \frac{(-13.6\text{ eV})(2)^2}{(1)^2} = \boxed{54.4\text{ eV}}$$

**28.29** $r_n = \dfrac{n^2}{Z}\left(\dfrac{\hbar^2}{m_e k_e e^2}\right) = \dfrac{n^2 a_0}{Z}$, so $r_1 = \dfrac{a_0}{Z} = \dfrac{0.052\,9\text{ nm}}{Z}$

(a) For $He^+$, $Z = 2$ and $r = \dfrac{0.052\,9\text{ nm}}{2} = \boxed{0.026\,5\text{ nm}}$

(b) For $Li^{2+}$, $Z = 3$ and $r = \dfrac{0.052\,9\text{ nm}}{3} = \boxed{0.017\,6\text{ nm}}$

(c) For $Be^{3+}$, $Z = 4$ and $r = \dfrac{0.052\,9\text{ nm}}{4} = \boxed{0.013\,2\text{ nm}}$

**28.30** (a) For hydrogen-like atoms having atomic number $Z$, the Rydberg constant is

$$R = \frac{m_e k_e^2 Z^2 e^4}{4\pi c \hbar^3}$$

Thus, for singly ionized helium with $Z = 2$, we have

$$R = \frac{(9.11 \times 10^{-31}\text{ kg})(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)^2 (2)^2 (1.60 \times 10^{-19}\text{ C})^4}{4\pi(3.00 \times 10^8)(1.05 \times 10^{-34}\text{ J}\cdot\text{s})^3} = \boxed{4.42 \times 10^7\text{ m}^{-1}}$$

(b) $$\frac{1}{\lambda} = R_{\text{He}}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(4.42\times10^7\ \text{m}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 3.32\times10^7\ \text{m}^{-1}$$

so $$\lambda = \left(\frac{1}{3.32\times10^7}\ \text{m}\right)\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = \boxed{30.1\ \text{nm}}$$

(c) This wavelength is in the deep $\boxed{\text{ultraviolet}}$ portion of the electromagnetic spectrum.

**28.31** From $L = m_e v_n r_n = n\hbar$ and $r_n = n^2 a_0$

we find that $p_n = mv_n = \dfrac{n\hbar}{r_n} = \dfrac{h}{(2\pi a_0)n}$

Thus, the de Broglie wavelength of the electron in the $n^{\text{th}}$ orbit is $\lambda = h/p_n = (2\pi a_0)n$. For $n = 4$, this yields

$$\lambda = 8\pi a_0 = 8\pi(0.052\,9\ \text{nm}) = \boxed{1.33\ \text{nm}}$$

**28.32** (a) For standing waves in a string fixed at both ends, $L = \dfrac{n\lambda}{2}$

or $\lambda = \dfrac{2L}{n}$. According to the de Broglie hypothesis, $p = \dfrac{h}{\lambda}$

Combining these expressions gives $p = mv = \boxed{\dfrac{nh}{2L}}$

(b) Using $E = \dfrac{1}{2}mv^2 = \dfrac{p^2}{2m}$, with $p$ as found in (a) above:

$$E_n = \frac{n^2h^2}{4L^2(2m)} = \boxed{n^2E_0 \quad \text{where } E_0 = \frac{h^2}{8mL^2}}$$

**28.33** In the 3$p$ subshell, $n = 3$ and $\ell = 1$. The 6 possible quantum states are

| | | | |
|---|---|---|---|
| $n = 3$ | $\ell = 1$ | $m_\ell = +1$ | $m_s = \pm\frac{1}{2}$ |
| $n = 3$ | $\ell = 1$ | $m_\ell = 0$ | $m_s = \pm\frac{1}{2}$ |
| $n = 3$ | $\ell = 1$ | $m_\ell = -1$ | $m_s = \pm\frac{1}{2}$ |

**28.34** (a) For a given value of the principle quantum number $n$, the orbital quantum number $\ell$ varies from 0 to $n-1$ in integer steps. Thus, if $n=4$, there are $\boxed{4}$ possible values of $\ell$: $\ell = 0, 1, 2,$ and 3

(b) For each possible value of the orbital quantum number $\ell$, the orbital magnetic quantum number $m_\ell$ ranges from $-\ell$ to $+\ell$ in integer steps. When the principle quantum number is $n=4$ and the largest allowed value of the orbital quantum number is $\ell = 3$, there are $\boxed{7}$ distinct possible values for $m_\ell$. These values are:

$$m_\ell = -3, -2, -1, 0, +1, +2, \text{ and } +3$$

**28.35** The 3$d$ subshell has $n=3$ and $\ell=2$. For $\rho$-mesons, we also have $s=1$. Thus, there are 15 possible quantum states as summarized in the table below.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| $\ell$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $m_\ell$ | +2 | +2 | +2 | +1 | +1 | +1 | 0 | 0 | 0 | -1 | -1 | -1 | -2 | -2 | -2 |
| $m_s$ | +1 | 0 | -1 | +1 | 0 | -1 | +1 | 0 | -1 | +1 | 0 | -1 | +1 | 0 | -1 |

**28.36** (a) The electronic configuration for oxygen $(Z=8)$ is $\boxed{1s^2\,2s^2\,2p^4}$

(b) The quantum numbers for the 8 electrons can be:

| | | | | |
|---|---|---|---|---|
| 1$s$ states | $n=1$ | $\ell=0$ | $m_\ell=0$ | $m_s=\pm\frac{1}{2}$ |
| 2$s$ states | $n=2$ | $\ell=0$ | $m_\ell=0$ | $m_s=\pm\frac{1}{2}$ |
| 2$p$ states | $n=2$ | $\ell=1$ | $m_\ell=0$<br>$m_\ell=1$ | $m_s=\pm\frac{1}{2}$<br>$m_s=\pm\frac{1}{2}$ |

**28.37** (a) For Electron #1 and also for Electron #2, $n=3$ and $\ell=1$. The other quantum numbers for each of the 30 allowed states are listed in the tables below.

| | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Electron #1 | +1 | $+\frac{1}{2}$ | +1 | $+\frac{1}{2}$ | +1 | $+\frac{1}{2}$ | +1 | $-\frac{1}{2}$ | +1 | $-\frac{1}{2}$ | +1 | $-\frac{1}{2}$ |
| Electron #2 | +1 | $-\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | -1 | $\pm\frac{1}{2}$ | +1 | $+\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | -1 | $\pm\frac{1}{2}$ |

| | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Electron #1 | 0 | $+\frac{1}{2}$ | 0 | $+\frac{1}{2}$ | 0 | $+\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $-\frac{1}{2}$ |
| Electron #2 | +1 | $\pm\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | -1 | $\pm\frac{1}{2}$ | +1 | $\pm\frac{1}{2}$ | 0 | $+\frac{1}{2}$ | -1 | $\pm\frac{1}{2}$ |

| | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ | $m_\ell$ | $m_s$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Electron #1 | -1 | $+\frac{1}{2}$ | -1 | $+\frac{1}{2}$ | -1 | $+\frac{1}{2}$ | -1 | $-\frac{1}{2}$ | -1 | $-\frac{1}{2}$ | -1 | $-\frac{1}{2}$ |
| Electron #2 | +1 | $\pm\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | -1 | $-\frac{1}{2}$ | +1 | $\pm\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | -1 | $+\frac{1}{2}$ |

There are $\boxed{\text{30 allowed states}}$, since Electron #1 can have any of three possible values of $m_\ell$ for both spin up and spin down, totaling six possible states. For each of these states, Electron #2 can be in either of the remaining five states.

(b) Were it not for the exclusion principle, there would be $\boxed{36}$ possible states, six for each electron independently.

**28.38** (a) For $n=1$, $\ell=0$ and there are $2(2\ell+1)$ states $=2(1)=\boxed{2}$ sets of quantum numbers

(b) For $n=2$, $\ell=0$ for $2(2\ell+1)$ states $=2(0+1)=2$ sets
and $\ell=1$ for $2(2\ell+1)$ states $=2(2+1)=6$ sets
total number of sets = $\boxed{8}$

(c) For $n=3$, $\ell=0$ for $2(2\ell+1)$ states $=2(0+1)=2$ sets
and $\ell=1$ for $2(2\ell+1)$ states $=2(2+1)=6$ sets
and $\ell=2$ for $2(2\ell+1)$ states $=2(4+1)=10$ sets
total number of sets = $\boxed{18}$

(d) For $n=4$, $\ell=0$ for $2(2\ell+1)$ states $=2(0+1)=2$ sets
and $\ell=1$ for $2(2\ell+1)$ states $=2(2+1)=6$ sets
and $\ell=2$ for $2(2\ell+1)$ states $=2(4+1)=10$ sets
and $\ell=3$ for $2(2\ell+1)$ states $=2(6+1)=14$ sets
total number of sets = $\boxed{32}$

(e) For $n=5$, $\ell=0$ for $2(2\ell+1)$ states $=2(0+1)=2$ sets
and $\ell=1$ for $2(2\ell+1)$ states $=2(2+1)=6$ sets
and $\ell=2$ for $2(2\ell+1)$ states $=2(4+1)=10$ sets
and $\ell=3$ for $2(2\ell+1)$ states $=2(6+1)=14$ sets
and $\ell=4$ for $2(2\ell+1)$ states $=2(8+1)=18$ sets
total number of sets = $\boxed{50}$

For $n=1$: $2n^2=2$ For $n=2$: $2n^2=8$
For $n=3$: $2n^2=18$ For $n=4$: $2n^2=32$
For $n=5$: $2n^2=50$

Thus, the number of sets of quantum states agrees with the $2n^2$ rule.

**28.39** (a) Zirconium, with 40 electrons, has 4 electrons outside a closed Krypton core. The Krypton core, with 36 electrons, has all states up through the $4p$ subshell filled. Normally, one would expect the next 4 electrons to go into the $4d$ subshell. However, an exception to the rule occurs at this point, and the $5s$ subshell fills (with 2 electrons) before the $4d$ subshell starts filling. The two remaining electrons in Zirconium are in an incomplete $4d$ subshell. Thus, $\boxed{n = 4, \text{ and } \ell = 2}$ for each of these electrons.

(b) For electrons in the $4d$ subshell, with $\ell = 2$, the possible values of $m_\ell$ are $\boxed{m_\ell = 0, \pm 1, \pm 2}$ and those for $m_s$ are $\boxed{m_s = \pm 1/2}$

(c) We have 40 electrons, so the electron configuration is:

$$\boxed{1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^{10}\,4s^2\,4p^6\,4d^2\,5s^2 = [\text{Kr}]4d^2\,5s^2}$$

**28.40** The photon energy is $E_\gamma = E_L - E_K = -951 \text{ eV} - (-8\,979 \text{ eV}) = 8\,028 \text{ eV}$, and the wavelength is

$$\lambda = \frac{hc}{E_\gamma} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(8\,028 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 1.55 \times 10^{-10} \text{ m} = \boxed{0.155 \text{ nm}}$$

To produce the $K_\alpha$ line, an electron from the K shell must be excited to the L shell or higher. Thus, a minimum energy of 8 028 eV must be given to the atom. A minimum accelerating voltage of $\Delta V = 8\,028 \text{ V} = \boxed{8.03 \text{ kV}}$ is required.

**28.41** For nickel, $Z = 28$ and

$$E_K \approx -(Z-1)^2 \frac{13.6 \text{ eV}}{(1)^2} = -(27)^2 (13.6 \text{ eV}) = -9.91\times10^3 \text{ eV}$$

$$E_L \approx -(Z-3)^2 \frac{13.6 \text{ eV}}{(2)^2} = -(25)^2 \frac{(13.6 \text{ eV})}{4} = -2.13\times10^3 \text{ eV}$$

Thus, $E_\gamma = E_L - E_K = -2.13 \text{ keV} - (-9.91 \text{ keV}) = 7.78 \text{ keV}$

and

$$\lambda = \frac{hc}{E_\gamma} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})}{7.78 \text{ keV}(1.60\times10^{-16} \text{ J/keV})} = 1.60\times10^{-10} \text{ m} = \boxed{0.160 \text{ nm}}$$

**28.42** The energies in the K and M shells are

$$E_K \approx -(Z-1)^2 \frac{13.6 \text{ eV}}{(1)^2} \text{ and } E_M \approx -(Z-9)^2 \frac{13.6 \text{ eV}}{(3)^2}$$

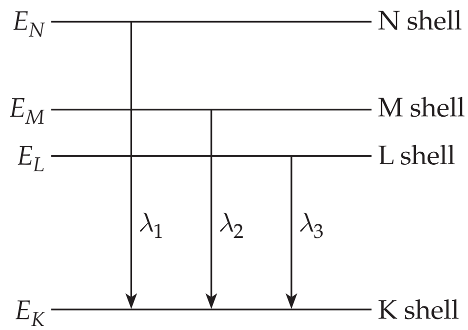
Thus, $E_\gamma = E_M - E_K \approx (13.6 \text{ eV})\left[-\frac{(Z-9)^2}{9} + (Z-1)^2\right] = (13.6 \text{ eV})\left(\frac{8}{9}Z^2 - 8\right)$

and $E_\gamma = \frac{hc}{\lambda}$ gives $Z^2 = \frac{9}{8}\left[8 + \frac{hc}{(13.6 \text{ eV})\lambda}\right]$, or

$$Z \approx \sqrt{9 + \frac{9(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})}{8(13.6 \text{ eV})(0.101\times10^{-9} \text{ m})}\left(\frac{1 \text{ eV}}{1.60\times10^{-19} \text{ J}}\right)} = 32.0$$

The element is $\boxed{\text{Germanium}}$

**28.43** The transitions that produce the three longest wavelengths in the K series are shown at the right. The energy of the K shell is $E_K = -69.5\ \text{keV}$.




Thus, the energy of the L shell is

$$E_L = E_K + \frac{hc}{\lambda_3}$$

or $$E_L = -69.5\ \text{keV} + \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{0.021\,5\times10^{-9}\ \text{m}}$$

$$= -69.5\ \text{keV} + 9.25\times10^{-15}\ \text{J}\left(\frac{1\ \text{keV}}{1.60\times10^{-16}\ \text{J}}\right)$$

$$= -69.5\ \text{keV} + 57.8\ \text{keV} = -11.7\ \text{keV}$$

Similarly, the energies of the M and N shells are

$$E_M = E_K + \frac{hc}{\lambda_2}$$

$$= -69.5\ \text{keV} + \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(0.020\,9\times10^{-9}\ \text{m})(1.60\times10^{-16}\ \text{J/keV})} = -10.0\ \text{keV}$$

and

$$E_N = E_K + \frac{hc}{\lambda_1}$$

$$= -69.5\ \text{keV} + \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(0.018\,5\times10^{-9}\ \text{m})(1.60\times10^{-16}\ \text{J/keV})} = -2.30\ \text{keV}$$

The ionization energies of the L, M, and N shells are

11.7 keV, 10.0 keV, and 2.30 keV respectively

**28.44** According to the Bohr model, the radii of the electron orbits in hydrogen are given by

$$r_n = n^2 a_0 \text{ with } a_0 = 0.052\,9 \text{ nm} = 5.29\times10^{-11} \text{ m}$$

Then, if $r_n \approx 1.00\ \mu\text{m} = 1.00\times10^{-6}\text{ m}$ , the quantum number is

$$n = \sqrt{\frac{r_n}{a_0}} = \sqrt{\frac{1.00\times10^{-6}\text{ m}}{5.29\times10^{-11}\text{ m}}} \approx \boxed{137}$$

**28.45** (a) $\Delta E = E_2 - E_1 = -13.6\text{ eV}/(2)^2 - \left(-13.6\text{ eV}/(1)^2\right) = \boxed{10.2\text{ eV}}$

(b) The average kinetic energy of the atoms must equal or exceed the needed excitation energy, or $\frac{3}{2}k_B T \ge \Delta E$ which gives

$$T \ge \frac{2(\Delta E)}{3k_B} = \frac{2(10.2\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}{3\left(1.38\times10^{-23}\text{ J/K}\right)} = \boxed{7.88\times10^4\text{ K}}$$

**28.46** (a) $L = c(\Delta t) = \left(3.00\times10^8\text{ m/s}\right)\left(14.0\times10^{-12}\text{ s}\right) = 4.20\times10^{-3}\text{ m} = \boxed{4.20\text{ mm}}$

(b) $N = \dfrac{E_{pulse}}{E_\gamma} = \dfrac{E_{pulse}}{hc/\lambda}$

$$= \frac{\left(694.3\times10^{-9}\text{ m}\right)(3.00\text{ J})}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^8\text{ m/s}\right)} = \boxed{1.05\times10^{19}\text{ photons}}$$

(c) $n = \dfrac{N}{V} = \dfrac{N}{L\left(\pi d^2/4\right)}$

$$= \frac{4\left(1.05\times10^{19}\text{ photons}\right)}{(4.20\text{ mm})\pi(6.00\text{ mm})^2} = \boxed{8.82\times10^{16}\text{ photons/mm}^3}$$

**28.47** (a) $$E_1 = E_\infty - \frac{hc}{\lambda_{limit}} = 0 - \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{152.0\times10^{-9}\ \text{m}\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{-8.18\ \text{eV}}$$

$$E_2 = E_1 + \frac{hc}{\lambda_1}$$

$$= -8.18\ \text{eV} + \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{202.6\times10^{-9}\ \text{m}\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{-2.04\ \text{eV}}$$

$$E_3 = E_1 + \frac{hc}{\lambda_2}$$

$$= -8.18\ \text{eV} + \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{170.9\times10^{-9}\ \text{m}\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{-0.904\ \text{eV}}$$

$$E_4 = E_1 + \frac{hc}{\lambda_3}$$

$$= -8.18\ \text{eV} + \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{162.1\times10^{-9}\ \text{m}\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{-0.510\ \text{eV}}$$

$$E_5 = E_1 + \frac{hc}{\lambda_4}$$

$$= -8.18\ \text{eV} + \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{158.3\times10^{-9}\ \text{m}\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{-0.325\ \text{eV}}$$

(b) From $\lambda = \frac{hc}{E_\gamma} = \frac{hc}{E_i - E_f}$, the longest and shortest wavelengths in the Balmer series for this atom are

$$\lambda_{long} = \frac{hc}{E_3 - E_2} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left[-0.904\ \text{eV} - \left(-2.04\ \text{eV}\right)\right]\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{1.09\times10^{3}\ \text{nm}}$$

and $$\lambda_{short} = \frac{hc}{E_\infty - E_2} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left[0 - \left(-2.04\ \text{eV}\right)\right]\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{609\ \text{nm}}$$

**28.48** (a) $$\frac{1}{\alpha}=\frac{\hbar c}{k_e e^2}=\frac{hc}{2\pi k_e e^2}=\frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\ \text{m/s}\right)}{2\pi\left(8.987\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602\times10^{-19}\ \text{C}\right)^2}=\boxed{137}$$

(b) $$\frac{a_0}{\lambda_C}=\frac{\hbar^2/m_e k_e e^2}{h/m_e c}=\frac{1}{2\pi}\left(\frac{\hbar c}{k_e e^2}\right)=\boxed{\frac{1}{2\pi\alpha}}$$

(c) $$\frac{1/R_H}{a_0}=\frac{4\pi\hbar^3 c/m_e k_e^2 e^4}{\hbar^2/m_e k_e e^2}=4\pi\left(\frac{\hbar c}{k_e e^2}\right)=\boxed{\frac{4\pi}{\alpha}}$$

**28.49** (a) $$E_\gamma=\frac{hc}{\lambda}=\frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\ \text{m/s}\right)}{\lambda\left(1.602\times10^{-19}\ \text{J/eV}\right)\left(10^{-9}\ \text{m/nm}\right)}=\frac{1\,240\ \text{eV}\cdot\text{nm}}{\lambda}$$

For:

$\lambda=253.7$ nm, $E_\gamma=4.888$ eV ;

$\lambda=185.0$ nm, $E_\gamma=6.703$ eV ;

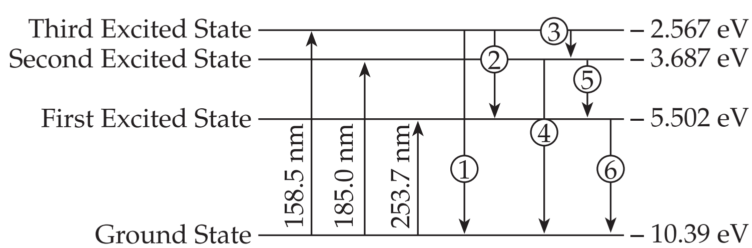
$\lambda=158.5$ nm, $E_\gamma=7.823$ eV

Thus, the energies of the first three excited states are:

$$E_1=-10.39\ \text{eV}+4.888\ \text{eV}=\boxed{-5.502\ \text{eV}}$$

$$E_2=-10.39\ \text{eV}+6.703\ \text{eV}=\boxed{-3.687\ \text{eV}}$$

and $$E_3=-10.39\ \text{eV}+7.823\ \text{eV}=\boxed{-2.567\ \text{eV}}$$



(b) From $\lambda = (1\,240 \text{ eV}\cdot\text{nm})/(E_i - E_f)$, the wavelengths of the emission lines shown are

$$\lambda_1 = \boxed{158.5 \text{ nm}},\ \lambda_2 = \boxed{422.5 \text{ nm}},\ \lambda_3 = \boxed{1\,107 \text{ nm}},\ \lambda_4 = \boxed{185.0 \text{ nm}},$$

$$\lambda_5 = \boxed{683.2 \text{ nm}}, \text{ and } \lambda_6 = \boxed{253.7 \text{ nm}}$$

(c) To have an inelastic collision, we must excite the atom from the ground state to the first excited state, so the incident electron must have a kinetic energy of at least $KE = 10.39 \text{ eV} - 5.502 \text{ eV} = 4.888 \text{ eV}$,

$$\text{so } v = \sqrt{\frac{2(KE)}{m_e}} = \sqrt{\frac{2(4.888 \text{ eV})(1.602\times10^{-19} \text{ J/eV})}{9.11\times10^{-31} \text{ kg}}} = \boxed{1.31\times10^{6} \text{ m/s}}$$

**28.50** $$E_\gamma = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^{8} \text{ m/s})}{\lambda(1.602\times10^{-19} \text{ J/eV})(10^{-9} \text{ m/nm})} = \frac{1\,240 \text{ eV}\cdot\text{nm}}{\lambda} = \Delta E$$

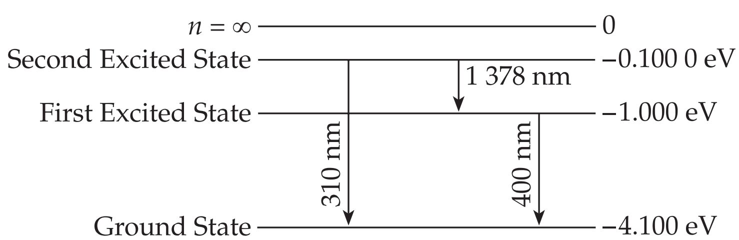
For:

$\lambda = 310.0 \text{ nm}$, $\Delta E = 4.000 \text{ eV}$

$\lambda = 400.0 \text{ nm}$, $\Delta E = 3.100 \text{ eV}$

and $\lambda = 1\,378 \text{ nm}$, $\Delta E = 0.900\,0 \text{ eV}$

The ionization energy is 4.100 eV. The energy level diagram having the fewest number of levels and consistent with these energy differences is shown below.




**28.51** (a) $$I = \frac{\mathcal{P}}{A} = \frac{(\Delta E/\Delta t)}{\pi d^2/4} = \frac{4(3.00\times10^{-3} \text{ J}/1.00\times10^{-9} \text{ s})}{\pi(30.0\times10^{-6} \text{ m})^2} = \boxed{4.24\times10^{15} \text{ W/m}^2}$$

(b) $E = IA(\Delta t)$

$$= \left(4.24\times10^{15} \frac{\text{W}}{\text{m}^2}\right)\left[\frac{\pi}{4}(0.600\times10^{-9} \text{ m})^2\right](1.00\times10^{-9} \text{ s}) = \boxed{1.20\times10^{-12} \text{ J}}$$

**28.52** (a) Given that the de Broglie wavelength is $\lambda = 2a_0$, the momentum is $p = h/\lambda = h/2a_0$. The kinetic energy of this non-relativistic electron is

$$KE = \frac{p^2}{2m_e} = \frac{h^2}{8m_e a_0^2}$$

$$= \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2\left(1\ \text{eV}/1.60\times10^{-19}\ \text{J}\right)}{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.052\,9\times10^{-9}\ \text{m}\right)^2} = \boxed{135\ \text{eV}}$$

(b) The kinetic energy of this electron is $\boxed{\approx 10\ \text{times}}$ the magnitude of the ground state energy of the hydrogen atom which is –13.6 eV.

**28.53** In the Bohr model,

$$f = \frac{\Delta E}{h} = \frac{E_n - E_{n-1}}{h}$$

$$= \frac{1}{h}\left[\frac{-m_e k_e^2 e^4}{2\hbar^2}\left(\frac{1}{n^2} - \frac{1}{(n-1)^2}\right)\right] = \frac{4\pi^2 m_e k_e^2 e^4}{2h^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

which reduces to $\boxed{f = \frac{2\pi^2 m_e k_e^2 e^4}{h^3}\left(\frac{2n-1}{(n-1)^2 n^2}\right)}$

**28.54** As $n\to\infty$, $2n-1\to 2n$ and $n-1\to n$. In this limit, the result of Problem 28.53 reduces to

$$f = \frac{2\pi^2 m_e k_e^2 e^4}{h^3}\left(\frac{2n}{n^4}\right) = \boxed{\frac{4\pi^2 m_e k_e^2 e^4}{h^3 n^3}}$$

Since the electrical force supplies the centripetal acceleration,

$$\frac{m_e v^2}{r} = \frac{k_e e^2}{r^2} \quad \text{or} \quad v = \sqrt{\frac{k_e e^2}{m_e r}}$$

The classical frequency is then

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi}\sqrt{\frac{k_e e^2}{m_e r^3}} \text{ where } r = n^2 a_0 = \frac{n^2 h^2}{4\pi^2 m_e k_e e^2}$$

This gives $$f = \frac{v}{2\pi r} = \sqrt{\frac{k_e e^2}{4\pi^2 m_e}\left(\frac{64\pi^6 m_e^3 k_e^3 e^6}{n^6 h^6}\right)} = \boxed{\frac{4\pi^2 m_e k_e^2 e^4}{h^3 n^3}}$$

Thus, the frequency from the Bohr model is the same as the classical frequency in the limit $n \to \infty$.

**28.55** (a) The energy levels in this atom are

$$E_n = -\frac{m_\pi Z^2 k_e^2 e^4}{2\hbar^2 n^2}$$

$$= -\frac{273(2)^2}{n^2}\left(\frac{m_e k_e^2 e^4}{2\hbar^2}\right) = -\frac{273}{n^2}\left[(2)^2(13.60 \text{ eV})\right] = \boxed{\frac{-1.485\times10^4 \text{ eV}}{n^2}}$$

The energies of the first six levels are:

$E_1 = -1.485\times10^4$ eV $\quad E_2 = -3.71\times10^3$ eV $\quad E_3 = -1.65\times10^3$ eV

$E_4 = -928$ eV $\quad E_5 = -594$ eV $\quad E_6 = -413$ eV

(b) From the Compton shift formula, the emitted wavelength was

$$\lambda_0 = \lambda' - \lambda_C(1-\cos\theta) = 0.089\,929\,3 \text{ nm} - (0.002\,43 \text{ nm})(1-\cos 42.68°)$$

$$= 0.089\,289 \text{ nm}$$

The energy radiated by the atom is then

$$\Delta E = E_i - E_f = \frac{hc}{\lambda_0}$$

$$= \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})}{(0.089\,289\times10^{-9} \text{ m})(1.60\times10^{-19} \text{ J/eV})} = 1.392\times10^4 \text{ eV}$$

Since $\Delta E > |E_2|$, the final state must be the ground state $E_1$. The energy of the initial state was

$$E_i = E_f + \Delta E = -1.485 \times 10^4 \text{ eV} + 1.392 \times 10^4 \text{ eV} = -928 \text{ eV}$$

This is seen to be $E_4$. Thus, the transition made by the pi meson was $\boxed{n = 4 \to n = 1}$

**28.56** (a) Using $a_0 = \dfrac{\hbar^2}{m_\mu k_e e^2}$, with $m_\mu = 207\, m_e$, gives the Bohr radius for the "muonic atom" as

$$a_0 = \frac{1}{207}\left(\frac{\hbar^2}{m_e k_e e^2}\right) = \frac{1}{207}(0.052\,9 \text{ nm}) = \boxed{2.56 \times 10^{-4} \text{ nm}}$$

(b) The energy levels in this atom are

$$E_n = -\frac{m_\mu Z^2 k_e^2 e^4}{2\hbar^2 n^2} = -\frac{207(1)^2}{n^2}\left(\frac{m_e k_e^2 e^4}{2\hbar^2}\right) = -\frac{207}{n^2}(13.6 \text{ eV}) = \frac{-2.82 \times 10^3 \text{ eV}}{n^2}$$

The energies of the three lowest levels are:

$$E_1 = \boxed{-2.82 \times 10^3 \text{ eV}} \qquad E_2 = \boxed{-704 \text{ eV}} \qquad E_3 = \boxed{-313 \text{ eV}}$$

**28.57** (a) From Newton's second law, $F = k_e |q_1 q_2| / r^2 = m_e a$, and the acceleration is

$$a = \frac{F}{m_e} = \frac{k_e e^2}{m_e a_0^2}$$

$$= \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(0.052\,9 \times 10^{-9} \text{ m})^2} = \boxed{9.03 \times 10^{22} \text{ m/s}^2}$$

(b) $\mathcal{P} = -\dfrac{2k_e e^2 a^2}{3c^3}$

$$= -\frac{2(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2 (9.03 \times 10^{22} \text{ m/s}^2)^2}{3(3.00 \times 10^8 \text{ m/s})^3}$$

$$\mathcal{P} = -4.63 \times 10^{-8} \text{ J/s} = \boxed{-4.63 \times 10^{-8} \text{ W}}$$

(c) With the electrical force supplying the centripetal acceleration,

$$\frac{m_e v^2}{r} = \frac{k_e e^2}{r^2} \text{ or } m_e v^2 = \frac{k_e e^2}{r} \text{ and } KE = \frac{1}{2} m_e v^2 = \frac{k_e e^2}{2r}$$

Thus,

$$KE = \frac{k_e e^2}{2a_0} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19} \text{ C}\right)^2}{2\left(0.052\,9\times10^{-9} \text{ m}\right)} = 2.17\times10^{-18} \text{ J}$$

The time required to radiate all this energy, and the estimated lifetime is

$$\Delta t = \frac{KE}{|\mathcal{P}|} = \frac{2.17\times10^{-18} \text{ J}}{4.63\times10^{-8} \text{ J/s}} = 4.69\times10^{-11} \text{ s or } \boxed{\Delta t \sim 10^{-11} \text{ s}}$$

**28.58** (a) The photon emitted by the hydrogen atom must have an energy $E_\gamma \geq 4.58$ eV if it is to eject a photoelectron from tungsten $(\phi = 4.58 \text{ eV})$. Thus, the electron in the hydrogen atom must give up at least 4.58 eV of energy, meaning that the energy of the final state must be $E_f \leq -4.58$ eV. The only state in the hydrogen atom satisfying this condition is the ground $(n=1)$ state, so it is necessary that $\boxed{n_f = 1}$

(b) If the stopping potential of the ejected photoelectron is $V_s = 7.51$ V, the kinetic energy of this electron as it leaves the tungsten is

$$KE_{\max} = eV_s = e(7.51 \text{ V}) = 7.51 \text{ eV}$$

and the photoelectric effect equation gives the photon energy as

$$E_\gamma = hf = \phi + KE_{\max} = 4.58 \text{ eV} + 7.51 \text{ eV} = 12.09 \text{ eV}$$

But, the photon energy equals the energy given up by the electron in the hydrogen atom. That is, $E_\gamma = E_i - E_f$. Since we determined in Part (a) that $n_f = 1$, then $E_f = -13.6$ eV and we have

$$E_\gamma = E_i - E_f = E_i + 13.6 \text{ eV} = 12.09 \text{ eV} \quad \text{or} \quad E_i = 12.09 \text{ eV} - 13.6 \text{ eV} = -1.51 \text{ eV}$$

Thus, from $E_n = \dfrac{-13.6 \text{ eV}}{n^2}$, $\quad n_i = \sqrt{\dfrac{-13.6 \text{ eV}}{E_i}} = \sqrt{\dfrac{-13.6 \text{ eV}}{-1.51 \text{ eV}}} = \boxed{3}$