Chapter 27
Quantum Physics

Quick Quizzes

1. (b). Some energy is transferred to the electron in the scattering process. Therefore, the scattered photon must have less energy (and hence, lower frequency) than the incident photon.

2. (c). Conservation of energy requires the kinetic energy given to the electron be equal to the difference between the energy of the incident photon and that of the scattered photon.

3. (c). Two particles with the same de Broglie wavelength will have the same momentum $p = mv$. If the electron and proton have the same momentum, they cannot have the same speed because of the difference in their masses. For the same reason, remembering that $KE = p^2/2m$, they cannot have the same kinetic energy. Because the kinetic energy is the only type of energy an isolated particle can have, and we have argued that the particles have different energies, the equation $f = E/h$ tells us that the particles do not have the same frequency.

4. (b). The Compton wavelength, $\lambda_C = h/m_e c$, is a combination of constants and has no relation to the motion of the electron. The de Broglie wavelength, $\lambda = h/m_e v$, is associated with the motion of the electron through its momentum.

Answers to Even Numbered Conceptual Questions

2. A microscope can see details no smaller than the wavelength of the waves it uses to produce images. Electrons with kinetic energies of several electron volts have wavelengths of less than a nanometer, which is much smaller than the wavelength of visible light (having wavelengths ranging from about 400 to 700 nm). Therefore, an electron microscope can resolve details of much smaller sizes as compared to an optical microscope.

4. Measuring the position of a particle implies having photons reflect from it. However, collisions between photons and the particle will alter the velocity of the particle.

6. Light has both wave and particle characteristics. In Young's double-slit experiment, light behaves as a wave. In the photoelectric effect, it behaves like a particle. Light can be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time, light can be characterized as a stream of photons, each carrying a discrete energy, hf.

8. **(a)** particle. Light behaves like a tiny, localized packet of energy, capable of being totally absorbed by a single electron.

(b) particle. Data from Compton scattering experiments can be fully explained by treating the scattering like a collision between two particles, conserving both energy and momentum.

(c) wave. The observed patterns when light passes through a pair of parallel slits have the same characteristics as the diffraction and interference patterns formed by water waves passing through closely spaced openings.

10. Ultraviolet light has a shorter wavelength and higher photon energy than visible light.

12. Increasing the temperature of the substance increases the average kinetic energy of the electrons inside the material. This makes it slightly easier for an electron to escape from the material when it absorbs a photon.

14. Most stars radiate nearly as blackbodies. Vega has a higher surface temperature than Arcturus. Vega radiates more intensely at shorter wavelengths.

16. The red beam. Each photon of red light has less energy (longer wavelength) than a photon of blue light, so the red beam must contain more photons to carry the same total energy.

Answers to Even Numbered Problems

2. **(a)** ~100 nm, ultraviolet **(b)** ~0.1 nm, γ-rays

4. red beam, 1.39×10^{3} photons ; blue beam, 845 photons

6. 9.66×10^{3} K

8. 5.7×10^{3} photons/sec

10. **(a)** 4.2×10^{35} **(b)** 3.3×10^{-34} J

12. 5.4 eV

14. **(a)** only lithium **(b)** 0.81 eV

16. 8.43×10^{-12} C

18. 8.7×10^{12} electrons/s

20. **(a)** 8.29×10^{-11} m **(b)** 1.24×10^{-11} m

22. 0.124 nm

24. 6.7°

26. 67.5°

28. 1.8 keV, 9.7×10^{-25} $\text{kg}\cdot\text{m/s}$

30. **(a)** 4.89×10^{-4} nm **(b)** 268 keV **(c)** 32 keV

32. **(a)** 0.002 87 nm **(b)** 100°

34. **(a)** 1.98×10^{-11} m **(b)** 1.98×10^{-14} m

36. 1.06×10^{-34} m

38. **(a)** $\sim10^{-34}$ m/s **(b)** $\sim10^{33}$ s
(c) No. The minimum transit time that could produce significant diffraction is $\sim10^{15}$ times the age of the Universe.

40. 547 eV

42. 3.5×10^{-32} m

44. **(a)** $0.250\ \text{m/s}$ **(b)** 2.25 m

46. **(b)** 5.2 MeV

48. **(a)** $2.49\times10^{-11}\ \text{m}$ **(b)** 0.285 nm

50. **(a)** $2.82\times10^{-37}\ \text{m}$ **(b)** $1.06\times10^{-32}\ \text{J}$ **(c)** $2.87\times10^{-37}\ \%$ or more

52. 191 MeV

54. 0.785 eV

56. **(a)** $v\geq0.999\,9c$ **(b)** $v\geq0.053c$

58. **(a)** $7.77\times10^{-12}\ \text{m}$ **(b)** 93.8° **(c)** 35.5

Problem Solutions

27.1 From Wien's displacement law,

(a) $$T=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{\lambda_{\max}}=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{970\times10^{-9}\text{ m}}$$

$$=2.99\times10^{3}\text{ K, or }\boxed{\approx 3\,000\text{ K}}$$

(b) $$T=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{\lambda_{\max}}=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{145\times10^{-9}\text{ m}}$$

$$=2.00\times10^{4}\text{ K, or }\boxed{\approx 20\,000\text{ K}}$$

27.2 Using Wien's displacement law,

(a) $$\lambda_{\max}=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{10^{4}\text{ K}}$$

$$=2.898\times10^{-7}\text{ m}\ \boxed{\sim100\text{ nm}}\ \boxed{\text{Ultraviolet}}$$

(b) $$\lambda_{\max}=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{10^{7}\text{ K}}=2.898\times10^{-10}\text{ m}\ \boxed{\sim10^{-1}\text{ nm}}\ \boxed{\gamma\text{-rays}}$$

27.3 The wavelength of maximum radiation is given by

$$\lambda_{\max}=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{5\,800\text{ K}}=5.00\times10^{-7}\text{ m}=\boxed{500\text{ nm}}$$

27.4 The energy of a photon having wavelength λ is $E_\gamma=hf=hc/\lambda$. Thus, the number of photons delivered by each beam must be:

Red Beam:

$$n_R=\frac{E_{total}}{E_{\gamma,R}}=\frac{E_{total}\lambda_R}{hc}=\frac{(2\,500\text{ eV})(690\times10^{-9}\text{ m})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)=\boxed{1.39\times10^{3}}$$

Blue Beam:

$$n_B=\frac{E_{total}}{E_{\gamma,B}}=\frac{E_{total}\lambda_B}{hc}=\frac{(2\,500\text{ eV})(420\times10^{-9}\text{ m})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)=\boxed{845}$$

27.5 $E = hf = \dfrac{hc}{\lambda} = \dfrac{\left(6.63\times10^{-34}\ \mathrm{J\cdot s}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)}{\lambda}\left(\dfrac{1.00\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right)f$

which yields $E = \dfrac{1.24\times10^{-6}\ \mathrm{m\cdot eV}}{\lambda}$

(a) $E = \dfrac{1.24\times10^{-6}\ \mathrm{m\cdot eV}}{5.00\times10^{-2}\ \mathrm{m}} = \boxed{2.49\times10^{-5}\ \mathrm{eV}}$

(b) $E = \dfrac{1.24\times10^{-6}\ \mathrm{m\cdot eV}}{500\times10^{-9}\ \mathrm{m}} = \boxed{2.49\ \mathrm{eV}}$

(c) $E = \dfrac{1.24\times10^{-6}\ \mathrm{m\cdot eV}}{5.00\times10^{-9}\ \mathrm{m}} = \boxed{249\ \mathrm{eV}}$

27.6 The wavelength of the peak radiation is

$$\lambda_{\mathrm{max}} = \frac{c}{f_{\mathrm{max}}} = \frac{3.00\times10^{8}\ \mathrm{m/s}}{1.00\times10^{15}\ \mathrm{Hz}} = 3.00\times10^{-7}\ \mathrm{m} = 300\ \mathrm{nm} \qquad \text{(ultra-violet)}$$

Wien's displacement law then gives the temperature of the blackbody radiator as

$$T = \frac{0.289\,8\times10^{-2}\ \mathrm{m\cdot K}}{\lambda_{\mathrm{max}}} = \frac{0.289\,8\times10^{-2}\ \mathrm{m\cdot K}}{3.00\times10^{-7}\ \mathrm{m}} = \boxed{9.66\times10^{3}\ \mathrm{K}}$$

27.7 The energy of a single photon is

$$E_\gamma = hf = \left(6.63\times10^{-34}\ \mathrm{J\cdot s}\right)\left(99.7\times10^{6}\ \mathrm{s^{-1}}\right) = 6.61\times10^{-26}\ \mathrm{J}$$

The number of photons emitted in $\Delta t = 1.00\ \mathrm{s}$ is

$$N = \frac{\Delta E}{E_\gamma} = \frac{\mathcal{P}\cdot(\Delta t)}{E_\gamma} = \frac{\left(150\times10^{3}\ \mathrm{J/s}\right)\left(1.00\ \mathrm{s}\right)}{6.61\times10^{-26}\ \mathrm{J}} = \boxed{2.27\times10^{30}}$$

27.8 The energy entering the eye each second is

$$\mathcal{P} = I \cdot A = \left(4.0\times10^{-11}\ \text{W/m}^2\right)\left[\frac{\pi}{4}\left(8.5\times10^{-3}\ \text{m}\right)^2\right] = 2.3\times10^{-15}\ \text{W}$$

The energy of a single photon is

$$E_\gamma = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{500\times10^{-9}\ \text{m}} = 3.98\times10^{-19}\ \text{J}$$

so the number of photons entering the eye in $\Delta t = 1.00\ \text{s}$ is

$$N = \frac{\Delta E}{E_\gamma} = \frac{\mathcal{P}\cdot(\Delta t)}{E_\gamma} = \frac{\left(2.3\times10^{-15}\ \text{J/s}\right)(1.00\ \text{s})}{3.98\times10^{-19}\ \text{J}} = \boxed{5.7\times10^{3}}$$

27.9 The frequency of the oscillator is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{20\ \text{N/m}}{1.5\ \text{kg}}} = 0.58\ \text{Hz}$

and its total energy is $E = \dfrac{1}{2}kA^2 = \dfrac{(20\ \text{N/m})\left(3.0\times10^{-2}\ \text{m}\right)^2}{2} = 9.0\times10^{-3}\ \text{J}$

(a) From $E = nhf$, the quantum number is

$$n = \frac{E}{hf} = \frac{9.0\times10^{-3}\ \text{J}}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(0.58\ \text{s}^{-1}\right)} = \boxed{2.3\times10^{31}}$$

(b) $\Delta E = (\Delta n)hf$, so the fractional change in energy is

$$\frac{\Delta E}{E} = \frac{(1)\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(0.58\ \text{s}^{-1}\right)}{9.0\times10^{-3}\ \text{J}} = \boxed{4.3\times10^{-32}}$$

27.10 (a) The total energy of the jungle hero is $E = \dfrac{1}{2}mv_{\text{max}}^2 = \dfrac{1}{2}(70.0\ \text{kg})(2.0\ \text{m/s})^2 = 1.4\times10^{2}\ \text{J}$

According to Planck's hypotheses, the energy of a harmonic oscillator is quantized according to $E_n = nhf$, where *f* is the frequency of oscillation. Hence, the quantum number for this system is

$$n = \frac{E}{hf} = \frac{1.4\times10^{2}\ \text{J}}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)(0.50\ \text{Hz})} = \boxed{4.2\times10^{35}}$$

(b) $\Delta E = (\Delta n)hf = (1)\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)(0.50\ \text{Hz}) = \boxed{3.3\times10^{-34}\ \text{J}}$

27.11 (a) From the photoelectric effect equation, the work function is

$$\phi = \frac{hc}{\lambda} - KE_{\text{max}}\text{, or}$$

$$\phi = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{350\times10^{-9}\ \text{m}}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) - 1.31\ \text{eV}$$

$$\phi = \boxed{2.24\ \text{eV}}$$

(b) $$\lambda_c = \frac{hc}{\phi} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{2.24\ \text{eV}}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{555\ \text{nm}}$$

(c) $$f_c = \frac{c}{\lambda_c} = \frac{3.00\times10^{8}\ \text{m/s}}{555\times10^{-9}\ \text{m}} = \boxed{5.41\times10^{14}\ \text{Hz}}$$

27.12 From Einstein's photoelectric effect equation,

$$e\left(\Delta V_s\right) = KE_{\text{max}} = hf - \phi$$

Thus, $$\phi = hf - e\left(\Delta V_s\right) = \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.0\times10^{15}\ \text{Hz}\right)\left(\frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{eV}}\right) - e(7.0\ \text{V})$$

or $$\phi = 12.4\ \text{eV} - 7.0\ \text{eV} = \boxed{5.4\ \text{eV}}$$

27.13 From Einstein's photoelectric effect equation,

$$KE_{\text{max}} = hf - \phi \qquad \text{or} \qquad KE_{\text{max}} = \frac{hc}{\lambda} - \phi$$

Thus, $$\lambda = \frac{hc}{KE_{\text{max}} + \phi} = \frac{hc}{\frac{1}{2}m_e v_{\text{max}}^2 + \phi}$$

or $$\lambda = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\frac{1}{2}\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.00\times10^{6}\ \text{m/s}\right)^2 + (2.46\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}$$

$$\lambda = 2.34\times10^{-7}\ \text{m} = \boxed{234\ \text{nm}}$$

27.14 (a) The energy of the incident photons is

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{400\times10^{-9}\ \text{m}}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = 3.11\ \text{eV}$$

For photo-electric emission to occur, it is necessary that $E_\gamma \geq \phi$. Thus, of the three metals given, $\boxed{\text{only lithium}}$ will exhibit the photo-electric effect.

(b) For lithium, $KE_{\max} = \dfrac{hc}{\lambda} - \phi = 3.11\ \text{eV} - 2.30\ \text{eV} = \boxed{0.81\ \text{eV}}$

27.15 The energy absorbed each second is

$$\mathcal{P} = I\cdot A = (500\ \text{W/m}^2)\left[\pi(2.82\times10^{-15}\ \text{m})^2\right] = 1.25\times10^{-26}\ \text{W}$$

The time required to absorb $E = 1.00\ \text{eV} = 1.60\times10^{-19}\ \text{J}$ is

$$t = \frac{E}{\mathcal{P}} = \frac{1.60\times10^{-19}\ \text{J}}{1.25\times10^{-26}\ \text{J/s}} = 1.28\times10^{7}\ \text{s}\left(\frac{1\ \text{d}}{8.64\times10^{4}\ \text{s}}\right) = \boxed{148\ \text{d}}$$

This prediction, based on classical theory, is $\boxed{\text{incompatible with observation}}$

27.16 Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $KE_{\max} = \dfrac{hc}{\lambda} - \phi$, or

$$KE_{\max} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{200\times10^{-9}\ \text{m}}\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) - 4.70\ \text{eV} = 1.52\ \text{eV}$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.52 V, no further electrons will be able to escape but will fall back onto the sphere. Its charge is then given by

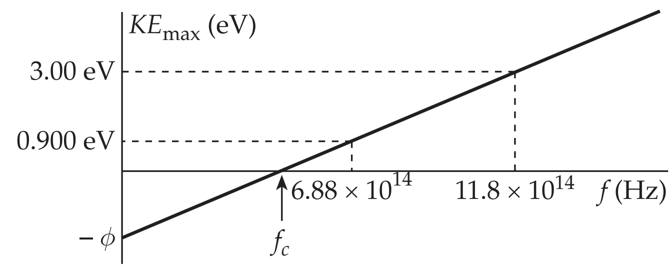
$$V = \frac{k_e Q}{r} \qquad \text{or} \qquad Q = \frac{rV}{k_e} = \frac{(5.00\times10^{-2}\ \text{m})(1.52\ \text{N}\cdot\text{m/C})}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.43\times10^{-12}\ \text{C}}$$

27.17 The two light frequencies allowed to strike the surface are

$$f_1 = \frac{c}{\lambda_1} = \frac{3.00\times10^8 \text{ m/s}}{254\times10^{-9} \text{ m}} = 11.8\times10^{14} \text{ Hz}$$

and $$f_2 = \frac{3.00\times10^8 \text{ m/s}}{436\times10^{-9} \text{ m}} = 6.88\times10^{14} \text{ Hz}$$

The graph you draw should look somewhat like that given at the right.



The desired quantities, read from the axis intercepts of the graph line, should agree within their uncertainties with

$$f_c = \boxed{4.8\times10^{14} \text{ Hz}} \text{ and } \phi = \boxed{2.0 \text{ eV}}$$

27.18 The total energy absorbed by an electron is

$$E_\gamma = \phi + KE_{\text{max}} = 3.44 \text{ eV}\left(\frac{1.60\times10^{-19} \text{ J}}{1 \text{ eV}}\right) + \frac{1}{2}\left(9.11\times10^{-31} \text{ kg}\right)\left(4.2\times10^5 \text{ m/s}\right)^2$$

or $$E_\gamma = 6.3\times10^{-19} \text{ J}$$

The energy absorbed by a square centimeter of surface in one second is

$$E = \mathcal{P}\cdot t = (I\cdot A)\cdot t = \left(0.055 \text{ W/m}^2\right)\left(1.00\times10^{-4} \text{ m}^2\right)(1.00 \text{ s}) = 5.5\times10^{-6} \text{ J}$$

so the number of electrons released per second is

$$N = \frac{E}{E_\gamma} = \frac{5.5\times10^{-6} \text{ J}}{6.3\times10^{-19} \text{ J}} = \boxed{8.7\times10^{12}}$$

27.19 Assuming the electron produces a single photon as it comes to rest, the energy of that photon is $E_\gamma = (KE)_i = eV$. The accelerating voltage is then

$$V = \frac{E_\gamma}{e} = \frac{hc}{e\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})\lambda} = \frac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{\lambda}$$

For $\lambda = 1.0\times10^{-8}\ \text{m}$, $V = \dfrac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{1.0\times10^{-8}\ \text{m}} = \boxed{1.2\times10^{2}\ \text{V}}$

and for $\lambda = 1.0\times10^{-13}\ \text{m}$, $V = \dfrac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{1.0\times10^{-13}\ \text{m}} = \boxed{1.2\times10^{7}\ \text{V}}$

27.20 A photon of maximum energy and minimum wavelength is produced when the electron gives up all its kinetic energy in a single collision.

$$\lambda_{\text{min}} = \frac{hc}{(E_\gamma)_{\text{max}}} = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})V} = \frac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{V}$$

(a) If $V = 15.0\ \text{kV}$, $\lambda_{\text{min}} = \dfrac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{15.0\times10^{3}\ \text{V}} = \boxed{8.29\times10^{-11}\ \text{m}}$

(b) If $V = 100\ \text{kV}$, $\lambda_{\text{min}} = \dfrac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{100\times10^{3}\ \text{V}} = \boxed{1.24\times10^{-11}\ \text{m}}$

27.21 $V = \dfrac{KE}{e} = \dfrac{E_\gamma}{e} = \dfrac{hc}{e\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.030\,0\times10^{-9}\ \text{m})}$

$= 4.14\times10^{4}\ \text{V} = \boxed{41.4\ \text{kV}}$

27.22 From Bragg's law, the wavelength of the reflected x-rays is

$$\lambda = \frac{2d\sin\theta}{m} = \frac{2(0.353\ \text{nm})\sin 20.5^\circ}{2} = \boxed{0.124\ \text{nm}}$$

27.23 Using Bragg's law, the wavelength is found to be

$$\lambda = \frac{2d\sin\theta}{m} = \frac{2(0.296\ \text{nm})\sin 7.6^\circ}{1} = \boxed{0.078\ \text{nm}}$$

27.24 The first-order constructive interference occurs at the smallest grazing angle. From Bragg's law, this angle is

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left[\frac{(1)(0.070\text{ nm})}{2(0.30)}\right] = \boxed{6.7^\circ}$$

27.25 The interplanar spacing in the crystal is given by Bragg's law as

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.140\text{ nm})}{2\sin 14.4^\circ} = \boxed{0.281\text{ nm}}$$

27.26 The scattering angle is given by the Compton shift formula as

$$\theta = \cos^{-1}\left(1 - \frac{\Delta\lambda}{\lambda_C}\right)$$ where the Compton wavelength is

$$\lambda_C = \frac{h}{m_e c^2} = 0.002\,43\text{ nm}$$

Thus, $$\theta = \cos^{-1}\left(1 - \frac{1.50\times10^{-3}\text{ nm}}{2.43\times10^{-3}\text{ nm}}\right) = \boxed{67.5^\circ}$$

27.27 $$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{700\times10^{-9}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{1.78\text{ eV}}$$

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{700\times10^{-9}\text{ m}} = \boxed{9.47\times10^{-28}\text{ kg}\cdot\text{m/s}}$$

27.28 Using the Compton shift formula, the wavelength is found to be

$$\lambda = \lambda_0 + \Delta\lambda = \lambda_0 + \lambda_C(1-\cos\theta)$$

$$= 0.68 \text{ nm} + (0.002\,43 \text{ nm})(1-\cos 45°) = 0.680\,7 \text{ nm}$$

Therefore,

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{0.680\,7\times10^{-9} \text{ m}}\left(\frac{1 \text{ keV}}{1.60\times10^{-16} \text{ J}}\right) = \boxed{1.8 \text{ keV}}$$

and $$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34} \text{ J}\cdot\text{s}}{0.680\,7\times10^{-9} \text{ m}} = \boxed{9.7\times10^{-25} \text{ kg}\cdot\text{m/s}}$$

27.29 If the scattered photon has energy equal to the kinetic energy of the recoiling electron, the energy of the incident photon is divided equally between them. Thus,

$$E_\gamma = \frac{(E_\gamma)_0}{2} \Rightarrow \frac{hc}{\lambda} = \frac{hc}{2\lambda_0}, \text{ so } \lambda = 2\lambda_0 \text{ and } \Delta\lambda = 2\lambda_0 - \lambda_0 = 0.001\,6 \text{ nm}$$

The Compton scattering formula then gives the scattering angle as

$$\theta = \cos^{-1}\left(1-\frac{\Delta\lambda}{\lambda_C}\right) = \cos^{-1}\left(1-\frac{0.001\,6 \text{ nm}}{0.002\,43 \text{ nm}}\right) = \boxed{70°}$$

27.30 (a) $\Delta\lambda = \lambda_C(1-\cos\theta) = (0.002\,43 \text{ nm})(1-\cos 37.0°) = \boxed{4.89\times10^{-4} \text{ nm}}$

(b) The wavelength of the incident x-rays is

$$\lambda_0 = \frac{hc}{(E_\gamma)_0} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{(300 \text{ keV})(1.60\times10^{-16} \text{ J/keV})}\left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = 4.14\times10^{-3} \text{ nm}$$

so the scattered wavelength is $\lambda = \lambda_0 + \Delta\lambda = 4.63\times10^{-3}$ nm

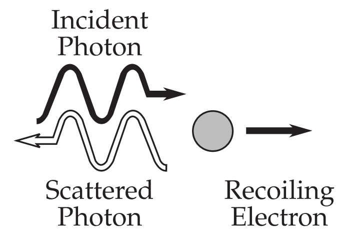
The energy of the scattered photons is then

$$E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{(4.63\times10^{-3} \text{ nm})(10^{-9} \text{ m/1 nm})}\left(\frac{1 \text{ keV}}{1.60\times10^{-16} \text{ J}}\right) = \boxed{268 \text{ keV}}$$

(c) The kinetic energy of the recoiling electrons is

$$KE = \left(E_\gamma\right)_0 - E_\gamma = 300 \text{ keV} - 268 \text{ keV} = \boxed{32 \text{ keV}}$$

27.31 This is Compton scattering with $\theta = 180°$. The Compton shift is $\Delta\lambda = \lambda_C\left(1 - \cos 180°\right) = 2\lambda_C = 0.004\,86 \text{ nm}$, and the scattered wavelength is



$$\lambda = \lambda_0 + \Delta\lambda = \left(0.110 + 0.004\,86\right) \text{ nm} = 0.115 \text{ nm}$$

The kinetic energy of the recoiling electron is then

$$KE = \left(E_\gamma\right)_0 - E_\gamma = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) = \frac{hc\left(\Delta\lambda\right)}{\lambda_0\lambda}$$

$$= \frac{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^{8} \text{ m/s}\right)\left(0.004\,86 \text{ nm}\right)}{\left(0.110 \times 10^{-9} \text{ m}\right)\left(0.115 \text{ nm}\right)} = 7.65 \times 10^{-17} \text{ J}$$

or $$KE = \left(7.65 \times 10^{-17} \text{ J}\right)\left(1 \text{ eV}/1.60 \times 10^{-19} \text{ J}\right) = \boxed{478 \text{ eV}}$$

The momentum of the recoiling electron (non-relativistic) is

$$p_e = \sqrt{2m_e\left(KE\right)} = \sqrt{2\left(9.11 \times 10^{-31} \text{ kg}\right)\left(7.65 \times 10^{-17} \text{ J}\right)} = \boxed{1.18 \times 10^{-23} \text{ kg} \cdot \text{m/s}}$$

27.32 The kinetic energy of the recoiling electron (non-relativistic) is

$$KE = \frac{1}{2}m_e v^2 = \frac{1}{2}\left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.40 \times 10^{6} \text{ m/s}\right)^2 = 8.93 \times 10^{-19} \text{ J}$$

Also, $KE = \left(E_\gamma\right)_0 - E_\gamma = hc\left(\dfrac{1}{\lambda_0} - \dfrac{1}{\lambda}\right) = \dfrac{hc\left(\Delta\lambda\right)}{\lambda_0\lambda} \approx \dfrac{hc\left(\Delta\lambda\right)}{\lambda_0^2}$

(a) The Compton shift is then

$$\Delta\lambda = \frac{\lambda_0^2\left(KE\right)}{hc} = \frac{\left(0.800 \times 10^{-9} \text{ m}\right)^2\left(8.93 \times 10^{-19} \text{ J}\right)}{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^{8} \text{ m/s}\right)}$$

$$= 2.87 \times 10^{-12} \text{ m} = \boxed{0.002\,87 \text{ nm}}$$

(b) From the Compton shift formula,

$$\theta = \cos^{-1}\left(1 - \frac{\Delta\lambda}{\lambda_C}\right) = \cos^{-1}\left(1 - \frac{0.002\,87\text{ nm}}{0.002\,43\text{ nm}}\right) = \boxed{100°}$$

27.33 (a) The kinetic energy of the recoiling electron is

$$KE = \left(E_\gamma\right)_0 - E_\gamma = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) = \frac{hc(\Delta\lambda)}{\lambda_0\lambda} \approx \frac{hc(\Delta\lambda)}{\lambda_0^2} = \frac{hc\left[\lambda_C(1-\cos\theta)\right]}{\lambda_0^2}$$

$$= \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)\left(0.002\,43\text{ nm}\right)\left(1-\cos 23°\right)}{\left(0.45\text{ nm}\right)^2\left(10^{-9}\text{ m/1 nm}\right)}$$

$$KE = 1.9\times10^{-19}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{1.2\text{ eV}}$$

(b) $v = \sqrt{\dfrac{2(KE)}{m_e}} = \sqrt{\dfrac{2\left(1.9\times10^{-19}\text{ J}\right)}{9.11\times10^{-31}\text{ kg}}} = \boxed{6.5\times10^{5}\text{ m/s}}$

27.34 The de Broglie wavelength of a particle of mass m is $\lambda = h/p$ where the momentum is given by $p = \gamma m v = mv\big/\sqrt{1-(v/c)^2}$. Note that when the particle is not relativistic, then $\gamma \approx 1$, and this relativistic expression for momentum reverts back to the classical expression.

(a) For a proton moving at speed $v = 2.00\times10^{4}\text{ m/s}$, $v << c$ and $\gamma \approx 1$ so

$$\lambda = \frac{h}{m_p v} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{4}\text{ m/s}\right)} = \boxed{1.98\times10^{-11}\text{ m}}$$

(b) For a proton moving at speed $v = 2.00\times10^{7}\text{ m/s}$

$$\lambda = \frac{h}{\gamma m_p v} = \frac{h}{m_p v}\sqrt{1-(v/c)^2}$$

$$= \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)}{\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{7}\text{ m/s}\right)}\sqrt{1-\left(\frac{2.00\times10^{7}\text{ m/s}}{3.00\times10^{8}\text{ m/s}}\right)^2} = \boxed{1.98\times10^{-14}\text{ m}}$$

27.35 (a) From $\lambda = h/p = h/mv$, the speed is

$$v = \frac{h}{m_e\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.00\times10^{-7}\ \text{m}\right)} = 1.46\times10^{3}\ \text{m/s} = \boxed{1.46\ \text{km/s}}$$

(b) $$\lambda = \frac{h}{m_e v} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.00\times10^{7}\ \text{m/s}\right)} = \boxed{7.28\times10^{-11}\ \text{m}}$$

27.36 After falling freely with acceleration $a_y = -g = -9.80\ \text{m/s}^2$ for 50.0 m, starting from rest, the speed of the ball will be

$$v = \sqrt{v_0^2 + 2a_y\left(\Delta y\right)} = \sqrt{0 + 2\left(-9.80\ \text{m/s}\right)^2\left(-50.0\ \text{m}\right)} = 31.3\ \text{m/s}$$

so the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(0.200\ \text{kg}\right)\left(31.3\ \text{m/s}\right)} = \boxed{1.06\times10^{-34}\ \text{m}}$$

27.37 (a) The momentum of the electron would be

$$p = \frac{h}{\lambda} \geq \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} \sim 7\times10^{-20}\ \text{kg}\cdot\text{m/s}$$

If the electron is nonrelativistic, then its speed would be

$$v = \frac{p}{m_e} \sim \frac{7\times10^{-20}\ \text{kg}\cdot\text{m/s}}{9.11\times10^{-31}\ \text{kg}} \sim 8\times10^{10}\ \text{m/s} \gg c$$

which is impossible. Thus, a relativistic calculation is required.

With a rest energy of $E_R = 0.511\ \text{MeV} \approx 8\times10^{-14}\ \text{J}$, its kinetic energy is

$$KE = E - E_R = \sqrt{p^2c^2 + E_R^2} - E_R$$

Thus, $KE \sim \sqrt{\left(7\times10^{-20}\ \text{kg}\cdot\text{m/s}\right)^2\left(3\times10^{8}\ \text{m/s}\right)^2 + \left(8\times10^{-14}\ \text{J}\right)^2} - 8\times10^{-14}\ \text{J}$

or $KE \sim 2\times10^{-11}\ \text{J}\left(\dfrac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) \rightarrow \boxed{\sim10^{2}\ \text{MeV}}$ or more

(b) The negative electrical potential energy of the electron (that is, binding energy) would be

$$PE = \frac{k_e q_1 q_2}{r} \sim \frac{\left(9\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(10^{-19}\ \mathrm{C}\right)(-e)}{10^{-14}\ \mathrm{m}} = -10^{5}\ \mathrm{eV} = -10^{-1}\ \mathrm{MeV}$$

With its kinetic energy much greater than the magnitude of its negative potential energy, the electron $\boxed{\text{would immediately escape}}$ from the nucleus.

27.38 (a) The de Broglie wavelength is $\lambda = h/p = h/mv$, so $v = h/m\lambda$

With $\lambda_{\min} = w/10 = 0.075\ \mathrm{m} \sim 10^{-1}\ \mathrm{m}$ and $m = 80\ \mathrm{kg} \sim 10^{2}\ \mathrm{kg}$, we find that the maximum speed allowing significant diffraction is

$$v_{\max} = \frac{h}{m\lambda_{\min}} \sim \frac{6\times10^{-34}\ \mathrm{J\cdot s}}{\left(10^{2}\ \mathrm{kg}\right)\left(10^{-1}\ \mathrm{m}\right)} = 6\times10^{-35}\ \mathrm{m/s} \text{ or } \boxed{\sim10^{-34}\ \mathrm{m/s}}$$

(b) With $d = 0.15\ \mathrm{m} \sim 10^{-1}\ \mathrm{m}$, the minimum time to pass through the door and have significant diffraction occur is

$$t_{\min} = \frac{d}{v_{\max}} \sim \frac{10^{-1}\ \mathrm{m}}{10^{-34}\ \mathrm{m/s}} \text{ or } \boxed{\sim10^{33}\ \mathrm{s}}$$

(c) $\boxed{\text{No.}}$ The minimum transit time that could produce significant diffraction is $\sim10^{15}$ times the age of the Universe.

27.39 For relativistic particles, $p = \dfrac{\sqrt{E^2 - E_R^2}}{c}$ and $\lambda = \dfrac{h}{p} = \dfrac{hc}{\sqrt{E^2 - E_R^2}}$

For 3.00 MeV electrons, $E = KE + E_R = 3.00\ \mathrm{MeV} + 0.511\ \mathrm{MeV} = 3.51\ \mathrm{MeV}$

so $$\lambda = \frac{\left(6.63\times10^{-34}\ \mathrm{J\cdot s}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)}{\sqrt{(3.51\ \mathrm{MeV})^2 - (0.511\ \mathrm{MeV})^2}}\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right) = \boxed{3.58\times10^{-13}\ \mathrm{m}}$$

27.40 From Chapter 24, the minima (or dark fringes) in a single slit diffraction pattern occur where $\sin\theta = m\lambda/a$ for $m = \pm1, \pm2, \pm3, \ldots$ Here, λ is the wavelength of the wave passing through the slit of width *a*. When the fringes are observed on a screen at distance *L* from the slit, the distance from the central maximum to the minima of order *m* is given by $y_m = L\tan\theta_m \approx L\sin\theta_m = m\lambda(L/a)$. The spacing between successive minima is then

$$\Delta y = y_{m+1} - y_m = \lambda\left(\frac{L}{a}\right)$$

Hence, if $\Delta y = 2.10$ cm when $L = 20.0$ cm and $a = 0.500$ nm, the de Broglie wavelength of the electrons passing through the slit must be

$$\lambda = \Delta y\left(\frac{a}{L}\right) = \left(2.10\times10^{-2}\text{ m}\right)\left(\frac{0.500\times10^{-9}\text{ m}}{20.0\times10^{-2}\text{ m}}\right) = 5.25\times10^{-11}\text{ m}$$

The momentum of one of these electrons is then

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{5.25\times10^{-11}\text{ m}} = 1.263\times10^{-23}\text{ kg}\cdot\text{m/s}$$

and, assuming the electron is non-relativistic, its kinetic energy is

$$KE = \frac{p^2}{2m_e} = \frac{\left(1.263\times10^{-23}\text{ kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\text{ kg}\right)}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{547\text{ eV}}$$

Note that if the particle had been relativistic, its kinetic energy would have been computed from $KE = E - E_R = \sqrt{p^2c^2 + E_R^2} - E_R$

27.41 (a) The required electron momentum is

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \mathrm{J\cdot s}}{1.0\times10^{-11}\ \mathrm{m}}\left(\frac{1\ \mathrm{keV}}{1.60\times10^{-16}\ \mathrm{J}}\right) = 4.1\times10^{-7}\ \frac{\mathrm{keV\cdot s}}{\mathrm{m}}$$

and the total energy is

$$E = \sqrt{p^2c^2 + E_R^2}$$

$$= \sqrt{\left(4.1\times10^{-7}\ \frac{\mathrm{keV\cdot s}}{\mathrm{m}}\right)^2 \left(3.00\times10^{8}\ \mathrm{m/s}\right)^2 + \left(511\ \mathrm{keV}\right)^2} = 526\ \mathrm{keV}$$

The kinetic energy is then,

$$KE = E - E_R = 526\ \mathrm{keV} - 511\ \mathrm{keV} = \boxed{15\ \mathrm{keV}}$$

(b) $E_\gamma = \dfrac{hc}{\lambda}$

$$= \frac{\left(6.63\times10^{-34}\ \mathrm{J\cdot s}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)}{1.0\times10^{-11}\ \mathrm{m}}\left(\frac{1\ \mathrm{keV}}{1.60\times10^{-16}\ \mathrm{J}}\right) = \boxed{1.2\times10^{2}\ \mathrm{keV}}$$

27.42 $p_x = mv_x$, and $\Delta p_x = m\left(\Delta v_x\right)$ assuming m is without uncertainty.

Since $\Delta v_x = 1.0\times10^{-3}\ v = 3.0\times10^{-2}\ \mathrm{m/s}$, we have

$$\Delta p_x = \left(50.0\times10^{-3}\ \mathrm{kg}\right)\left(3.0\times10^{-2}\ \mathrm{m/s}\right) = 1.5\times10^{-3}\ \mathrm{kg\cdot m/s}$$

and $$\Delta x \geq \frac{h}{4\pi\left(\Delta p_x\right)} = \frac{6.63\times10^{-34}\ \mathrm{J\cdot s}}{4\pi\left(1.5\times10^{-3}\ \mathrm{kg\cdot m/s}\right)} = \boxed{3.5\times10^{-32}\ \mathrm{m}}$$

27.43 From the uncertainty principle, the minimum uncertainty in the momentum of the electron is

$$\Delta p_x = \frac{h}{4\pi(\Delta x)} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(0.10\times10^{-9}\ \text{m}\right)} = 5.3\times10^{-25}\ \text{kg}\cdot\text{m/s}$$

so the uncertainty in the speed of the electron is

$$\Delta v_x = \frac{\Delta p_x}{m} = \frac{5.3\times10^{-25}\ \text{kg}\cdot\text{m/s}}{9.11\times10^{-31}\ \text{kg}} = 5.8\times10^{5}\ \text{m/s}\quad\text{or}\quad\sim10^{6}\ \text{m/s}$$

Thus, if the speed is on the order of the uncertainty in the speed, then $\boxed{v\sim10^{6}\ \text{m/s}}$

27.44 (a) With uncertainty Δx in position, the minimum uncertainty in the speed is

$$\Delta v_x = \frac{\Delta p_x}{m} \geq \frac{h}{4\pi m(\Delta x)} = \frac{2\pi\ \text{J}\cdot\text{s}}{4\pi(2.00\ \text{kg})(1.00\ \text{m})} = 0.250\ \text{m/s}$$

(b) If we knew Fuzzy's initial position and velocity exactly, his final position after an elapsed time t would be given by $x_f = x_i + v_x t$. Since we have uncertainty in both the initial position and the speed, the uncertainty in the final position is

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = 1.00\ \text{m} + (0.250\ \text{m/s})(5.00\ \text{s}) = \boxed{2.25\ \text{m}}$$

27.45 With $\Delta x = 5.00\times10^{-7}\ \text{m/s}$, the minimum uncertainty in the speed is

$$\Delta v_x = \frac{\Delta p_x}{m_e} \geq \frac{h}{4\pi m_e(\Delta x)} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.00\times10^{-7}\ \text{m}\right)} = \boxed{116\ \text{m/s}}$$

27.46 (a) For a non-relativistic particle, $KE = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \boxed{\frac{p^2}{2m}}$

(b) From the uncertainty principle,

$$\Delta p_x \geq \frac{h}{4\pi(\Delta x)} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(1.0\times10^{-15}\ \text{m}\right)} = 5.3\times10^{-20}\ \text{kg}\cdot\text{m/s}$$

Since the momentum must be at least as large as its own uncertainty, the minimum kinetic energy is

$$KE_{\min} = \frac{p_{\min}^2}{2m} = \frac{\left(5.3\times10^{-20}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(1.67\times10^{-27}\ \text{kg}\right)}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{5.2\ \text{MeV}}$$

27.47 The peak radiation occurs at approximately 560 nm wavelength. From Wien's displacement law,

$$T = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{560\times10^{-9}\ \text{m}} \approx \boxed{5\,200\ \text{K}}$$

Clearly, a firefly is not at this temperature,

so $\boxed{\text{this is not blackbody radiation}}$

27.48 (a) Minimum wavelength photons are produced when an electron gives up all its kinetic energy in a single collision. Then, $E_\gamma = 50\,000\ \text{eV}$ and

$$\lambda_{\min} = \frac{hc}{E_\gamma} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(5.00\times10^{4}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{2.49\times10^{-11}\ \text{m}}$$

(b) From Bragg's law, the interplanar spacing is

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)\left(2.49\times10^{-11}\ \text{m}\right)}{2\sin(2.50°)} = 2.85\times10^{-10}\ \text{m} = \boxed{0.285\ \text{nm}}$$

27.49 The x-ray wavelength is $\lambda = hc/E_\gamma$, so Bragg's law yields

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left(\frac{mhc}{2dE_\gamma}\right)$$

or $$\theta = \sin^{-1}\left[\frac{2\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{2\left(0.352\times10^{-9}\ \text{m}\right)(11.3\ \text{keV})\left(1.60\times10^{-16}\ \text{J/keV}\right)}\right] = \boxed{18.2°}$$

27.50 (a) From $v^2 = v_0^2 + 2a_y(\Delta y)$, Johnny's speed just before impact is

$$v = \sqrt{2g|\Delta y|} = \sqrt{2(9.80\ \text{m/s})(50.0\ \text{m})} = 31.3\ \text{m/s}$$

and his de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63\times 10^{-34}\ \text{J}\cdot\text{s}}{(75.0\ \text{kg})(31.3\ \text{m/s})} = \boxed{2.82\times 10^{-37}\ \text{m}}$$

(b) The energy uncertainty is

$$\Delta E \geq \frac{h}{4\pi(\Delta t)} = \frac{6.63\times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi(5.00\times 10^{-3}\ \text{s})} = \boxed{1.06\times 10^{-32}\ \text{J}}$$

(c) $\%\ \text{error} = \dfrac{\Delta E}{mg|\Delta y|}(100\%)$

$$\geq \frac{(1.06\times 10^{-32}\ \text{J})(100\%)}{(75.0\ \text{kg})(9.80\ \text{m/s}^2)(50.0\ \text{m})} \geq \boxed{2.87\times 10^{-35}\ \%}$$

27.51 The magnetic force supplies the centripetal acceleration for the electrons, so $m\dfrac{v^2}{r} = qvB$, or $p = mv = qrB$

The maximum kinetic energy is then $KE_{\text{max}} = \dfrac{p^2}{2m} = \dfrac{q^2r^2B^2}{2m}$

or $$KE_{\text{max}} = \frac{(1.60\times 10^{-19}\ \text{J})^2(0.200\ \text{m})^2(2.00\times 10^{-5}\ \text{T})^2}{2(9.11\times 10^{-31}\ \text{kg})} = 2.25\times 10^{-19}\ \text{J}$$

The work function of the surface is given by $\phi = E_\gamma - KE_{max} = hc/\lambda - KE_{max}$

or $$\phi = \frac{(6.63\times 10^{-34}\ \text{J}\cdot\text{s})(3.00\times 10^{8}\ \text{m/s})}{450\times 10^{-9}\ \text{m}} - 2.25\times 10^{-19}\ \text{J}$$

$$= 2.17\times 10^{-19}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times 10^{-19}\ \text{J}}\right) = \boxed{1.36\ \text{eV}}$$

27.52 The incident wavelength is $\lambda_0 = hc/E_\gamma$, and the Compton shift is

$$\Delta\lambda = \frac{h}{m_p c}(1-\cos\theta) = \frac{hc}{m_p c^2}(1-\cos\theta) = \frac{hc}{(E_R)_{proton}}(1-\cos\theta)$$

The scattered wavelength is $\lambda = \lambda_0 + \Delta\lambda = hc\left[\frac{1}{E_\gamma} + \frac{1-\cos\theta}{(E_R)_{proton}}\right]$, or

$$\lambda = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{1.60\times10^{-13}\ \text{J/MeV}}\left[\frac{1}{200\ \text{MeV}} + \frac{1-\cos(40.0^\circ)}{939\ \text{MeV}}\right] = 6.53\times10^{-15}\ \text{m}$$

The energy of the scattered photon is then

$$E'_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{6.53\times10^{-15}\ \text{m}}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{191\ \text{MeV}}$$

27.53 From the photoelectric effect equation, $KE_{\max} = E_\gamma - \phi = \dfrac{hc}{\lambda} - \phi$

For $\lambda = \lambda_0$, $KE_{\max} = 1.00\ \text{eV}$ so $1.00\ \text{eV} = \dfrac{hc}{\lambda_0} - \phi$ (1)

For $\lambda = \dfrac{\lambda_0}{2}$, $KE_{\max} = 4.00\ \text{eV}$ giving $4.00\ \text{eV} = \dfrac{2hc}{\lambda_0} - \phi$ (2)

Multiplying equation (1) by a factor of 2 and subtracting

the result from equation (2) gives the work function as $\phi = \boxed{2.00\ \text{eV}}$

27.54 From the photoelectric effect equation, $KE_{\max} = E_\gamma - \phi = \dfrac{hc}{\lambda} - \phi$

For $\lambda = 670\ \text{nm}$, $KE_{\max} = E_1$ so $E_1 = \dfrac{hc}{670\ \text{nm}} - \phi$ (1)

For $\lambda = 520\ \text{nm}$, $KE_{\max} = 1.50\,E_1$ giving $1.50\,E_1 = \dfrac{hc}{520\ \text{nm}} - \phi$ (2)

Multiplying equation (1) by a factor of 1.50 and subtracting the result from equation (2) gives the work function as

$$\phi = hc\left(\frac{3.00}{670\text{ nm}} - \frac{2.00}{520\text{ nm}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)$$

$$\phi = 1.26\times10^{-19}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{0.785\text{ eV}}$$

27.55 (a) If $KE = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.00\times10^{-8}\text{ m}} = 1.99\times10^{-17}\text{ J} = 124\text{ eV}$

the electron is non-relativistic and

$$v = \sqrt{\frac{2(KE)}{m_e}} = \sqrt{\frac{2(1.99\times10^{-17}\text{ J})}{9.11\times10^{-31}\text{ kg}}}$$

$$= \left(6.61\times10^{6}\ \frac{\text{m}}{\text{s}}\right)\left(\frac{c}{3.00\times10^{8}\text{ m/s}}\right) = \boxed{0.022\,0c}$$

(b) When

$$KE = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.00\times10^{-13}\text{ m}}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = 12.4\text{ MeV}$$

the electron is highly relativistic and $KE = (\gamma - 1)E_R$

or $\gamma = 1 + \dfrac{KE}{E_R} = 1 + \dfrac{12.4\text{ MeV}}{0.511\text{ MeV}} = 25.3$

Then, $v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(25.3)^2} = \boxed{0.999\,2c}$

27.56 $\Delta p \geq \dfrac{h}{4\pi(\Delta x)} = \dfrac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(2.0\times10^{-15}\text{ m})}\left(\dfrac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = 1.65\times10^{-7}\ \dfrac{\text{MeV}\cdot\text{s}}{\text{m}}$

Since the momentum must be at least as large as its own uncertainty,

$$p \geq 1.65\times10^{-7}\ \frac{\text{MeV}\cdot\text{s}}{\text{m}}$$

(a) If the confined particle is an electron, $E_R = 0.511\text{ MeV}$ and

$$E = \sqrt{(pc)^2 + E_R^2} \geq \sqrt{\left(1.65\times10^{-7}\ \frac{\text{MeV}\cdot\text{s}}{\text{m}}\right)^2 \left(3.00\times10^{8}\ \text{m/s}\right)^2 + (0.511\text{ MeV})^2}$$

or $E \geq 49.5\text{ MeV}$. Then, $\gamma = \dfrac{E}{E_R} \geq \dfrac{49.5\text{ MeV}}{0.511\text{ MeV}} = 96.8$ and

$$v = c\sqrt{1-1/\gamma^2} \geq c\sqrt{1-1/(96.8)^2} = \boxed{0.999\,9c} \text{ (highly relativistic)}$$

(b) If the confined particle is a proton, $E_R = 939\text{ MeV}$ and

$$E = \sqrt{(pc)^2 + E_R^2} \geq \sqrt{\left(1.65\times10^{-7}\ \frac{\text{MeV}\cdot\text{s}}{\text{m}}\right)^2 \left(3.00\times10^{8}\ \text{m/s}\right)^2 + (939\text{ MeV})^2}$$

or $E \geq 940\text{ MeV}$. Then, $\gamma = \dfrac{E}{E_R} \geq \dfrac{940\text{ MeV}}{939\text{ MeV}} = 1.001$ and

$$v = c\sqrt{1-1/\gamma^2} \geq c\sqrt{1-1/(1.001)^2} = \boxed{0.053c} \text{ (non-relativistic)}$$

27.57 (a) From conservation of energy, $E + E_R = \phi + \sqrt{(pc)^2 + E_R^2}$, where E is the photon energy and $E_R = m_e c^2$.

The de Broglie wavelength of the electron is $\lambda = \dfrac{h}{p}$, giving $pc = \dfrac{hc}{\lambda}$

If λ is also the wavelength of the incident photon, then

$E = hc/\lambda$ and $pc = E$

The energy conservation equation then becomes

$$E + E_R - \phi = \sqrt{E^2 + E_R^2}$$

Squaring both sides and simplifying yields $2E_R E - 2\phi(E + E_R) + \phi^2 = 0$, which reduces to

$$E = \frac{\phi(2E_R - \phi)}{2(E_R - \phi)} = \boxed{\frac{\phi(m_e c^2 - \phi/2)}{m_e c^2 - \phi}}$$

(b) From the photoelectric effect equation, $KE_{\max} = E - \phi$. Using the result from above, and the fact that $m_e c^2 = 0.511\ \text{MeV}$, gives

$$KE_{\max} = \frac{\phi\left(m_e c^2 - \phi/2\right)}{m_e c^2 - \phi} - \phi = \frac{\phi^2}{2\left(m_e c^2 - \phi\right)} = \frac{(6.35\ \text{eV})^2}{2\left(5.11\times10^5\ \text{eV} - 6.35\ \text{eV}\right)}$$

or $$KE_{\max} = 3.95\times10^{-5}\ \text{eV} = 6.31\times10^{-24}\ \text{J}$$

Therefore,

$$v \le \sqrt{\frac{2\left(KE_{\max}\right)}{m_e}} = \sqrt{\frac{2\left(6.31\times10^{-24}\ \text{J}\right)}{9.11\times10^{-31}\ \text{kg}}} = 3.72\times10^3\ \frac{\text{m}}{\text{s}} = \boxed{3.72\ \text{km/s}}$$

27.58 (a) From conservation of energy, $\left(E_\gamma\right)_0 + E_R = E_\gamma + \left(E_R + KE\right)$ or

$$\left(E_\gamma\right)_0 = E_\gamma + KE = 120\ \text{keV} + 40.0\ \text{keV} = 160\ \text{keV}$$

Therefore, the wavelength of the incident photon is

$$\lambda_0 = \frac{hc}{\left(E_\gamma\right)_0} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{(160\ \text{keV})\left(1.60\times10^{-16}\ \text{J/keV}\right)} = \boxed{7.77\times10^{-12}\ \text{m}}$$

(b) The wavelength of the scattered photon is

$$\lambda = \frac{hc}{E_\gamma} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{(120\ \text{keV})\left(1.60\times10^{-16}\ \text{J/keV}\right)} = 1.04\times10^{-11}\ \text{m}$$

so the Compton shift is $\Delta\lambda = \lambda - \lambda_0 = 2.59\times10^{-12}\ \text{m}$

The Compton shift formula then gives the photon scattering angle as

$$\theta = \cos^{-1}\left(1 - \frac{\Delta\lambda}{\lambda_C}\right) = \cos^{-1}\left(1 - \frac{2.59\times10^{-12}\ \text{m}}{2.43\times10^{-12}\ \text{m}}\right) = \boxed{93.8^\circ}$$

(c) The momentum of the scattered photon is $p_\gamma = \dfrac{E_\gamma}{c} = 120 \ \dfrac{\text{keV}}{c}$

The rest energy of an electron is $E_R = 0.511 \text{ MeV} = 511 \text{ keV}$, so the total energy of the recoiling electron is

$$E = E_R + KE = 511 \text{ keV} + 40.0 \text{ keV} = 551 \text{ keV}$$

The momentum of the electron is then

$$p_e = \frac{\sqrt{E^2 - E_R^2}}{c} = \frac{\sqrt{(551 \text{ keV})^2 - (511 \text{ keV})^2}}{c} = 206 \ \frac{\text{keV}}{c}$$

Taking the direction of the incident photon to be the x-axis, conservation of momentum in the y direction requires that $p_\gamma \sin\theta = p_e \sin\phi$, where ϕ is the recoil angle of the electron. Thus,

$$\phi = \sin^{-1}\left(\frac{p_\gamma \sin\theta}{p_e}\right) = \sin^{-1}\left(\frac{(120 \ \text{keV}/c)\sin 93.8°}{206 \ \text{keV}/c}\right) = \boxed{35.5°}$$

27.59 (a) The woman tries to release the pellets from rest directly above the spot on the floor. However, there is some uncertainty Δx_i in their horizontal position when released. Also, the uncertainty principle $(\Delta x)(\Delta p_x) \geq \hbar/2$ states that the minimum uncertainty that can exist in their horizontal velocity at the instant of release is given by

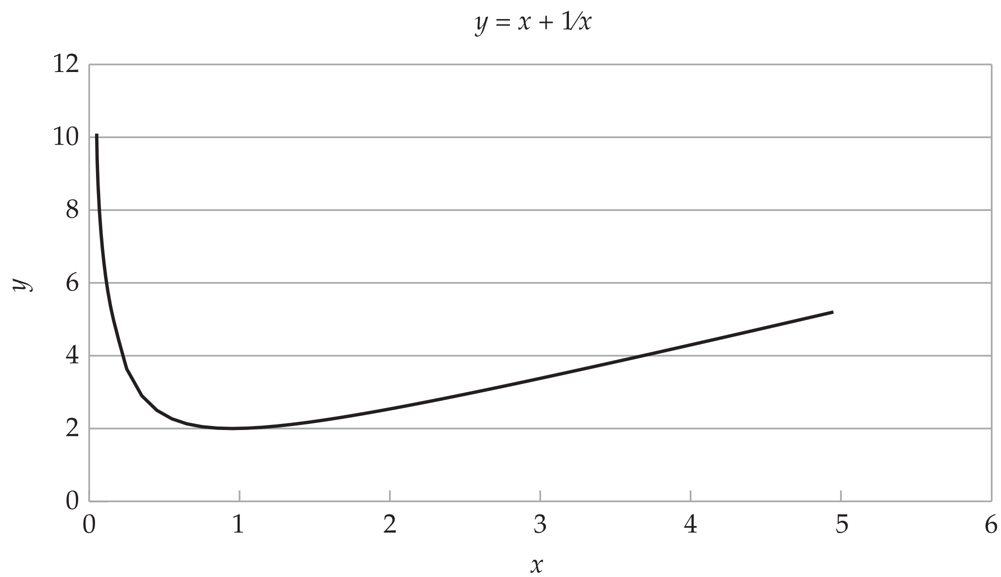
$$\Delta v_x = \frac{\Delta p_x}{m} = \frac{\hbar}{2m(\Delta x_i)}$$

Thus, the uncertainty in the horizontal position of the pellet when it reaches the floor after falling for a time $t = \sqrt{2H/g}$ is

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \frac{A}{\Delta x_i} \quad \text{where} \quad A \equiv \frac{\hbar}{2m}\sqrt{\frac{2H}{g}}$$

To determine the minimum value Δx_f can take on, rewrite this equation in the form

$$\frac{\Delta x_f}{\sqrt{A}} = \frac{\Delta x_i}{\sqrt{A}} + \frac{\sqrt{A}}{\Delta x_i} \quad \text{or} \quad y = x + \frac{1}{x} \quad \text{where} \quad y \equiv \frac{\Delta x_f}{\sqrt{A}} \text{ and } x \equiv \frac{\Delta x_i}{\sqrt{A}}$$



The graph of y vs x for $x > 0$ given above shows that y has a minimum value of 2 at $x = 1$. Thus,

$$\left(\frac{\Delta x_f}{\sqrt{A}}\right)_{\min} = 2 \quad \text{or} \quad \left(\Delta x_f\right)_{\min} = 2\sqrt{A}$$

We then have that

$$\left(\Delta x_f\right)_{\min} = 2\left[\frac{\hbar}{2m}\sqrt{\frac{2H}{g}}\right]^{1/2}$$

or $$\left(\Delta x_f\right)_{\min} = \boxed{\left(\frac{2\hbar}{m}\right)^{1/2}\left(\frac{2H}{g}\right)^{1/4}}$$

(b) If $H = 2.00\ \text{m}$ and $m = 0.500\ \text{g}$, then

$$\Delta x \geq \left[\frac{2\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)}{0.500\times10^{-3}\ \text{kg}}\right]^{\frac{1}{2}}\left[\frac{2\left(2.00\ \text{m}\right)}{9.80\ \text{m/s}^2}\right]^{\frac{1}{4}} = \boxed{5.19\times10^{-16}\ \text{m}}$$

27.60 The Compton wavelength is $\lambda_C = \dfrac{h}{mc}$, where m is the mass of the particle imagined to be scattering the photon.

The de Broglie wavelength of the particle is

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} \text{ , where } \gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Therefore, $h = (mc)\lambda_C = (\gamma mv)\lambda$ which gives $\gamma(v/c) = \lambda_C/\lambda$

Squaring this result yields $\dfrac{(v/c)^2}{1-(v/c)^2} = \left(\dfrac{\lambda_C}{\lambda}\right)^2$, which simplifies to

$$\left(\frac{v}{c}\right)^2 = \frac{(\lambda_C/\lambda)^2}{(\lambda_C/\lambda)^2+1} = \frac{1}{1+(\lambda/\lambda_C)^2} \quad \text{or} \quad \boxed{v = \frac{c}{\sqrt{1+(\lambda/\lambda_C)^2}}}$$

27.61 (a) The density of iron is $\rho_{Fe} = 7.86\times10^3 \text{ kg/m}^3$, so the mass of the sphere must be

$$m = \rho_{Fe}V = \rho_{Fe}\left(\frac{4}{3}\pi r^3\right) = \left(7.86\times10^3 \text{ kg/m}^3\right)\frac{4}{3}\pi\left(2.00\times10^{-2} \text{ m}\right)^3 = \boxed{0.263 \text{ kg}}$$

(b) From Stefan's law (see Chapter 11), the rate at which the sphere radiates energy is $\mathcal{P} = \sigma AeT^4$ where $\sigma = 5.669\,6\times10^{-8} \text{ W/m}^2\cdot\text{K}^4$, $A = 4\pi r^2$ is the surface area, $e = 0.860$ is the emissivity, and $T = 20+273 = 293 \text{ K}$ is the absolute temperature. Thus,

$$\mathcal{P} = \left(5.669\,6\times10^{-8} \text{ W/m}^2\cdot\text{K}^4\right)4\pi\left(2.00\times10^{-2} \text{ m}\right)^2(0.860)(293 \text{ K})^4 = \boxed{1.81 \text{ W}}$$

(c) If the sphere radiates a quantity of energy ΔQ, its temperature will decrease by $\Delta T = \Delta Q/mc_{Fe}$, where $c_{Fe} = 448 \text{ J/kg}\cdot{}^\circ\text{C}$ is the specific heat of iron. The rate that the temperature of the isolated sphere would change is then

$$\frac{\Delta T}{\Delta t} = -\frac{\Delta Q/\Delta t}{mc_{Fe}} = -\frac{\mathcal{P}}{mc_{Fe}} = -\frac{1.81 \text{ J/s}}{(0.263 \text{ kg})(448 \text{ J/kg}\cdot{}^\circ\text{C})} = \boxed{-0.015\,3\ {}^\circ\text{C/s}}$$

or $\dfrac{\Delta T}{\Delta t} = (-0.015\,3\ {}^\circ\text{C/s})\left(\dfrac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{-0.919\ {}^\circ\text{C/min}}$

(d) From Wien's displacement law, the wavelength of peak radiation from an object with an absolute temperature of 293 K is

$$\lambda_{\text{max}} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{T} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{293\ \text{K}} = 9.89\times10^{-6}\ \text{m} = \boxed{9.89\ \mu\text{m}}$$

(e) The energy of a photon having this wavelength is

$$E_\gamma = hf = \frac{hc}{\lambda_{\text{max}}} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{9.89\times10^{-6}\ \text{m}} = \boxed{2.01\times10^{-20}\ \text{J}}$$

(f) If all of the radiated energy was at this wavelength, the number of photons the sphere would radiate each second would be

$$N = \frac{\mathcal{P}}{E_\gamma} = \frac{1.81\ \text{J/s}}{2.01\times10^{-20}\ \text{J/photon}} = \boxed{8.98\times10^{19}\ \text{photons/s}}$$