7.2 Wave Nature of Matter

De Broglie Wavelength
Diffraction of electrons
Uncertainty Principle
Wave Function
Tunneling

Wave properties of matter
Material particles behave as waves with a wavelength given by the De Broglie wavelength (Planck’s constant/momentum)

\[ \lambda = \frac{h}{p} \]

The particles are diffracted by passing through an aperture in a similar manner as light waves.

The wave properties of particles mean that when you confine it in a very small space its momentum (and kinetic energy) must increase. (uncertainty principle) This is responsible for the size of the atom.

Wave properties are only dominant in very small particles.

De Broglie Wavelength

Momentum of a photon - inverse to wavelength.

\[ p = \frac{E}{c} \]

Einstein’s special relativity theory

De Broglie proposed that this wavelength applied to material particles as well as for photons. (1924)

Wavelength of a particle - inverse to momentum.

\[ \lambda = \frac{h}{p} \]

De Broglie wavelength of a baseball

A baseball with a mass of 0.15 kg is pitched at 45 m/s. What is its De Broglie wavelength?

\[ \lambda = \frac{h}{p} = \frac{6.6x10^{-34} J\cdot s}{\frac{0.15 \text{ kg}}{45 \text{ m/s}}} = 9.8x10^{-36} \text{ m} \]

Diffraction effects of a baseball are negligible

de Broglie wavelength of an electron

Find the de Broglie wavelength of a 1000 eV electron.

\[ \lambda = \frac{h}{p} = \frac{\frac{h}{\sqrt{2mV}}} {\frac{6.6x10^{-34} J\cdot s}{1000 \text{ eV}}} = 4.0x10^{-11} \text{ m} \]
Proof of the wave nature of electrons by Electron Diffraction

• Davisson – Germer Experiment
• Thompson Experiment

Showed the wave nature of light by diffraction of electrons by crystals.

Diffraction of light from crystals

Crystals act as a three-dimensional diffraction grating. Light with wavelength close to the inter-atomic spacing (x-rays) is diffracted.

Diffraction of electrons from crystals

Davisson-Germer Experiment (1927)
An electron beam is scattered from a crystal

Ni crystal

The scattered beam shows a diffraction pattern expected for the crystal spacing.

Comparison between electron diffraction and x-ray diffraction

George Thomson (1927)

Either x-rays or electrons

The electron wavelength was adjusted to the same value as the x-ray by varying the voltage.
The diffraction pattern for x-rays and electrons are very similar.

Material particles have wave properties.

Electron microscopy

\[ \lambda = \frac{h}{mv} \]

Shorter wavelength can be obtained by increasing \( v \), the speed of the electron.

Electron microscopy

Electron microscopy can be used to image structures of molecules.
Diffraction of particles
Probabilistic Interpretation of the wave amplitude.

Intensity \propto \text{Amplitude}^2 of the diffracted wave.

But each particle hits a certain point on the screen.

The amplitude^2 is interpreted as the probability of the particle hitting the screen at a certain position. This is true for electrons as well as photons.

Wavefunction
In quantum mechanics the result of an experiment is given in terms of a wavefunction $\Psi$. The square of the wavefunction $\Psi^2$ is the probability of the particle being at a certain position.

The wavefunction can be calculated using the Schrödinger Equation. For instance for electrons in an atom.

Wave property of particles

Decreasing the slit reduces $\Delta x$
But increases the width of the diffraction, $\Delta v_x$

When $\Delta x$ decreases, $\Delta p_x$ increases.

Uncertainty Principle
Particle passing through a slit –
The uncertainty in position is $\Delta x$
The uncertainty in the x component of momentum is $\Delta p_x = m \Delta v_x$

The particle is diffracted

most often written as an inequality

$\Delta x \Delta p_x \geq \frac{h}{4\pi}$

The position and velocity cannot be know with unlimited certainty.

The size of an atom
What accounts for the size of the hydrogen atom?

Classical electrostatics predicts that the potential energy of the hydrogen atom should go to – infinity

The finite size of the atom is a quantum mechanical effect.

As the size of the atom $r$ decreases – The uncertainty in the momentum increases until the uncertainty in momentum limits the momentum. Then the kinetic energy must increase due to the uncertainty principle.

Use linear momentum as a rough estimate.

$\Delta x \Delta p_x = h$

$\Delta p_x = \frac{h}{\Delta x}$

$\Delta x = \Delta r$

$\Delta p_x = \frac{h}{2r}$

$\rho_x$ cannot be smaller than $\Delta p_x$

KE increases as $\frac{1}{4r^2}$

$E = KE + PE$ goes through a minimum as a function of $r$
Tunneling across a barrier

A macroscopic object impinging on a barrier the object cannot penetrate within the barrier.

A wave particle impinging on a barrier can penetrate within the barrier for distance and go through the barrier if it is thin enough.

The probability of tunneling decreases exponentially with the width of the barrier.

Electron Tunneling Microscope

Electron Tunneling Probability \( P \) and wave function decay as a function of distance. The probability falls off exponentially with distance and is a sensitive function of distance.

\[
E_e \propto \alpha e^{-\frac{r}{\alpha}}
\]

Tunneling in Photosynthesis

Electron transfer is due to tunneling. Electron transfer time vs distance. The electron transfer time decays exponentially as a function of distance.