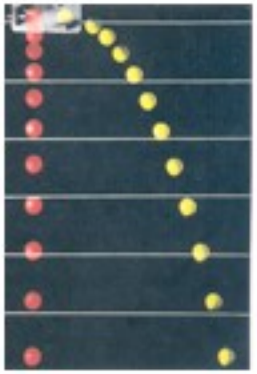
Motion in 1D With Constant Acceleration

Reminder

$$v_x = v_{0x} + a_x t \; ; \; x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$



When $\vec{a}_x = 0, \; \vec{a}_y = -g = -9.80 m/s^2$

and $\Rightarrow v_x = v_{0x} \; ; \; x = x_0 + v_{0x} t$

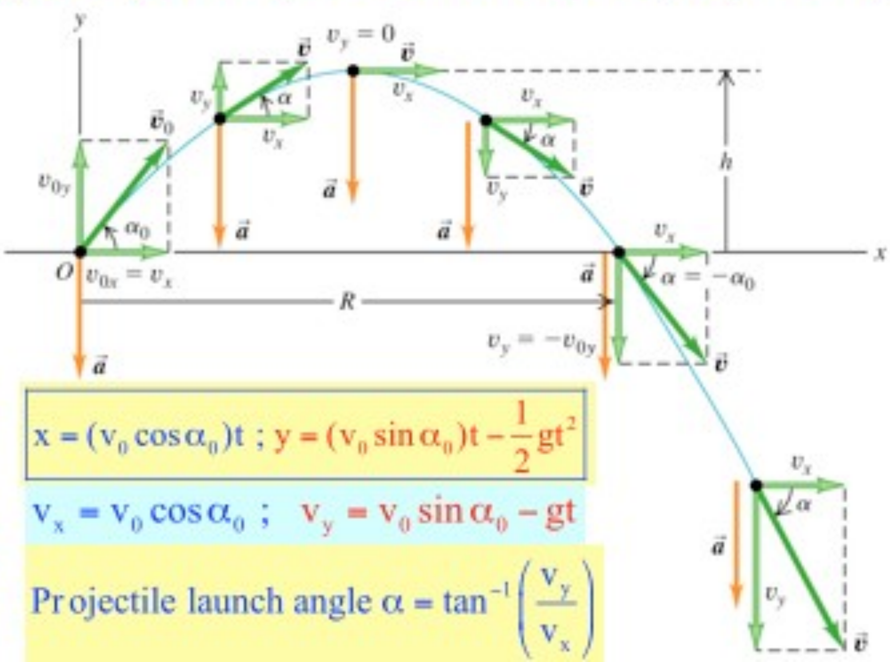
$$v_y = v_{0y} - gt \; ; \; y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

Thus

Displacement : $\vec{r} = x\hat{i} + y\hat{j}$

Velocity : $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Trajectory of Projectile with Velocity $\vec{v}_0$ at t=0



$$x = (v_0 \cos\alpha_0)t \; ; \; y = (v_0 \sin\alpha_0)t - \frac{1}{2} g t^2$$

$$v_x = v_0 \cos\alpha_0 \; ; \quad v_y = v_0 \sin\alpha_0 - gt$$

Projectile launch angle $\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Projectile Trajectory is Parabolic

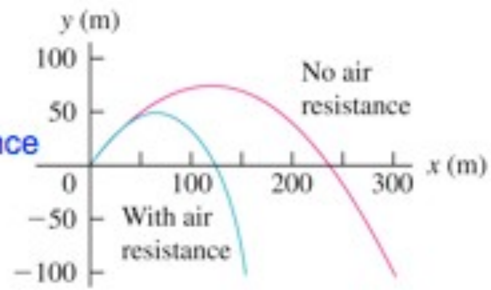
Equation for trajectory along y axis ?

Use $t = x/(v_0 \cos\alpha_0)$

$$\Rightarrow \boxed{y = (\tan\alpha_0)x - \frac{g}{2v_0\cos^2\alpha_0}x^2}$$

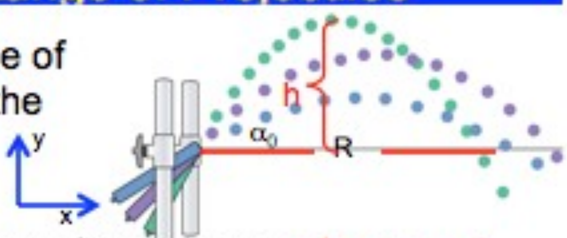
Trajectory is always *parabolic* in x

Effect of (neglected) air resistance



Height & Range of Projectiles

Max. height & the range of projectile depends on the firing angle α_0



Projectile at its highest point at time t_1 when $v_y = 0$

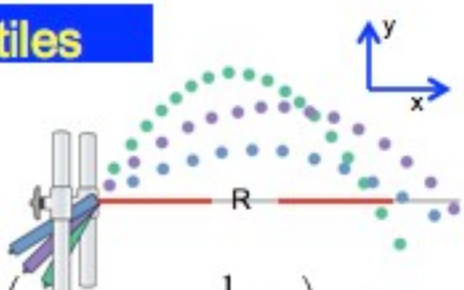
$$\Rightarrow v_y = v_0\sin\alpha_0 - gt_1 = 0 \Rightarrow t_1 = \frac{v_0\sin\alpha_0}{g}$$

at this time, $y = h = v_0\sin\alpha_0\frac{v_0\sin\alpha_0}{g} - \frac{1}{2}g\left(\frac{v_0\sin\alpha_0}{g}\right)^2$

$$\Rightarrow \boxed{h = \frac{v_0^2\sin^2\alpha_0}{2g}};\ \text{largest at } \alpha_0 = 90^0 \text{ (vertical launch)}$$

Range of Projectiles

R is the projectile's x location at some $t=t_2$ when $y = 0$



$$0 = (v_0 \sin\alpha_0)\, t_2 - \frac{1}{2} g t_2^2 = t_2 \left(v_0 \sin\alpha_0 \; - \frac{1}{2} g t_2 \right) = 0$$

Two solutions for t_2 : $\boxed{t_2 = 0}$ & $\boxed{t_2 = \frac{2 v_0 \sin\alpha_0}{g}}$

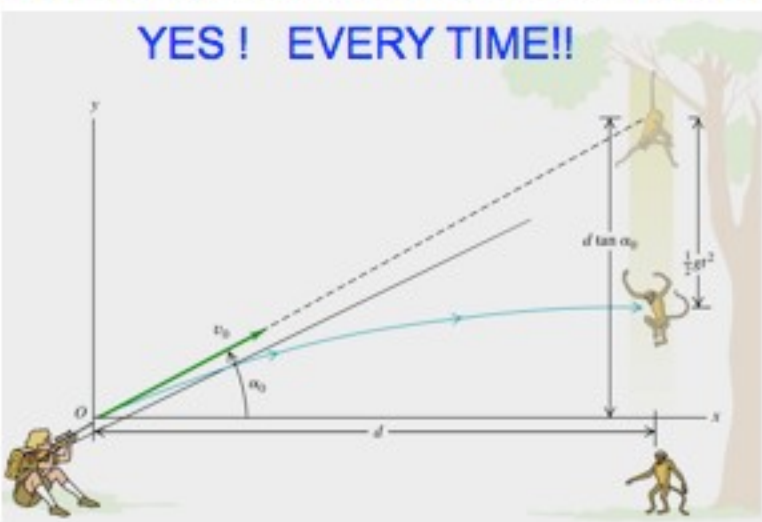
$$\text{Range } R = v_0 \cos\alpha_0 . \frac{2 v_0 \sin\alpha_0}{g} = \boxed{\frac{v_0^2 \sin 2\alpha_0}{g} = R}$$

$R = R_{max}$ when $\alpha_0 = 45^o$

(Using trig. identity: $2\sin\theta\cos\theta = \sin 2\theta$)

Shoot The Monkey !

Monkey escapes from SD zoo. Climbs up a tree. Wont come back to Zoo Zookeeper aims tranquilizer gun directly at monkey and shoots! At that instant monkey (did not take PHYS2A) jumps down. Will the dart hit the monkey?



Trajectory Of Monkey & The Dart

- Two projectiles released simultaneously
  - the dart (travels in x & y)
  - the monkey (travels in y)
- Place reference axes (x=0,y=0) on the dart gun
- At some time t if Monkey is hit by dart ⇒
  - show that $(x_{monkey}, y_{monkey}) = (x_{dart}, y_{dart})$

Trajectory Of Monkey & The Dart

Monkey drops straight down ⇒ $\boxed{x_{monkey} = d}$

Monkey falls from height $\boxed{y_{0monkey} = d\tan\alpha_0}$

at any time t since jump, $\boxed{y_{monkey} = d\tan\alpha_0 - \frac{1}{2}gt^2}$

Dart's initial velocity $(v_{0x}, v_{0y}) = (v_0\cos\alpha_0, v_0\sin\alpha_0)$

at time t, dart's position in $\boxed{x_{dart} = v_0\cos\alpha_0 . t}$

at time t, dart's position in $\boxed{y_{dart} = v_0\sin\alpha_0 . t - \frac{1}{2}gt^2}$

Shooting The Monkey: Everytime

When Dart intercepts monkey:

$$x_{monkey} = x_{dart} \Rightarrow d = v_0 \cos\alpha_0 . t \Rightarrow t_x^{hit} = \frac{d}{v_0 \cos\alpha_0}$$

$$y_{monkey} = y_{dart} \Rightarrow d \tan\alpha_0 - \frac{1}{2} g t^2 = v_0 \sin\alpha_0 . t - \frac{1}{2} g t^2$$

$$\Rightarrow d \tan\alpha_0 = v_0 \sin\alpha_0 . t \Rightarrow t_y^{hit} = \frac{d \tan\alpha_0}{v_0 \sin\alpha_0} = \frac{d}{v_0 \cos\alpha_0}$$

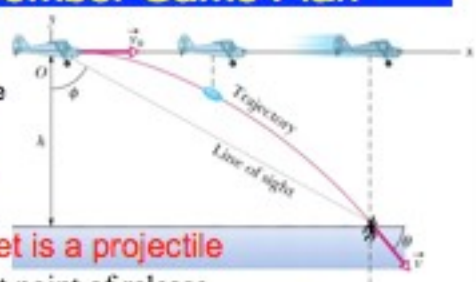
$\Rightarrow$ Dart & Monkey's coordinates coincide at time t^{hit}

Dart aimed at monkey on tree, always hits monkey irrespective of dart's initial velocity or free fall acc. g

$\Rightarrow$ If monkey runs off to moon, keeper will still get him !

Rescue Plane/ B-2 Bomber Game Plan

Rescue plane flies at 198km/h at height of 500m towards a point directly over a boating accident victim in water. Pilot wants to release lifejacket so that it hits water close to victim. What should be the angle φ of pilot's line of sight to the victim when the release is made?



Once released from plane, lifejacket is a projectile

Attach ref. frame to plane, with origin at point of release

Lifejacket, released (not shot) from plane $\Rightarrow \vec{v}_0 = \vec{v}_{plane}; \theta_0 = 0$

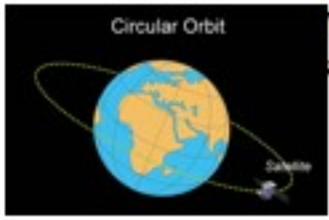
Must travel dist x in some time t: $x = (v_{plane} \cos\theta_0) t$

In time t since release, lifejacket travels $y = -500m$

$$\Rightarrow y = -500m = (v_{plane} \sin\theta_0) t - 0.5 g t^2 \Rightarrow t = 10.1s$$

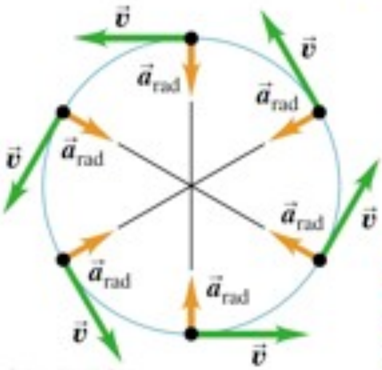
$$\Rightarrow x = (v_{plane}) 10.1s = 555.5m \text{ and } \varphi = \tan^{-1}\left(\frac{x}{h}\right) = 48.0^0$$

Uniform Circular Motion



Very different from projectile motion where accel. was const. and **always** in 1 direction

In uniform circular motion the speed of object is constant but velocity is always changing

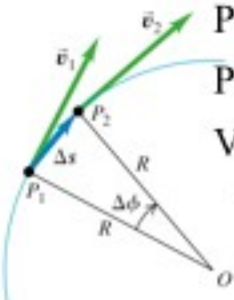


No component of accel. parallel (or tangent) to path so no change in speed

$$\boxed{\vec{a} = \vec{a}_{\perp} = \vec{a}_r}$$

Component of accel $\perp$ to path causes direction of velocity to change

Radial Component of Acceleration



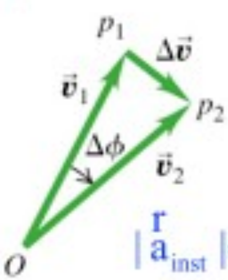
Particle moving in circle of radius R around O

Particle moves from $P_1 \rightarrow P_2$ in time Δt

Velocity changes from $\vec{v}_1 \rightarrow \vec{v}_2$ in time Δt

Triangles OP_1P_2 & Op_1p_2 are *similar* $\Rightarrow$

Ratio of corresponding sides are equal



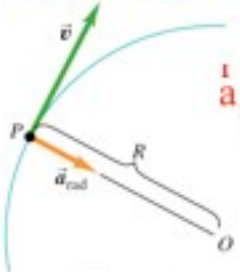
$$\Rightarrow \frac{\Delta\vec{v}}{|\vec{v}_1|} = \frac{\Delta s}{R} \Rightarrow \Delta\vec{v} = \frac{\Delta s}{R}|\vec{v}_1|$$

$$\Rightarrow \quad |\vec{a}_{av}| = \frac{\Delta\vec{v}}{\Delta t} = \frac{|\vec{v}_1|}{R}\frac{\Delta s}{\Delta t}$$

$$|\vec{a}_{inst}| = \lim_{\Delta t \to 0}\frac{|\vec{v}_1|}{R}\frac{\Delta s}{\Delta t} = \frac{|\vec{v}_1|}{R}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t} = \frac{|\vec{v}_1||\vec{v}_1|}{R} = \boxed{\frac{v^2}{R}}$$

Radial Component of Acceleration

$$|\vec{a}_{inst}| = |\vec{a}_{rad}| = \boxed{\frac{v^2}{R}}$$ also called Centripetal Acceleration

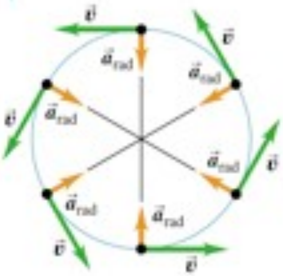


Centripetal = "seeking the center"

$\vec{a}_{rad}$ is $\perp$ to $\vec{v}$ and directed radially *inwards*

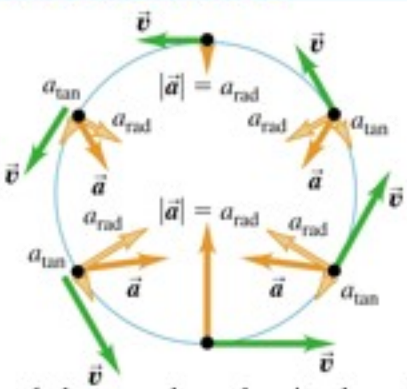
Period of Motion :T

In time T, particle makes one full trip around circle. So its speed $\boxed{|\vec{v}| = \frac{2\pi R}{T}}$



But since $|\vec{a}_{rad}| = \frac{v^2}{R} \Rightarrow \boxed{|\vec{a}_{rad}| = \frac{4\pi^2 R}{T^2}}$

Non-Uniform Circular Motion



Here speed changes along the circular path

$$\boxed{a_P = a_{tangent} = \frac{d|\vec{v}|}{dt} \neq 0 \ \ \& \ a_r = \frac{v^2}{R}}$$

a_{rad} is largest when speed is largest, smallest when speed is smallest.

a_{tan} is || to $\vec{v}$ (going downhill) and anti-|| $\vec{v}$ when object going uphill !