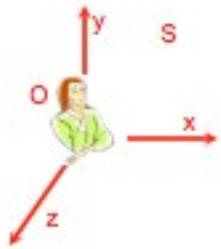
Relative Velocity

Blue Angel pilots must keep track of their velocity w.r.t air so as to maintain enough airflow over their wings to sustain the "lift" & not crash

They must also be aware of relative velocity of their aircraft w.r.t another !

Frames of Reference, Observers & Motion

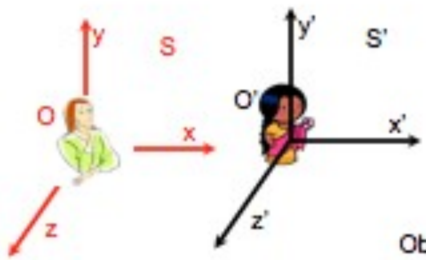
Event: Some thing happening, some where at some time



Frame of reference S = a coordinate system + clock

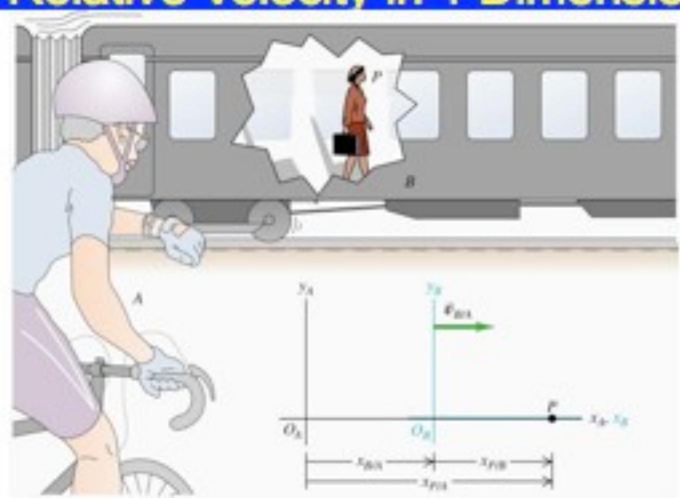
Observer O : sits in S, measures events with ruler, clock

Observers in diff frames of refs, depending on relative location may measure different positions for an event but measure same time. Their clocks are synchronized !



Observers can move w.r.t each other

Relative Velocity in 1 Dimension



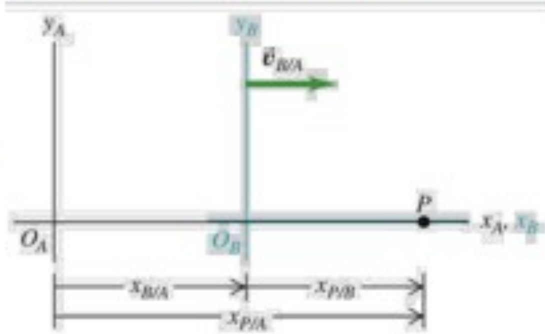
Woman walks with velocity of 1.0 m/s along train's aisle

Train is moving with velocity of 3.0 m/s to the right

What is the woman's velocity ?

According to which Observer ???

Relative Velocity in 1 Dimension



At any instant (take a snapshot)

$x_{P/A}$ = pos. of P rel. to frame A ; $x_{P/B}$ = pos. of P rel. to frame B (train)

$x_{B/A}$ = distance from origin of A to origin of B

Clearly $x_{P/A} = x_{P/B} + x_{B/A}$ & $\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt} \Rightarrow v_{P/A} = v_{P/B} + v_{B/A}$

(non-relativistic! you'll see in 2d it's not quite so simple as this!)

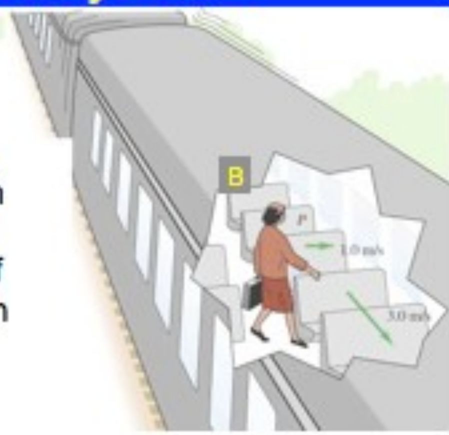
So woman's velocity as seen by cyclist (A)

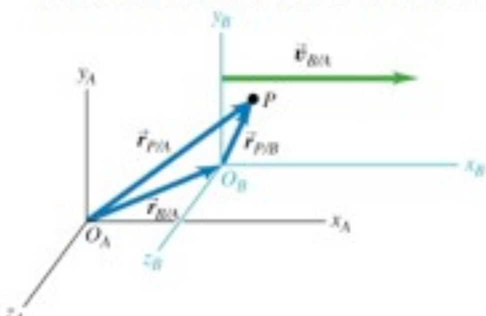
$v_{P/A} = 1.0\text{m/s} + 3.0\text{m/s} = 4.0\text{m/s}$ but If woman was walking in opp. dir. in train

$v_{P/A} = -1.0\text{m/s} + 3.0\text{m/s} = 2.0\text{m/s}$

Relative Velocity in 3D

Suppose woman walks from one side of car to another with speed of 1.0m/s . Her motion is perpendicular to direction of the aisle and the train's motion

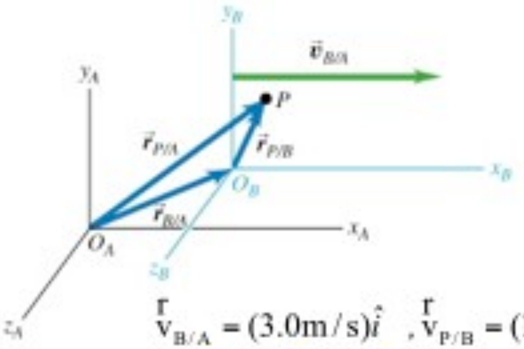


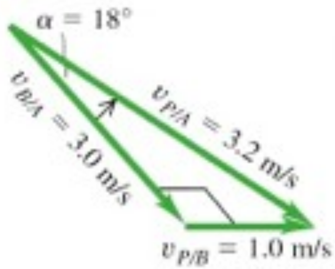


$$\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A}$$

$$\Rightarrow \vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

Relative Velocity in 3D





$\vec{v}_{B/A} = (3.0\text{m/s})\hat{i}$, $\vec{v}_{P/B} = (1.0\text{m/s})\hat{j}$

$\Rightarrow \boxed{\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} = (3.0\hat{i} + 1.0\hat{j})\text{m/s}}$

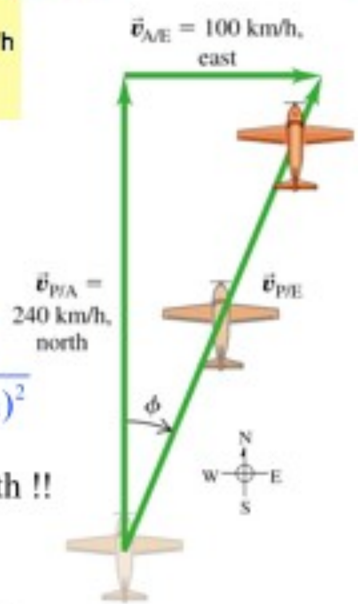
speed of P seen by A= $|\vec{v}_{P/A}| = \sqrt{(3.0\text{m/s})^2 + (1.0\text{m/s})^2}$

Cyclist on ground sees woman moving at angle α w.r.t.

train's motion: $\tan\alpha = \frac{v_{P/B}}{v_{B/A}} = \frac{1.0\text{m/s}}{3.0\text{m/s}} \Rightarrow \alpha = 18°$

Flying In Crosswind

compass indicates, plane heading due North
airspeed indicator shows it moving thru air at 240km/h
If there is wind of 100km/h west to east,
What is velocity of plane relative to earth?



Two observers: In air (A), on earth (E)

Watch plane (P)'s motion

$\vec{v}_{P/A} = (240\text{km/h})\hat{j}$; $\vec{v}_{A/E} = (100\text{km/h})\hat{i}$

$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E} = (100\hat{i} + 240\hat{j})\text{km/h}$

speed $|\vec{v}_{P/E}| = \sqrt{(240\text{km/h})^2 + (100\text{km/h})^2}$

$\phi = \tan^{-1}\left(\frac{100\text{km/h}}{240\text{km/h}}\right) = 23^0$ East of North !!

Must make course correction if

he wants to land on an airport due north !

Course Correction: How Much, Which Way ?

Because of crosswind what direction should the pilot be headed to travel **due North**. What will be his velocity relative to earth?

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$$

pilot must point plane's nose at angle ϕ w.r.t wind to makeup.

Angle ϕ tells direction of $\vec{v}_{P/A}$

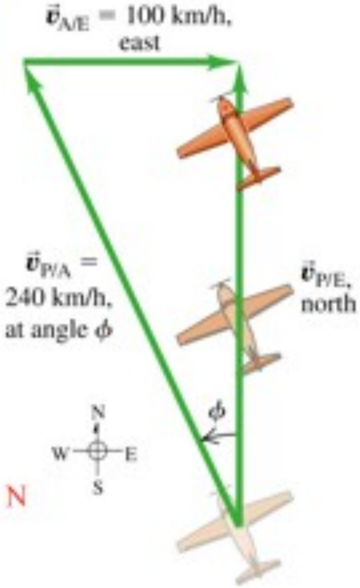
$\vec{v}_{P/E}$: speed ? but due N

$\vec{v}_{P/A}$: speed = 240km/h, dir?

$\vec{v}_{A/E}$: speed = 100km/h, due E

$$\phi = \sin^{-1}\left(\frac{v_{A/E}}{v_{P/A}}\right) = \sin^{-1}\left(\frac{100\text{km/h}}{240\text{km/h}}\right) = 25^{\circ}\text{ W of N}$$

$$v_{P/E} = \sqrt{(v_{P/A})^2 - (v_{A/E})^2} = 218\text{km/h}$$



Archimedes & Weapon of Mass Destruction

Epic account of Syracusan's defense of their city from Roman invaders
Thanks to Archimedes (and his eureka moment !)

The Catapult As A War Machine

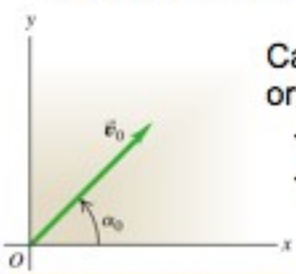
Catapults were invented in many civilizations. Earliest known record is from 9th century BC in Nimrud (modern Day Iraq !). But Archimedes's catapults were fantastic. They could throw 100kg boulder +200m away → sank roman ships

Archimedes genius was in his exquisite control of projectile trajectories !

Projectile Motion

Projectile: An object launched with some initial velocity that follows a path determined entirely by effects of gravitational acceleration g = - 9.8 m/s^2 (and air resistance)

Trajectory: path of a projectile (such as from a catapult)



All projectile motion occurs in the vertical plane containing initial $\vec{v}_0$ vector

Can decompose any projectile motion into two orthogonal and independent components

- along x axis with constant velocity, $a_x=0$
- along y axis with constant accel., $a_y= g$

Can express all relations for $\vec{r}$, $\vec{v}$ and $\vec{a}$ in terms of *seperate* components along x, y axis.

Projectile motion is superposition of these seperate and independent motions.

Motion Along X & Y Are Independent

Two Actions:

A Red ball ***dropped*** from rest

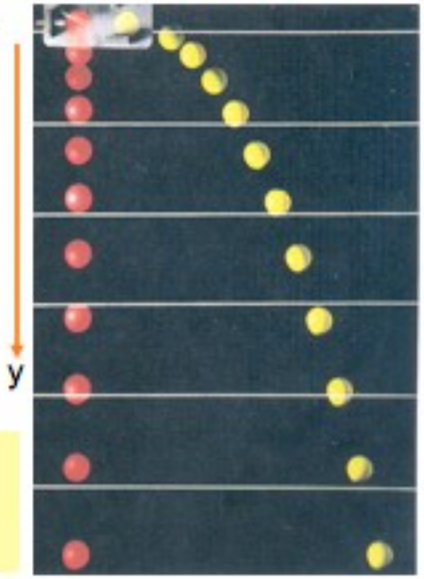
Simultaneously a Yellow ball ***projected*** horizontally

What do you see ?

At any time both balls have same y position, velocity & acceleration

But balls have ***different*** locations in x and velocity along x axis !

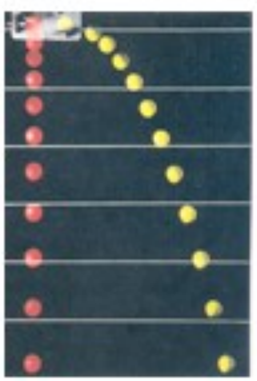
Stroboscopic pictures:



Motion in 1D With Constant Acceleration

Reminder

$$v_x = v_{0x} + a_x t\;;\quad x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$



When $\vec{a}_x = 0,\ \vec{a}_y = -g = -9.80\,\mathrm{m/s^2}$

$$\Rightarrow v_x = v_{0x}\ ;\ x = x_0 + v_{0x}t$$

and

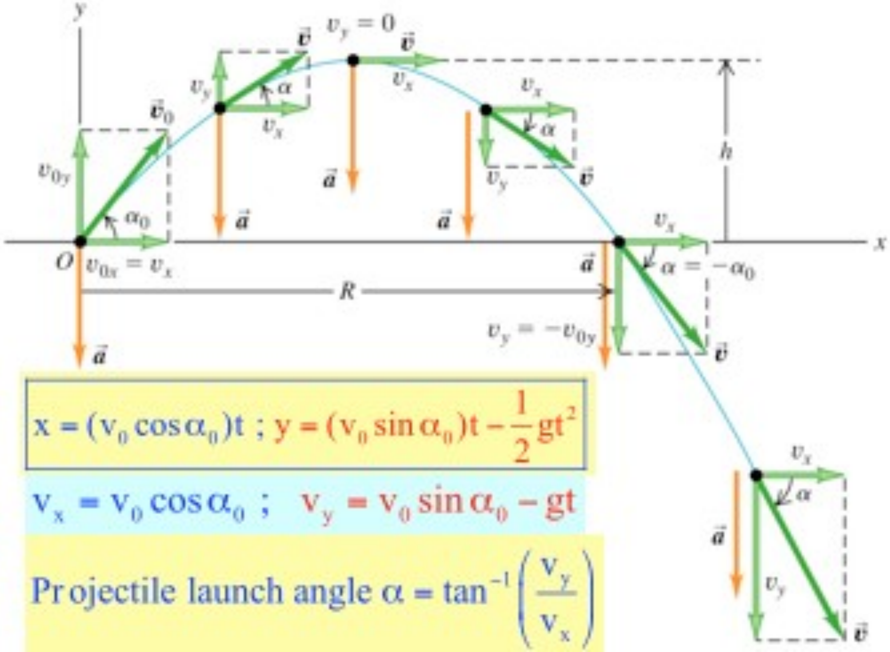
$$v_y = v_{0y} - gt\ ;\ y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

Thus

Displacement : $\vec{r} = x\hat{i} + y\hat{j}$

Velocity : $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Trajectory of Projectile with Velocity $\vec{v}_0$ at t=0



$$x = (v_0 \cos\alpha_0)t \;;\; y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$$

$$v_x = v_0 \cos\alpha_0 \;;\quad v_y = v_0 \sin\alpha_0 - gt$$

Projectile launch angle $\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)$