Physics 1B: Electricity & Magnetism, Fall 2010
Quiz #3, Nov. 2, 2010

This is version **A**!

Useful coefficients: $g = 9.8$ m s^{-2}; Unit of elementary charge $e = 1.6 \times 10^{-19}$ C
Mass of proton, m_p = Mass of neutron, $m_n = 1.67 \times 10^{-27}$ kg; Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
Coulomb's constant, $k_e = 8.99 \times 10^9$ N m^2 C^{-2}
Permittivity of free space $\epsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}
Prefixes: milli (m) = 10^{-3}; micro (μ) = 10^{-6}; nano (n) = 10^{-9}; pico (p) = 10^{-12}; fempto (f) = 10^{-15}
1 eV = 1.6 $\times 10^{-19}$ J

Questions 1–3: You are given a circuit consisting of 7 identical capacitors, each with capacitance 14 mF, connected IN SERIES with a 9-V battery.

1. What is the voltage drop across each individual capacitor?
a. 0.78 V
b. 1.29 V
c. 2.0 V
d. 7.0 V
e. 9.0 V

2: Calculate the equivalent capacitance.
a. 0.14 mF
b. 0.5 mF
c. 2 mF
d. 49 mF
e. 14 mF

3: While the capacitors are all kept connected to the battery, a dielectric material with a dielectric constant $\kappa = 2$ is inserted into the gaps of each capacitor. What is the ratio of the equivalent capacitance now to that derived in problem 1, i.e., what is $C_{eq,2}$ / $C_{eq,1}$?
a. 2
b. 2/7
c. 1/2
d. 7/2
e. 14

Solution for No. 1: The sum of the voltage drops across the capacitors must equal 9 V. With each capacitor being equal, $\Delta V = 9/7 =$ **1.29 V** for each.

Solution for No. 2: Let C denote the capacitance on each individual capacitor.
$\frac{1}{C_{eq,1}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{7}{C}$

$\frac{1}{C_{eq,1}} = \frac{1}{14mF} + \frac{1}{14mF} + \frac{1}{14mF} + \frac{1}{14mF} + \frac{1}{14mF} + \frac{1}{14mF} + \frac{1}{14mF} = \frac{7}{14mF} = \frac{1}{2mF}$

$$\mathbf{C_{eq,1} = \frac{C}{7} = 2\ mF.}$$

Solution for No. 3: Let C denote the original capacitance, before the dielectrics are inserted. When the dielectrics are inserted, we have $C \to \kappa C$ on each capacitor.

$\frac{1}{C_{eq,2}} = \frac{1}{\kappa C} + \frac{1}{\kappa C} + \frac{1}{\kappa C} + \frac{1}{\kappa C} + \frac{1}{\kappa C} + \frac{1}{\kappa C} + \frac{1}{\kappa C} = \frac{1}{\kappa} \times \frac{7}{C}$

Taking the reciprocal of both sides, we get $C_{eq,2} = \kappa \frac{C}{7} = \kappa C_{eq,1}$

$$\mathbf{So\ C_{eq,2} / C_{eq,1} = \kappa = 2}$$

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4. Consider an electron moving in a circuit which consists of five resistors connected in series with a battery. The five resistors, R_a, R_b, R_c, R_d, R_e, have resistances of 19Ω, 15Ω, 10Ω, 25Ω, and 1Ω respectively, and the battery supplies an EMF of 19 V. The drop in potential energy is greatest when traversing which resistor?

a. R_a
b. R_b
c. R_c
d. R_d
d. R_e

Solution: ΔPE is related to ΔV as $\Delta U = q\Delta V$, so we need to to identify the largest drop in ΔV. This will occur for the resistor with the largest value of resistance, $\mathbf{R_d}$**.** Furthermore, the current is identical at all points in the circuit, so $\Delta V = IR$ holds for each resistor separately, and the largest ΔV occurs at the resistors with the largest value of R.

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5. What is the maximum number of hair dryers (each consuming power at a rate of 1000W) you can connect in parallel in a 120-V home circuit without tripping the home's 45-Amp circuit breaker?
a. 3
b. 4
c. 5
d. 6
e. 7

Solution: Each hair draw must draw a current of I = P/ΔV = 1000W/120V = 8.33 A. The total current needed to operate N hair dryers is 8.33N Amps. 5 hair dryers draws 41.67 Amps, but 6 hair dryers would draw 50 A and thus trip the circuit breaker. The answer is **5**.

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6. You have two cylindrical resistors, A and B. Both are made of platinum. Their lengths are identical. The radius of A is one half that of B. Resistor A is to be operated at 20°C. Resistor B is to be operated at 120°. α for platinum is a constant in this temperature range: 3.92×10^{-3} $(°C)^{-1}$. Find the ratio of their resistance values, R(A)/R(B).
a) 0.35
b) 0.70
c) 1.43
d) 2.87
e) 5.56

SOLUTION: We use R = ρ l / A for each resistor, and can form their ratio of resistance values $\frac{R(A)}{R(B)} = \frac{\rho_A L_A/A_A}{\rho_B L_B/A_B} = \frac{\rho_A L_A A_B}{\rho_B L_B A_A}$.
We are given that $L_A = L_B$.
When you halve the radius, you decrease the cross-sectional area by a factor of 4. So the area of A is one-fourth that of B: $A_B = 4\ A_A$
Now we must calculate ρ_A/ρ_B:
We can use $\rho_B = \rho_A[1 + \alpha(T - T_0)]$
where T = 120°C and T_0 is 20°C (reference temperature).
$(T - T_0) = 100°C$.
$\alpha(T - T_0) = 0.392$
$\rho_B/\rho_A = [1 + \alpha(T - T_0)] = 1.392$
$\rho_A/\rho_B = 1 / 1.392$

Finally, we have $\frac{R(A)}{R(B)} = \frac{1}{1.392} \times \frac{4}{1} = \mathbf{2.87}$

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7. You are given three samples of different metals, and you can determine the resistance R for each of them at temperatures of 65°F and at 458°F (R_0 and R, respectively):

Sample	R_0 at T=65°F	R at T=458°F
Palladium (Pd)	1.00 Ω	1.81 Ω
Nickel (Ni)	10.00 Ω	16.00 Ω
Brass	2.00 Ω	2.44 Ω
Tin (Sn)	2.00 Ω	3.96 Ω

Rank these metals in order from the highest temperature coefficient of resistivity α to the lowest (assume α is constant over this temperature range).

a. Sn, Pd, Ni, Brass
b. Brass, Pd, Ni, Sn

c. Ni, Sn, Brass, Pd
d. Ni, Brass, Sn, Pd
e. Sn, Ni, Pd, Brass

Solution: Consider the equation for the dependence of resistance as a function of temperature,
$R = R_0\ [1 + \alpha(T - T_0)]$: it can be re-written as
$R/R_0 = [1 + \alpha(T - T_0)]$

We are given T and T_0, which are the same for all three samples. Whichever metal has the highest ratio of R/R_0 must have the highest value of $[1 + \alpha(T - T_0)]$ and thus the highest values of α.
R/R_0 for Pd is 1.81.
R/R_0 for Ni is 1.60.
R/R_0 for brass is 1.22.
R/R_0 for Sn is 1.98.
These are the values of $[1 + \alpha(T - T_0)]$. So,
$\alpha\Delta T$ for Pd is 0.81
$\alpha\Delta T$ for Ni is 0.60
$\alpha\Delta T$ for brass is 0.22
$\alpha\Delta T$ for Sn is 0.98
And $(T - T_0)$ is the same for all four samples. We conclude that
α is highest for Sn. Pd is second-highest. Ni is third-highest.
α is lowest for brass.

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8. The dielectric strength of “substance X” is 4.0×10^7 N/C and its dielectric constant is 4. You are given a parallel plate capacitor whose plates are separated by 1 mm. When filled with substance X between its plates, this capacitor can hold a charge of 0.01 C before dielectric breakdown occurs. Find the maximum amount of energy that can be stored in the capacitor without dielectric breakdown occuring.

a. 50 J
b. 800 J
c. 2 J
d. 15 J
e. 200 J

Solution: We can use Energy U = 1/2 Q (ΔV_{max}) where ΔV_{max} is the maximum voltage difference which can be applied before dielectric breakdown occurs.
$\Delta V_{max} = E_{max}$ d = (4.0×10^7 V/m)(0.001m) = 4.0×10^4 V.
U = 1/2 Q (ΔV_{max}) = 1/2 (0.01 C) (40000V) = **200 J**

An alternate solution would have been to calculate the capacitance:
C = Q/(ΔV) = (0.01C) / 40000 V = 2.5 $\times 10^{-7}$ F = 250 nF.
We can then use U = 1/2 C $(\Delta V_{max})^2$ = 1/2 (2.5×10^{-7} F)$(40000\text{V})^2$ = **200 J.**

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9. Two capacitors are connected in parallel with a battery: capacitor A has capacitance C_A; capacitor B has capacitance C_B; C_A is greater than C_B. Which of the following statements is true?
a. There is a larger potential difference across capacitor A.
b. There is a larger potential difference across capacitor B.
c. There is the same quantity of charge stored on each capacitor.
d. There exists the same potential difference across both capacitors.
e. There is more charge stored on capacitor B.

Solution: For circuit elements connected in parallel with a battery, the voltage drop across them must be identical (**confirming choice D** and ruling out choices A and B). Because the values of capacitance are not equal, and using C = Q/ΔV, there must be differing amounts of charge stored on each capacitor (ruling out C). Finally, because $C_A > C_B$, and because Q = CΔV, we have $Q_A > Q_B$ (in contradiction to choice e).

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10. If 1.0×10^{20} electrons enter a 20Ω resistor in a duration of one minute and four seconds, what is the potential difference ΔV across the resistor?
a) 0.31 V
b) 5.0 V
c) 32 V
d) 50 V
e) 80 V

Solution: We can calculate the current using I = ΔQ/ΔT.
ΔQ = 1.0×10^{20} electrons $\times$ (1.6 $\times 10^{-19}$ C per electron) = 1.6 $\times 10^1$ C = 16 C.
I = ΔQ/ΔT = 16C / 64 seconds = 0.25 Amps.
ΔV = IR = (0.25A)(20Ω) = **5.0 V**.

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