## Practice problems for the final exam (with solutions)

There will be at least a couple of problems on the final which will be identical to, or at least very similar to, problems listed here. Not all the questions here are multiple choice.

Useful coefficients: $\quad g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$; Unit of elementary charge $e=1.6 \times 10^{-19} \mathrm{C}$
Mass of proton, $\mathrm{m}_{\mathrm{p}}=$ Mass of neutron, $\mathrm{m}_{\mathrm{n}}=1.67 \times 10^{-27} \mathrm{~kg}$
Mass of electron, $\mathrm{m}_{\mathrm{e}}=9.11 \times 10^{-31} \mathrm{~kg}$
Coulomb's constant, $k_{e}=8.99 \times 10^{9} \mathrm{~N} \mathrm{~m}^{2} \mathrm{C}^{-2}$
Permittivity of free space $\epsilon_{0}=\frac{1}{4 \pi k_{e}}=8.85 \times 10^{-12} \mathrm{C}^{2} \mathrm{~N}^{-1} \mathrm{~m}^{-2}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J} \quad 1 \mathrm{~T}=10^{4} \mathrm{G}$
Permeability of free space $\mu_{0}=4 \pi \times 10^{-7} \mathrm{~T} \mathrm{~m} \mathrm{~A}{ }^{-1}$.
Prefixes: milli $(\mathrm{m})=10^{-3} ; \operatorname{micro}(\mu)=10^{-6} ;$ nano $(\mathrm{n})=10^{-9} ;$ pico $(\mathrm{p})=10^{-12}$; fempto (f) $=10^{-15}$

19-1: Excess charge on any conductor
a. is always evenly distributed. b. is in the center. c. is on the surface. d. is fixed in space. e. cannot be detected.

ANSWER:C

19-2: Electric field lines start
a. at positive charges or at infinity.
b. at negative charges or infinity.
c. at infinity and end at positive charges.
d. midway between positive charges and negative charges.
e. midway between positive charges and infinity.

ANSWER: A

19-3: The electric flux through a surface of fixed area is maximum when the surface is
a. parallel to the electric field.
b. antiparallel to the electric field.
c. perpendicular to the electric field.
d. at an angle of $45^{\circ}$ to the electric field.
e. closed, but does not contain the charge.

ANSWER: C: Perpendicular. Note that this holds for any arbitrary E-field or surface- as long as the local electric field is perpendicular to the surface at a given location, then $\Phi_{E}$ is maximized at that location (maximum number of field lines penetrating the surface).
$\Phi_{E}=\mathrm{E} \mathrm{A} \cos (\theta)$, and this is maximized when $\cos (\theta)=1$ and $\theta=0^{\circ}$.
If $\vec{E}$ can be made to be perpendicular to the surface at all points on the surface, then the sum total $\Phi_{E}$ will be maximized.

19-4: When a conductor is in electrostatic equilibrium, the electric field inside the conductor is (Choices: 0, $\quad \sigma / \epsilon_{0}, \quad 2 \sigma / \epsilon_{0}, \quad \sigma /\left(2 \epsilon_{0}\right)$ ).
ANSWER: E-field inside a conductor at equilibrium is zero.

19-5: When a conductor is in electrostatic equilibrium, the electric field a point just outside the conductor is
(Choices: 0, $\quad \sigma / \epsilon_{0}, \quad 2 \sigma / \epsilon_{0}, \quad \sigma /\left(2 \epsilon_{0}\right)$ ).
ANSWER: E-field just outside a conductor is $\sigma / \epsilon_{0}$.
(Compare to the case of an infinite plane of uniform charge, where both above \& below the plane, the magnitude of $\vec{E}$ is $\left.\sigma /\left(2 \epsilon_{0}\right)\right)$

19-6: The $x y$ plane is "painted" with a uniform surface charge density of $40 \mathrm{nC} / \mathrm{m}^{2}$. Consider a spherical surface with a 0.04 m radius that has a point in the $x y$ plane as its center. What is the total electric flux $\Phi_{E}$ in $\mathrm{Nm}^{2} / \mathrm{C}$ ?
ANSWER: Use Gauss' Law: $\Phi_{E}=Q_{\text {encl }} / \epsilon_{0}$.
The amount of charge enclosed by the sphere will be that contained on a circle of radius 0.04 m , since that's the area inside the Gaussian spherical surface.
$Q_{\text {encl }}=\pi(0.04 \mathrm{~m})^{2} \times 40 \mathrm{nC} / \mathrm{m}^{2}=2.0 \times 10^{-10} \mathrm{C}$.
$\Phi_{E}=Q_{\mathrm{encl}} / \epsilon_{0}=\left(2.0 \times 10^{-10} \mathrm{C}\right) /\left(8.85 \times 10^{-12} \mathrm{C}^{2} \mathrm{~N}^{-1} \mathrm{~m}^{-2}\right)=$
$22.7 \mathrm{Nm}^{2} / \mathrm{C}$

20-1. The electrical potential energy of a pair of like charges is
a. positive. b. negative. c. zero. d. proportional to the square of the distance. e. inversely proportional to the square of the distance.

ANSWER: $U=\frac{k_{e} q_{1} q_{2}}{r}$.
If both $q_{1} \& q_{2}$ are positive, $U$ is positive.
If both $q_{1} \& q_{2}$ are negative, $U$ is again positive.
For a pair of charges, $U$ will be negative only when the charges have opposite signs. So the answer here is A .
Choice E is rejected because $U$ and $V$ are both proportional to $1 / \mathrm{r}$, and the magnitudes of $\vec{F}$ and $\vec{E}$ are inversely proportional to the square of the distance.

20-2. Equipotential surfaces are
a. perpendicular to electric field lines. b. parallel to electric field lines.

ANSWER: perpendicular: see section 20.2

20-3: When the potential difference between the plates of a parallel-plate capacitor is doubled (while keeping the area and plate separation distance constant), the magnitude of the charge stored on one of the plates
a. is doubled. b. is increased by a factor of 4. c. is tripled. d. remains the same. e. is halved.
ANSWER: If $\Delta \mathrm{V}$ is doubled, and $d$ is kept constant, $\Delta \mathrm{V}=E d$ tells us that $E$ must also double. Also, the magnitude of the electric field inside the capacitor is given by $E=\sigma / \epsilon_{0}$. That is, if $E$ doubles, then the charge density $\sigma$ must also double. $\sigma=\mathrm{Q} / \mathrm{A}$, and the area A does not change. So Q must also double. This extra charge is drawn from the extra source of potential difference to reside on the plates.

20-4: Four positive point charges, each with identical amounts of charge $Q$, are placed $(+1,0)$, $(-1,0),(0,+1)$, and $(0,-1)$. Which is not zero at the origin?
a. V
b. $\vec{F}_{e}$ on a positively-charged particle placed at the origin
c. $\vec{F}_{e}$ on a negatively-charged particle placed at the origin
d. $\vec{E}$

ANSWER: Due to symmetry, the total $\vec{E}$ at the origin must be zero. Similarly, a negatively-(positively-) charged particle placed at the origin will experience a net electrostatic force of 0 because it attracted to (repelled by) each of the four positively-charged particles equally. Also, since $\vec{E}$ is zero at the origin, for any test particle with charge $q_{0}$ placed at the origin, $\vec{F}_{e}=q_{0} \vec{E}$ will also be zero for any test charge. The total potential at the origin will be equal to $4\left(k_{e} Q / r\right)$, where $r$ is the distance from the origin to each charge. This cannot be equal to zero.

20-5. When the electric field is zero at all points in any region of space
a. the potential must be zero at all points in that region.
b. the potential must have a constant value at all points in that region.
c. the potential cannot be found unless we know its value at a minimum of two points in the region.
d. charged bodies cannot be present on the boundaries of the region.
e. the region might be found between the plates of a charged capacitor.

ANSWER: b: The electric field can be found by taking the partial derivative of the potential, i.e., $E_{x}=\frac{\partial V_{x}}{\partial x}, E_{y}=\frac{\partial V_{y}}{\partial y}$, etc. If $V$ is a constant, then $E$ will be zero. If $E$ is to be non-zero, then $V$ must have at least a linear variation in at least one direction in space.

20-6. When a cavity is present inside a conductor in equilibrium
a. the potential in the cavity must be zero.
b. the electric field in the cavity must be zero.
c. the electric field in the cavity must have a constant non-zero value.
d. the electric field in the cavity must decrease inversely as the square of the distance from the walls.
e. no two points on the surface of the cavity can be at the same potential.

ANSWER: B: The electric field is zero, as described in section 20.6.

20-7. Three point charges are positioned on the x-axis: $Q_{1}=+3 \mu \mathrm{C}$, located at $\mathrm{x}=+0 \mathrm{~cm}$, $Q_{2}=-2 \mu \mathrm{C}$, located at $\mathrm{x}=+10 \mathrm{~cm}$, and $Q_{3}=+8 \mu \mathrm{C}$, located at $\mathrm{x}=+25 \mathrm{~cm}$. What is the magnitude and direction of the net electrostatic force acting on each particle? How much work is required by us to to assemble this system by simultaneously bringing in all three points from infinity?

ANSWER: The net electrostatic force acting on particle 1 due to the electric fields set up by the other two particles will be $\overrightarrow{F_{1}}=\overrightarrow{F_{21}}+\overrightarrow{F_{31}}$, where $\overrightarrow{F_{21}}$ denotes the force on particle 1 due to particle 2, etc.
$\vec{F}_{1}=\frac{k_{e} q_{1} q_{2}}{r_{21}^{2}} \hat{r_{21}}+\frac{k_{e} q_{1} q_{3}}{r_{31}^{2}} \hat{r_{31}}$, where $r_{21}$ is the distance from particle 2 to particle 1 , and $\hat{r_{21}}$ is the unit vector directed from the position of particle 2 to that of particle 1 , etc. $\hat{r_{21}}$ and $\hat{r_{11}}$ in this case are both directed in the $-\hat{x}$ direction.
$\vec{F}_{1}=\frac{\left(k_{e}\right)(+3 \mu C)(-2 \mu C)}{(0.1 m)^{2}}(-\hat{x})+\frac{\left(k_{e}\right)(+3 \mu C)(+8 \mu C)}{(0.25 m)^{2}}(-\hat{x})$
Collecting the minus signs, we get:
$\vec{F}_{1}=\frac{\left(k_{e}\right)(3 \mu C)(2 \mu C)}{(0.1 m)^{2}}(\hat{x})+\frac{\left(k_{e}\right)(3 \mu C)(8 \mu C)}{(0.25 m)^{2}}(-\hat{x})$
So $\overrightarrow{F_{21}}$ points towards $+\hat{x}$ (attractive force pulling 1 towards 2), and $\overrightarrow{F_{31}}$ points towards $-\hat{x}$ (repelling force pushing 1 away from 3).
The NET force on particle 1 is thus
$\vec{F}_{1}=(5.4 \mathrm{~N})(+\hat{x})+(3.5 \mathrm{~N})(-\hat{x})=(+1.9 \mathrm{~N})(+\hat{x})$.
For particle 2:
$\overrightarrow{F_{2}}=\overrightarrow{F_{12}}+\overrightarrow{F_{32}}$
$\overrightarrow{F_{2}}=\frac{k_{e} q_{2} q_{1}}{\left(r_{12}\right)^{2}} r \hat{12}+\frac{k_{e} q_{2} q_{3}}{\left(r_{32}\right)^{2}} r_{32}$
$\vec{F}_{2}=\frac{\left(k_{e}\right)(-2 \mu C)(+3 \mu C)}{(0.1 m)^{2}}(+\hat{x})+\frac{\left(k_{e}\right)(-2 \mu C)(+8 \mu C)}{(0.15 m)^{2}}(-\hat{x})$
Collecting the minus signs, we get:
$\vec{F}_{2}=\frac{\left(k_{e}\right)(2 \mu C)(+3 \mu C)}{(0.1 m)^{2}}(-\hat{x})+\frac{\left(k_{e}\right)(2 \mu C)(+8 \mu C)}{(0.15 m)^{2}}(+\hat{x})$
So $\overrightarrow{F_{12}}$ points towards $-\hat{x}$ (attractive force pulling 2 towards 1 ), and $\overrightarrow{F_{32}}$ points towards $+\hat{x}$ (attractive force pulling 2 towards 3).
The NET force on particle 2 is thus
$\vec{F}_{2}=(5.4 \mathrm{~N})(-\hat{x})+(6.4 \mathrm{~N})(+\hat{x})$
$\overrightarrow{F_{2}}=(1.0 \mathrm{~N})(+\hat{x})$
For particle 3:
$\overrightarrow{F_{3}}=\overrightarrow{F_{13}}+\overrightarrow{F_{23}}$
$\overrightarrow{F_{3}}=\frac{k_{e} q_{3} q_{1}}{\left(r_{13}\right)^{2}} \hat{r_{13}}+\frac{k_{e} q_{3} q_{2}}{\left(r_{23}\right)^{2}} \hat{r_{23}}$
$\vec{F}_{3}=\frac{\left(k_{e}\right)(+8 \mu C)(+3 \mu C)}{(0.25 m)^{2}}(+\hat{x})+\frac{\left(k_{e}\right)(+8 \mu C)(-2 \mu C)}{(0.15 m)^{2}}(+\hat{x})$
Collecting the minus signs, we get:
$\vec{F}_{3}=\frac{\left(k_{e}\right)(+8 \mu C)(+3 \mu C)}{(0.25 m)^{2}}(+\hat{x})+\frac{\left(k_{e}\right)(+8 \mu C)(2 \mu C)}{(0.15 m)^{2}}(-\hat{x})$
So $\overrightarrow{F_{12}}$ points towards $+\hat{x}$ (repelling force pushing 3 away from 1 ), and $\overrightarrow{F_{23}}$ points towards $-\hat{x}$ (attractive force pulling 3 towards 2 ).
The NET force on particle 3 is thus
$\vec{F}_{3}=(3.5 \mathrm{~N})(+\hat{x})+(6.4 \mathrm{~N})(-\hat{x})$
$\vec{F}_{3}=(2.9 \mathrm{~N})(-\hat{x})$

Notice that $\overrightarrow{F_{13}}=-\overrightarrow{F_{31}}, \quad \overrightarrow{F_{12}}=-\overrightarrow{F_{21}}, \quad$ and $\overrightarrow{F_{32}}=-\overrightarrow{F_{23}}$;
in each case, vectors are equal in magnitude but opposite in direction.
To determine the amount of work needed to assemble this 3-particle system by moving the points in from infinity, we must calculate the system's total potential energy, considering all three pairs of interactions:
$U_{\text {tot }}=U_{12}+U_{23}+U_{13}$
$U_{\text {tot }}=\frac{k_{e} q_{1} q_{2}}{r_{12}}+\frac{k_{e} q_{2} q_{3}}{r_{23}}+\frac{k_{e} q_{1} q_{3}}{r_{13}}$
$U_{\text {tot }}=\frac{\left(k_{e}\right)(+3 \mu C)(-2 \mu C)}{0.1 m}+\frac{\left(k_{e}\right)(-2 \mu C)(+8 \mu C)}{0.15 m}+\frac{\left(k_{e}\right)(+3 \mu C)(+8 \mu C)}{0.25 m}$
$U_{\text {tot }}=(-0.54 \mathrm{~J})+(-0.96 \mathrm{~J})+(+0.86 \mathrm{~J})=-0.64 \mathrm{~J}$.
Note that because the potential energy is negative, this is a relatively stable system. To assemble it, we (as an external agent) don't even have to put in any work; the electric fields can do all the work ( +0.64 J of work, to be exact) for us in this case.
21.1. A 9.0 -volt battery moves 100 milliCoulombs of charge through a circuit connecting the battery's terminals. How much energy was delivered to the circuit?
a. 90.0 J
b. 9.0 J
c. 0.9 J
d. 3.0 J
e. 0.3 J

The energy can calculated via $\Delta P E=q \Delta V=(0.1 \mathrm{C})(9.0 \mathrm{~V})=0.9 \mathrm{~J}$.
21.2: Find $R_{e q}$ and the magnitude of the currents through each resistor.


ANSWER: Solution: The three resistors in the right
 branches are all connected in parallel.
$\frac{1}{R_{e q}}=\frac{1}{5 \Omega}+\frac{1}{10 \Omega}+\frac{1}{5 \Omega}=\frac{5}{10 \Omega}=\frac{5}{10 \Omega}$
$R_{e q 1}=2 \Omega$
We then take this resistance and combine it in series with the $5 \Omega$ resistor on the bottom to obtain $R_{e q}$ for the whole circuit:
$R_{e q}=2 \Omega+5 \Omega=7 \Omega$
Let's call $I_{\text {tot }}$ the current through the battery, and $I_{1}, I_{2}$, \& $I_{3}$ the currents through each of the branches on the
right side; in accordance with the Junction Rule, $I_{t o t}=I_{1}+I_{2}+I_{3}$.
Now we can use $\epsilon=I_{t o t} R_{e q}$ to solve for $I_{t o t}$ :
$I_{t o t}=21 \mathrm{~V} / 7 \Omega=3 \mathrm{~A}$.
To solve for $I_{1}, I_{2}, \& I_{3}$, we note that the voltage drop across each of the resistors in the right branch is identical - but it will not be equal to 21 V . In accordance with the loop rule: It will be equal to 21 V minus the $\Delta \mathrm{V}$ associated with traversing the bottom $5 \Omega$ resistor. That $\Delta \mathrm{V}$ is $(5 \Omega)\left(I_{t o t}\right)=(5 \Omega)(3 \mathrm{~A})=15 \mathrm{~V}$.
The voltage drop across each branch is thus 6 V . We now solve for the current in each branch: The current flowing through the $10 \Omega$ resistor is $\Delta \mathrm{V} / \mathrm{R}=6 \mathrm{~V} / 10 \Omega=0.6 \mathrm{~A}$. The current flowing through each $5 \Omega$ resistor on the right side is $\Delta \mathrm{V} / \mathrm{R}=6 \mathrm{~V} / 5 \Omega=1.2 \mathrm{~A}$.
Notice that the Junction Rule, $I_{\text {tot }}=I_{1}+I_{2}+I_{3}$, is confirmed: $3 \mathrm{~A}=1.2 \mathrm{~A}+1.2 \mathrm{~A}+0.6$ A!

22-1. The magnetism of ferromagnetic materials, such as iron, has its origin in the intrinsic magnetic moment of electrons that is associated with their
a. orbital angular momentum. b. spin angular momentum. c. electric charge. d. being magnetically soft. e. being magnetically hard.
ANSWER: B (see section 22.11)

22-2. When two parallel current-carrying wires have currents in the same direction, the wires a. attract one another.
b. repel one another.
c. exert no forces on one another.
d. exert equal and opposite repulsive forces on one another perpendicular to the plane of the wires.
e. exert equal and opposite attractive forces on one another perpendicular to the plane of the wires.
ANSWER: A (see section 22.8)

22-3 The head end of a nail is attracted to the north pole of a magnet and also to its south pole.
a. The nail is magnetized.
b. The nail is initially not magnetized.
c. The nail is aluminum.
d. The nail is copper.
e. The magnet is aluminum.

ANSWER: B - the nail must be composed of a soft magnetic material, since it responds to whatever B-field it is placed in (near either pole of a magnet), then loses its magnetism when taken away from the pole of the magnet.

22-4. The head of a nail is attracted to the north pole of a magnet and repelled by the magnet's south pole.
a. The nail is magnetized.
b. The nail is initially not magnetized.
c. The nail is aluminum.
d. The nail is copper.
e. The magnet is aluminum.

ANSWER: A - the nail must be a permanent magnet, comprised of a hard magnetic material, and having its own permanent magnetic field.
$22-5$. If an electron moves from right to left across this page and suddenly is subjected to a magnetic field directed into the page, what is the direction of the magnetic force on the electron?
a. Into the page b. Out of the page c. From the bottom to the top of the page d. From the top to the bottom of the page e. From the left to the right of the page
ANSWER: C. Right hand rule, for a positive charge: Thumb $=\vec{v}=$ left. Fingers $=\vec{B}=$ into page. Out of palm $=\vec{F}=$ down.
Negative charge: reverse direction.

22-6: An electron is located on the Earth's magnetic equator at an altitude of several km . In which direction can the velocity vector be pointing such that the deflection due to magnetic force is zero?
Choices are: magnetic north, upward, downard, magnetic south, magnetic east, magnetic west. Note that 2 answers are acceptable.
ANSWER: If $\vec{v}$ and $\vec{B}$ are parallel to each other, then $|\vec{F}|=0$. At the magnetic equator, the local magnetic field lines are oriented magnetic-north-south. Answers $=$ magnetic $\mathrm{N} \& \mathrm{~S}$.

22-7: Suppose you have in your possession a gigantic single copper loop that has a radius $r$ of 1.1 km . You want to run a current through the loop to generate a magnetic field whose strength at the center of the loop is ten times the strength of the Earth's magnetic field at the surface (roughly 0.3 to 0.5 G , depending on location. Let's choose 0.3 G ). The total resistance of the loop is $0.12 \Omega$.
A: What EMF do you need to supply?
B: If the loop is horizontal, in which direction should the current flow if you want the B-field generated by the current loop to be directed downward?
The loop is made from pure copper, which, as per Table 21.1, has a resistivity $\rho$ of $1.7 \times 10^{-8} \Omega \mathrm{~m}$ at an operating temperature $T_{0}=20^{\circ} \mathrm{C}$.
C: The loop has a square cross section. What is the cross-sectional area of the loop and the size of the square?
D: How much power is dissipated in the resistor?
E: Suppose a friend of yours (who has not taken Physics 1B yet) thinks that making the loop hotter will "somehow" make the magnetic field strength larger. He/she heats up the loop uniformly (I dunno, a giant blowtorch?), so that its temperature is $80^{\circ} \mathrm{C}$. Assume the EMF supplied does not change. What effect will this really have on the B-field at the center
of the loop? Explain to your friend what effect increasing the temperature will have on the current (at the microscopic level) and thus the final magnetic field strength at the loop's center.
BTW, Copper has a temperature coefficient $\alpha$ of $3.9 \times 10^{-3}\left[\left({ }^{\circ} \mathrm{C}\right)^{-1}\right]$, as per Table 21.1. Ignore any thermal expansion in the copper. On an exam, you would be given $\alpha \& \rho$.

ANSWERS:
A: Our target B-field strength is 3 G .
The magnitude of the B-field at the center of a single current loop of radius $r$ is given by $B=\frac{\mu_{0} I}{2 r}$ (eqn. 22.24). Rearrange to solve for I, using SI units (Tesla, not Gauss. m, not $\mathrm{km})$ :
$I=\frac{2 B r}{\mu_{0}}=\frac{2\left(3 \times 10^{-4} T\right)(1100 \mathrm{~m})}{4 \pi \times 10^{-7} \mathrm{Tm} / \mathrm{A}}=5.3 \times 10^{5}$ Amps.
This is a very large current! But bear in mind that the B-field strength at the loop's center is proportional to $1 / r$; the farther away from the center the current is, the lower the B-field strength will be at the center.

To drive this current, you need to supply an EMF of $\epsilon=I R=\left(5.3 \times 10^{5} \mathrm{~A}\right)(0.12 \Omega)=63600$ Volts.

B: For the direction, use the right-hand rule: fingers, indicating direction of current, are clockwise to have B-field oriented downward.

C: To solve for the cross-sectional area of the loop, we go back into chapter 21 and use the equation which defines resistance of a resistor in terms of its geometric properties and resistivity: $R=\rho L / A$. Rearrange to solve for $A$ :
$A=\rho L / R$
The length $L$ is the circumference of the loop, $2 \pi r$.
$A=(\rho)(2 \pi r) /(R)$
$A=\left(1.7 \times 10^{-8} \Omega m\right)(2 \pi 1100 \mathrm{~m}) /(0.12 \Omega)=9.8 \times 10^{-4} \mathrm{~m}^{2}$.
So each side of the square has length $0.031 \mathrm{~m}=3.1 \mathrm{~cm}$.
D: The power dissipated in the resistor can be calculated via $P=I \Delta V=(530000 \mathrm{~A})(63600 \mathrm{~V})$ $=3.4 \times 10^{10} \mathrm{~W}$.
You can also use $P=I^{2} R=(530000 \mathrm{~A})^{2}(0.12 \Omega)=3.4 \times 10^{10} \mathrm{~W}$.
$P=(\Delta V)^{2} / R$ gives the same result, too.
E: When the temperature is increased from $20^{\circ} \mathrm{C}$ to $80^{\circ} \mathrm{C}$, the atoms in the conducting lattice vibrate much more strongly, thereby increasing the frequency of collisions between the charge carriers (electrons) and the lattice of the conductor. The resistance of the copper conductor increases. (this is of course just a quick summary, see 21.1 and 21.4 for more details).
For a given EMF, the resulting current is thus lower (in accordance with Ohm's Law).
$\rho_{o}=1.7 \times 10^{-8} \Omega \mathrm{~m}$ at $T_{0}=20^{\circ} \mathrm{C}$.
$\rho=? ? ? ? ? \Omega \mathrm{~m}$ at $T=80^{\circ} \mathrm{C}$.
Using equation 21.10,
$\rho=\rho_{0}\left[1+\alpha\left(T-T_{0}\right)\right]$
$\rho=1.7 \times 10^{-8} \Omega \mathrm{~m}\left[1+3.9 \times 10^{-3}\left[\left({ }^{\circ} \mathrm{C}\right)^{-1}\right]\left(80^{\circ} \mathrm{C}-20^{\circ} \mathrm{C}\right)\right]$
$\rho=1.7 \times 10^{-8} \Omega \mathrm{~m}\left[1+3.9 \times 10^{-3}\left[\left({ }^{\circ} \mathrm{C}\right)^{-1}\right]\left(60^{\circ} \mathrm{C}\right)\right]$
$\rho=1.7 \times 10^{-8} \Omega \mathrm{~m}[1+0.23]$
$\rho=1.7 \times 10^{-8} \Omega \mathrm{~m}[1.23]$
$\rho=2.1 \times 10^{-8} \Omega \mathrm{~m}$
So your friend has increased the resistivity and thus the resistance by a factor of 1.23 ; $R=0.148 \Omega$. The final current will be a factor of 1.23 lower at the same EMF supplied earlier: $I=63600 \mathrm{~V} / 0.148 \Omega=430000 \mathrm{~A}$.
The final B-field strength at the center of the loop will also be a factor of 1.23 lower, because $B$ is proportional to $I$.
$B=\frac{\mu_{0} I}{2 r}=2.44$ Gauss.

22-8: What does a commutator do?
ANSWER: Electric motors rely on AC current to rotate loops non-stop, but commutators are used in electric motors which rely on DC power sources. A commutator is a split-ring device used to switch the sign of the current every half-cycle of the rotating loop, thereby transforming DC current into AC current in the loop. (11/23 lecture)
In electrical generators, which naturally produce AC current, commutators can be used to transform AC into DC ( $12 / 2$ lecture).

23-1: The magnetic flux $\Phi_{\mathrm{B}}$ through a 100 -turn coil increases from 0.04 Wb to 0.09 Wb in a time $\Delta t$ of 0.5 sec . Calculate the average $\epsilon_{\text {ind }}$. Then, assuming that the total resistance in the entire coil is $2 \Omega$, calculate the magnitude of the average induced current.

ANSWER:
$\Phi_{\mathrm{B} 1}=0.04 \mathrm{~Wb}$.
$\Phi_{\mathrm{B} 2}=0.09 \mathrm{~Wb}$.
$\Delta \Phi_{\mathrm{B}}=+0.05 \mathrm{~Wb}$.
From Faraday's Law: $\epsilon_{\text {ind }}=-N \frac{\Delta \Phi_{B}}{\Delta t}=-100 \frac{0.05 \mathrm{~Wb}}{0.5 \mathrm{sec}}=-10 \mathrm{~V}$.
Neglect minus sign, since we're just concerned with magnitude here:
$\mathrm{I}=\epsilon_{\text {ind }} / R=10 \mathrm{~V} / 2 \Omega=5 \mathrm{~A}$.

23-2: The magnetic field in a certain region of space is uniform, but is decreasing exponentially at a rate quantified as $B(t)=e^{-c t}$ where $c$ is a constant. Derive an expression for the induced EMF as a function of time through a single conducting loop with area A.

ANSWER: From Faraday's Law: $\epsilon_{\text {ind }}(t)=-N \frac{d \Phi_{B}(t)}{d t}$
Here, $N$ can be set to 1 .
$\Phi_{\mathrm{B}}(t)=A B(t)$
$\frac{d \Phi_{\mathrm{B}}(t)}{d t}=A \frac{d B(t)}{d t}$
$\epsilon_{\text {ind }}(t)=-A \frac{d B(t)}{d t}$
$\epsilon_{\text {ind }}(t)=(-1)-A c e^{-c t}$
$\epsilon_{\text {ind }}(t)=+A c e^{-c t}$

23-3: A 100 -turn coil with cross-section $0.1 \mathrm{~m}^{2}$ is immersed in a magnetic field of 0.01 T . Initially, the plane of the loops is perpendicular to the magnetic field lines. Then, in a time $\Delta t$, the coil is turned $90^{\circ}$ such that the magnetic flux is zero. You would like to obtain an induced EMF in the coil whose magnitude is 2 V . Calculate the $\Delta t$ necessary to accomplish this.

ANSWER: From Faraday's Law: $\epsilon_{\text {ind }}=-N \frac{\Delta \Phi_{B}}{\Delta t}$
$\Delta \Phi_{B}=B \Delta A$.
$\Delta A=A-0$
Rearrange Faraday's Law to solve for $\Delta t$ (and let's drop the negative sign since time cannot be negative, and we were just given the magnitude, not direction, of the induced emf) : $\Delta t=\left(N \Delta \Phi_{B}\right) / \epsilon_{\text {ind }}=\left((100)(0.01 \mathrm{~T})\left(0.1 \mathrm{~m}^{2}\right)\right) /(2 \mathrm{~V})=0.05 \mathrm{~s}$.

23-4: A bar magnet approaches, and passes through, a conducting loop from the left with its north pole directed towards the right. It moves with a constant velocity; the loop, which has radius $r$, is held stationary. The current in the loop has its greatest magnitude AND a clockwise direction a. as the north pole is approaching the loop, a distance $3 r$ away
b. just as the north pole of the magnet is entering the loop
c. When the magnet is exactly halfway through the loop
d. as the south pole is leaving the loop
e. after the south pole has left the loop and is a distance $3 r$ away

ANSWER: The CHANGE in magnetic flux is maximum when one of the poles of the magnet is passing through the loop.
When the north pole is first entering the loop, the magnet's B-field lines are pointing from left to right and the magnetic flux through the loop is increasing. Consistent with Lenz's law, the induced B-field will try to counteract this increase, and will be oriented right-toleft. The induced current in the loop needed to generate right-to-left induced B-fields will go counterclockwise.
Later, when the south pole is passing through the loop, the magnet's B-field lines are still pointing from left to right, though now the magnetic flux through the loop is decreasing. Consistent with Lenz's law, the induced B-field will try to counteract this decrease, and will be oriented left-to-right to attempt to bolster the decreasing original field. The induced current in the loop needed to generate left-to-right induced B-fields will go clockwise.
Final answer $=$ D.

23-5. An ideal transformer is one that:
a. has a turn ratio, $\mathrm{N}_{2} / \mathrm{N}_{1}$, equal to 1 .
b. works with direct current.
c. experiences no power loss.
d. has an output frequency of 60 Hz .

ANSWER $=$ No power loss.

23-6. An AC power source produces a peak voltage of 7200 V and a peak current of 100 A in a circuit. This circuit is attached to an ideal transformer whose job is to step-up the voltage to $200,000 \mathrm{~V}$ (peak) for energy transmission in a long-distance power line. What is the peak current in the long-distance line?
a. 1.0 A
b. 2.8 A
c. 24 A
d. 2800 A
e. 3.6 A

Solution: Power input at primary $=$ power output at secondary: $I_{1} \Delta V_{1}=I_{2} \Delta V_{2}$
$I_{2}=I_{1} \Delta V_{1} / \Delta V_{2}=(100 \mathrm{~A})(7200 \mathrm{~V}) /\left(2 \times 10^{5} \mathrm{~V}\right)=3.6 \mathrm{~A}$

