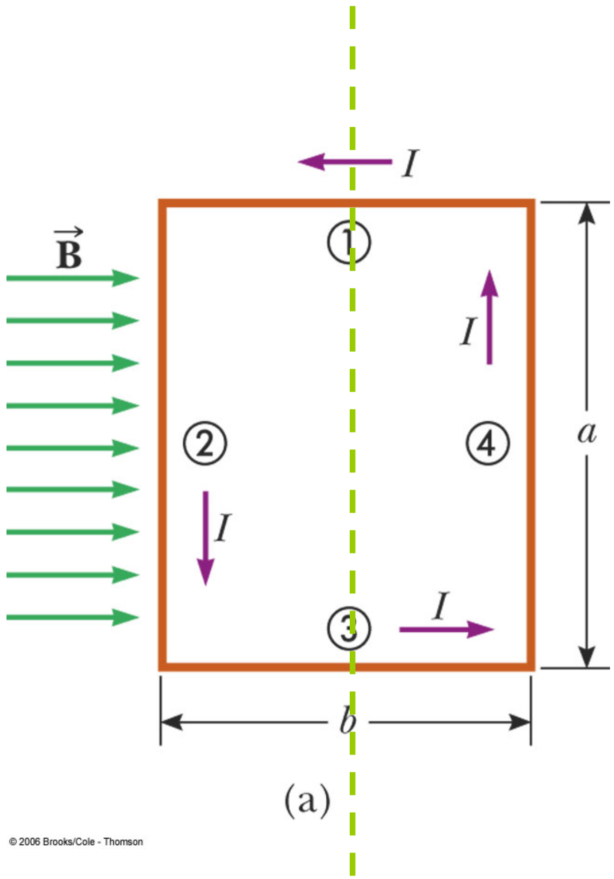
Force on a square loop of current in a uniform B-field.

$F_{top} = 0$ $\theta = 0$; $\sin\theta = 0$; so $F_B = 0$
$F_{bottom} = 0$

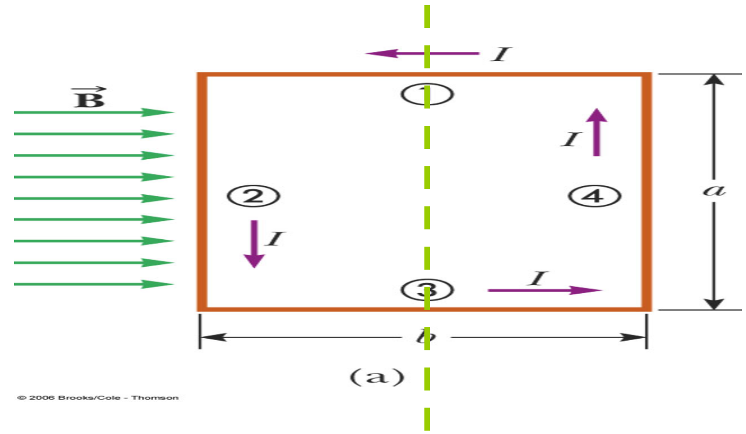
$F_{left} = I\,a\,B$ (out of page)
$F_{right} = I\,a\,B$ (into page)



Assume loop is on a frictionless axis

What's the TORQUE on the current loop?

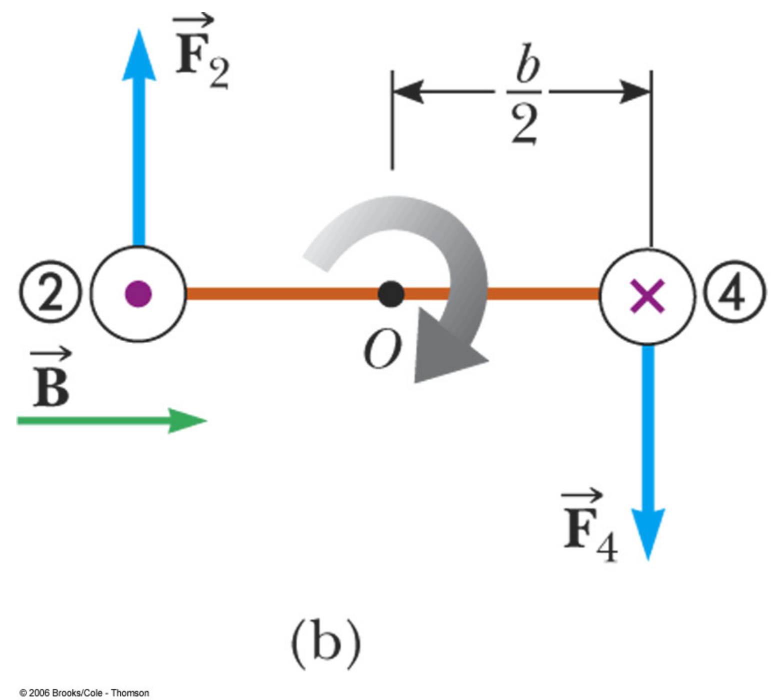
Fig. 22.19b in text is the view along the axis, from the bottom towards the top.



Reminder: torque = $\vec{F} \times \vec{r}$ = F r sinθ

$\tau = F_{left}\, b/2 + F_{right}\, b/2 =$
(B a I + B a I) b/2 = B I A

A = area; θ=90º here

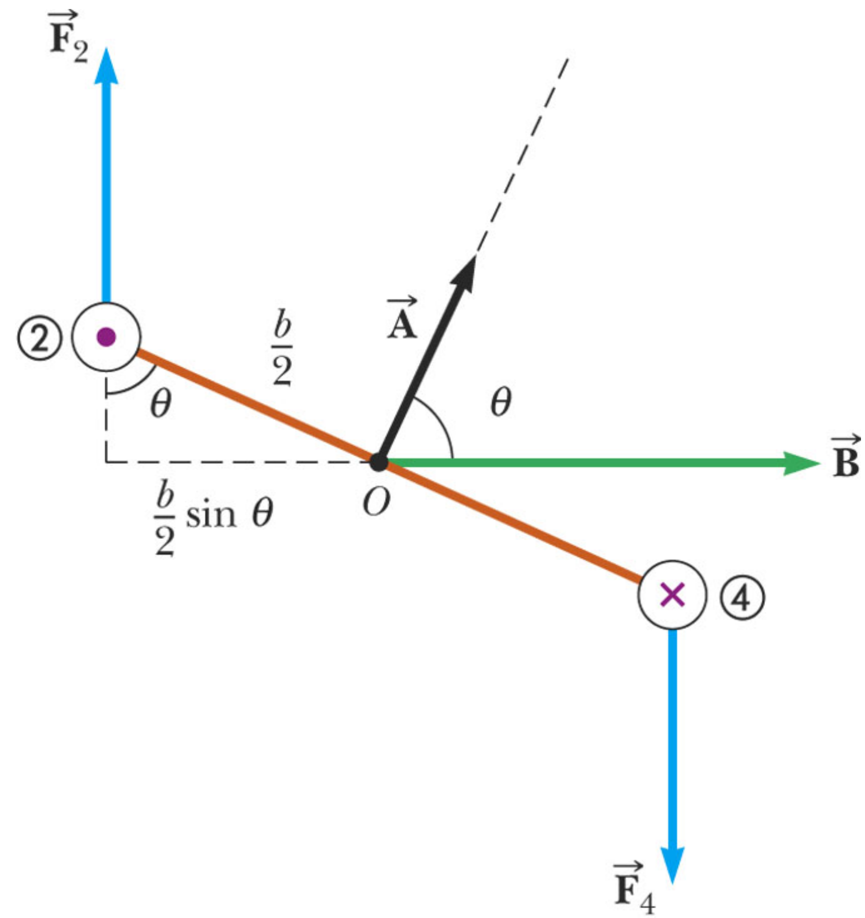


Note direction of torque: clockwise

For 1 loop:

$\tau = BIA \sin\theta$

$\tau_{max} = BIA$



For N turns: Total current = NI

$\tau = BIAN \sin\theta$

Magnetic Moment $\vec{\mu} = I\vec{A}N$

$\vec{\mu}$ always points perp. to the plane of the loops (points along the normal)

$\tau = \mu B \sin\theta$

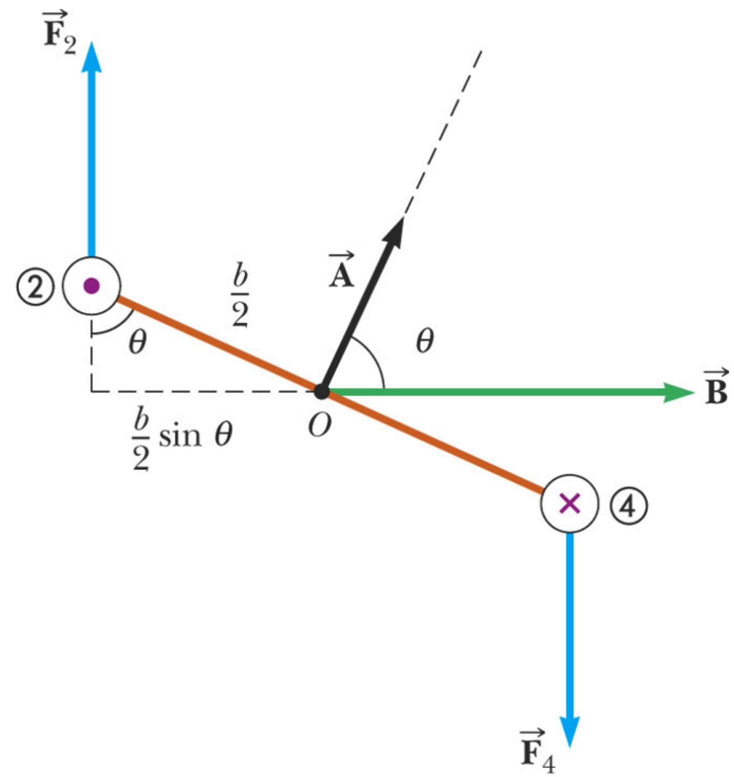
$$\vec{\tau} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

- The product $I\vec{A}$ is defined as the ***magnetic dipole moment,*** $\vec{\mu}$ of the loop (for ANY loop shape)
- SI units: A m^2
- Torque in terms of magnetic moment:

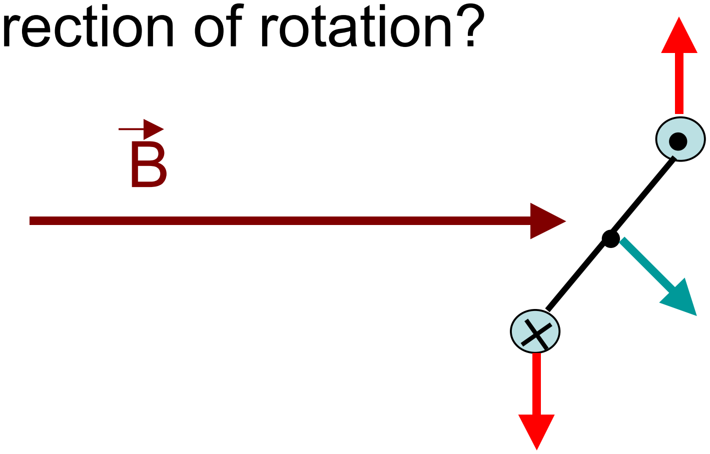
$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$$

For a coil with N turns of wire:

$\vec{\mu} = NI\vec{A}$

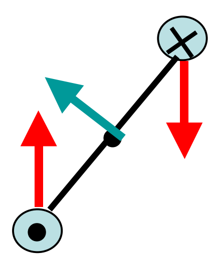


A coil consisting of 100 turns, each carrying 3A of current and having an area 0.2 m^2, is oriented such that its normal makes a angle of 45º with a B-field of 0.5T. Find the total torque on the coil. What's the direction of rotation?

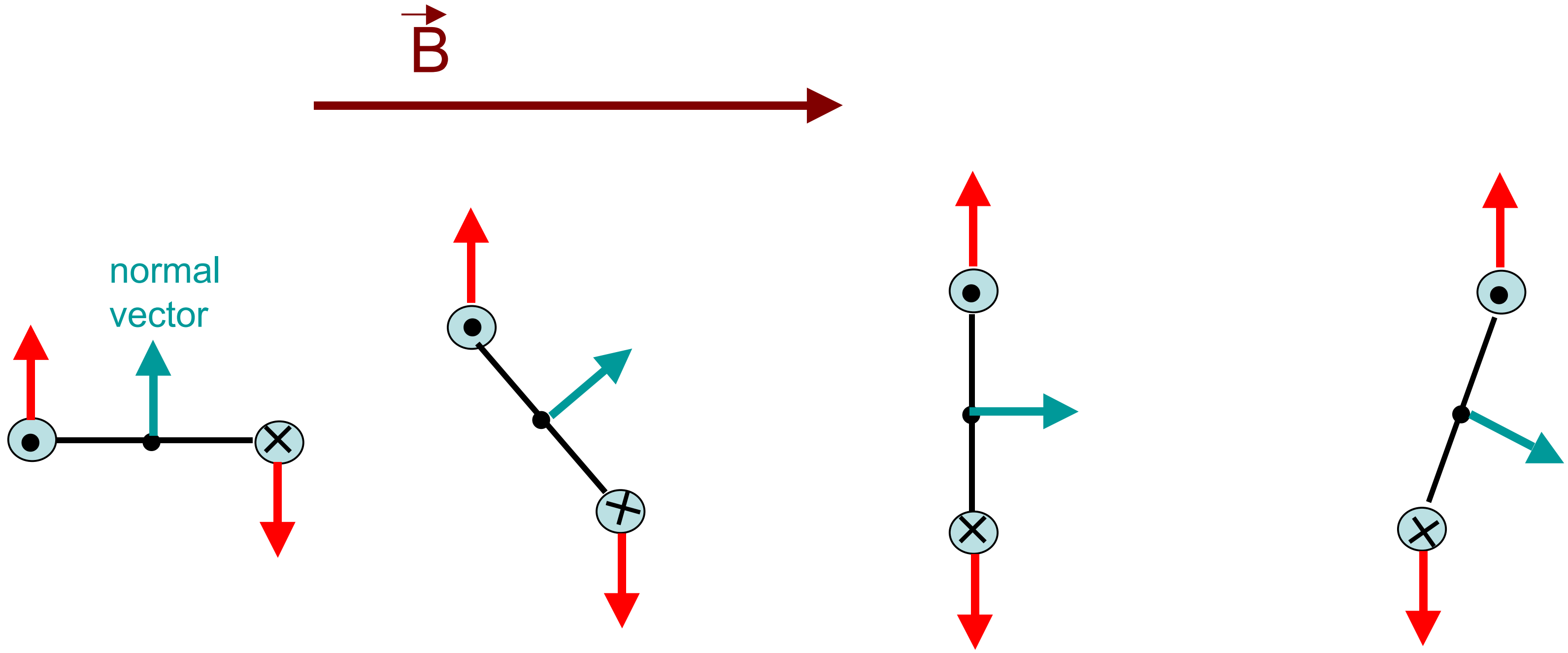


τ = BIANsinθ = (0.5T)(3A)(100)(0.2m^2)sin45º = 21.2 Nm

What would happen if the current were flowing in the opposite direction?



Same magnitude of τ, but rotation is now CW



#1: θ=90º

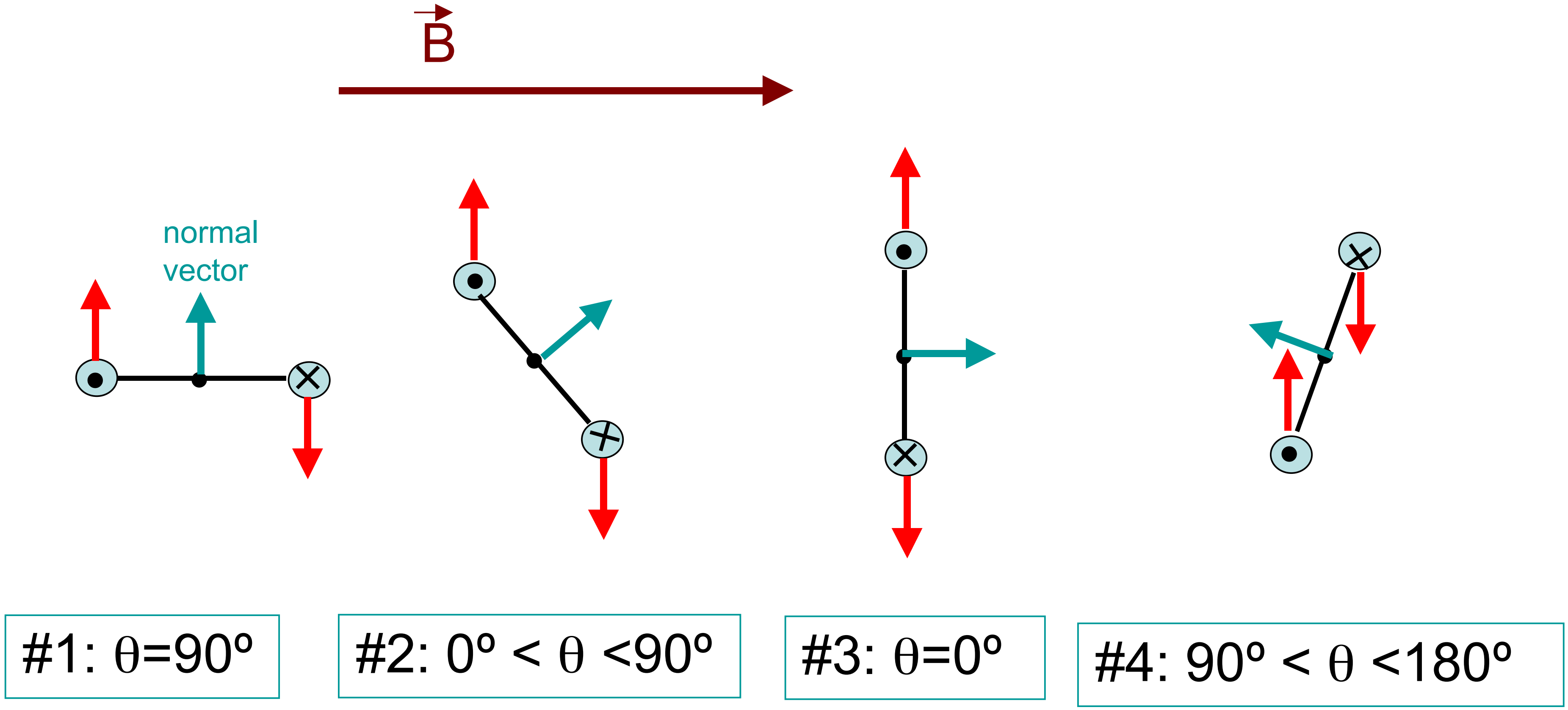
#2: 0º < θ <90º

#3: θ=0º

#4: 90º < θ <180º

torque acts to align plane of loop perpendicular to B-field (align normal vector with B-field), as in #3

(if released from rest in this position, it won't rotate)



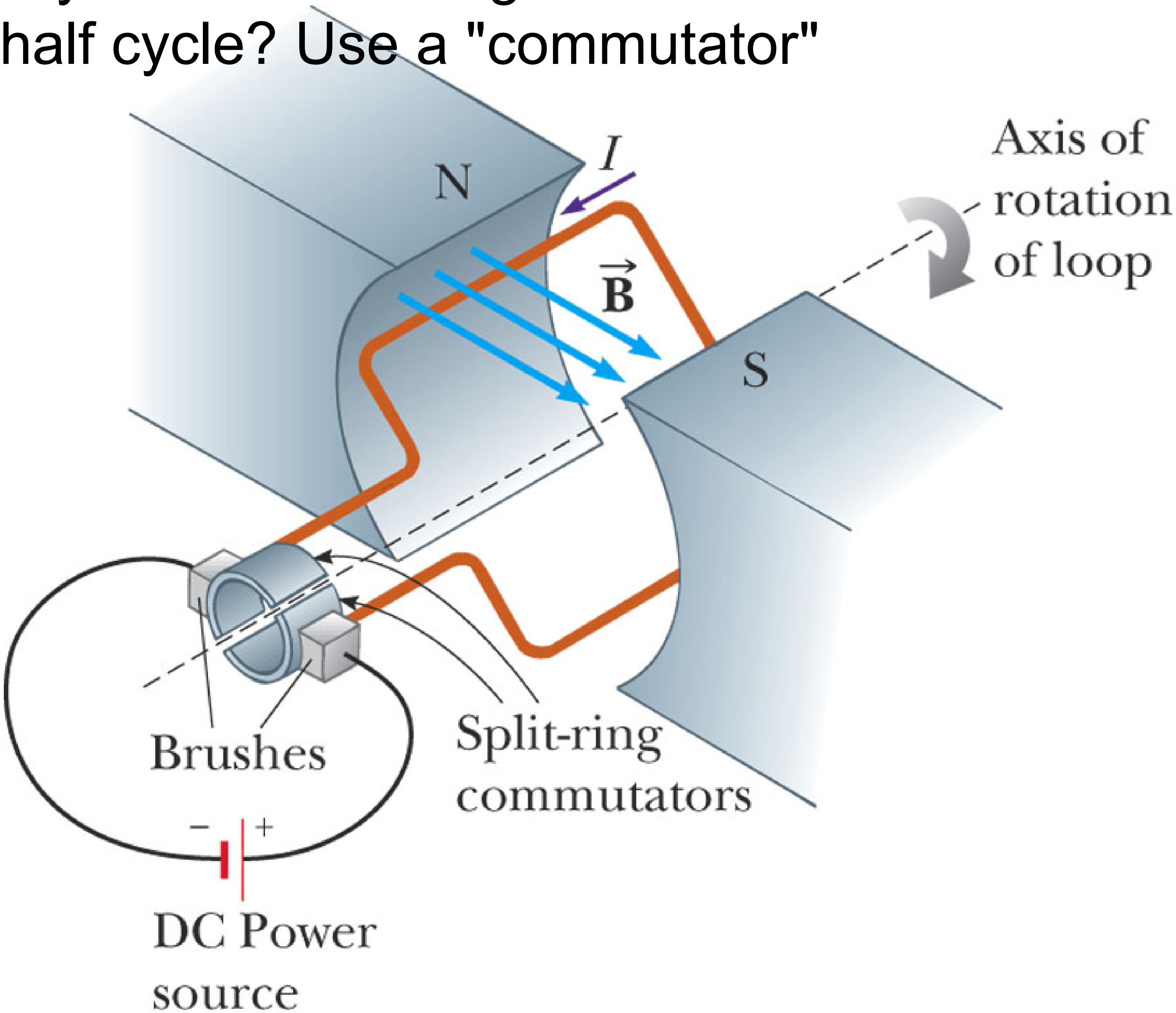
As loop is rotating, what would happen if we switched the direction of current immediately after #3?

The loop would continue to rotate clockwise!

Electric motors

- If direction of current is switched every time τ is about to change sign, then τ will never change sign!
- Loop will rotate nonstop: we have an electric motor (electrical energy converted to mechanical (rotational) energy)!
- Fans, blenders, power drills, etc.
- Use AC current (sign changes naturally), or if you only have DC current available….

How do you switch the sign of current every half cycle? Use a "commutator"



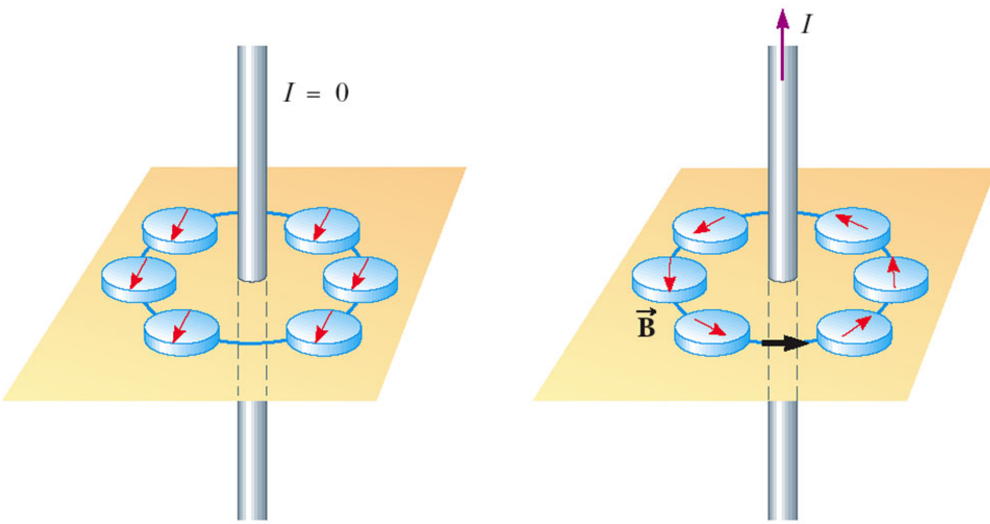
22.7 & 22.9 Biot-Savart Law / Ampere's Law

Context: Previous sections discussed what happens when moving charges are placed in a previously-existing B-fields: charged particles/current-carrying wires experience magnetic force; a loop of wire experiences a torque.

But what can GENERATE a magnetic field?

Magnetic Field of a long straight wire

In 1819, Oersted (Denmark) noticed that a magnet (compass needle) was deflected when current was drawn through a nearby wire. 1820: compasses in a horizontal plane:



Magnetic Field of a long straight wire

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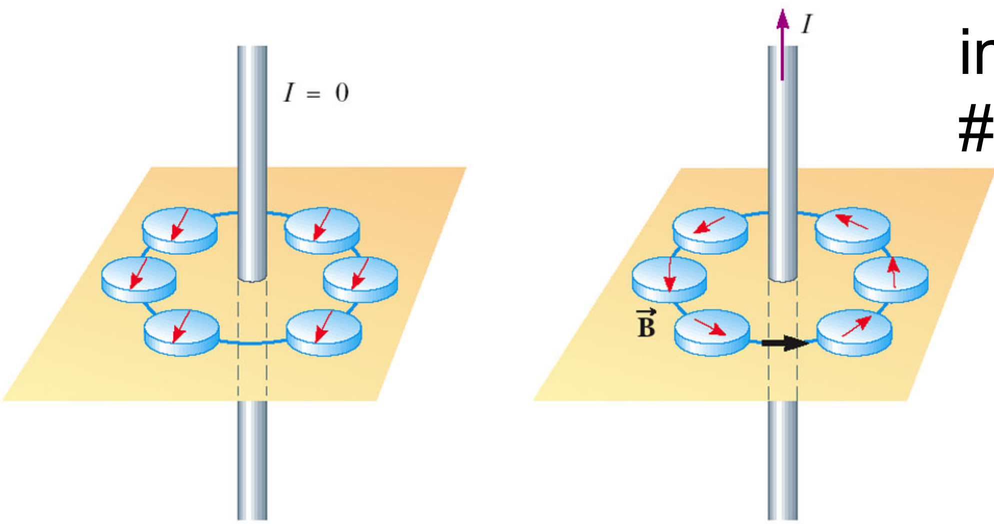
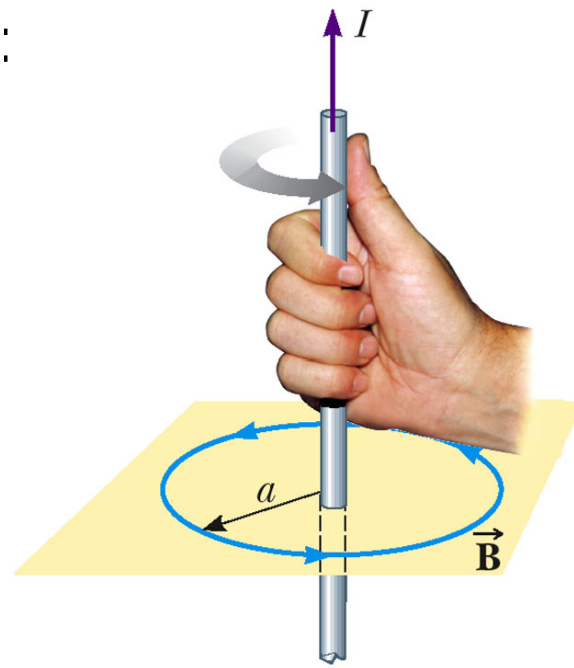


Direction of deflection indicated “right-hand rule #2”:

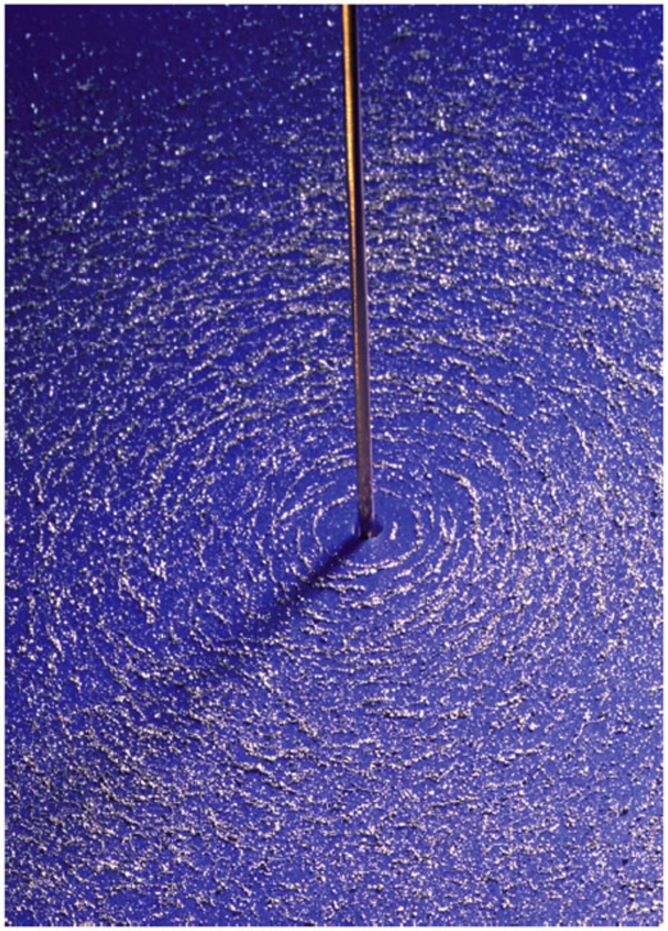


Magnetic Field of a long straight wire

B-field lines form concentric circles:

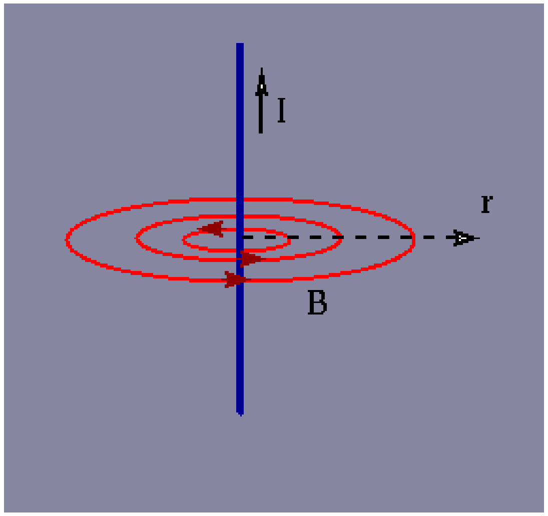
Notice that the iron filings are more strongly aligned closer to the wire

Magnitude of $\vec{B}$ is the same everywhere along a given radius: $|\vec{B}|$ depends only on r (& physical constants)



Magnetic Field of a long straight wire

The magnitude of the field at a distance r from a wire carrying a current of I is



$$B = \frac{\mu_o I}{2\pi r}$$

$\mu_o = 4\,\pi \times 10^{-7}$ T·m / A

μ_o is called the *permeability of free space*

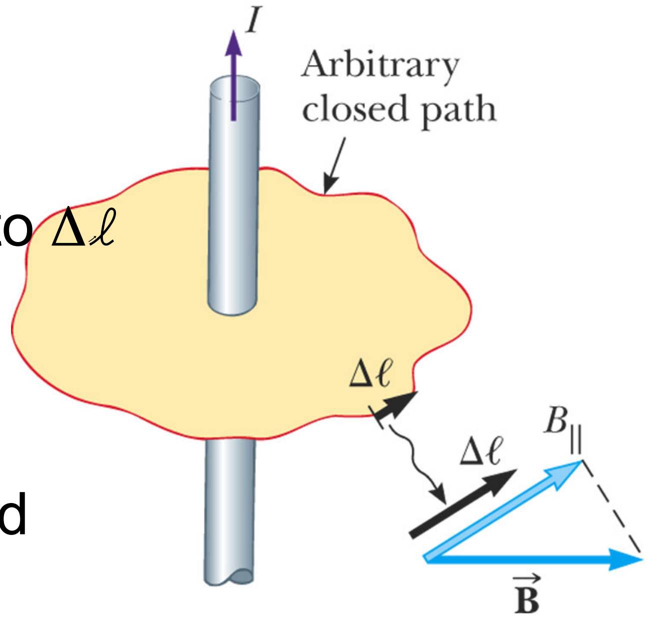
Ampere's Law: A general method for deriving magnitude of B-field due to sources of current

1. Construct a closed path consisting of short segments, each of length $\Delta\ell$

2. B = magnetic field
$B_{||}$ = Component of B-field PARALLEL to $\Delta\ell$

Consider the product $B_{||} * \Delta\ell$

3. Sum of these products over the closed path $= \mu_0$ I



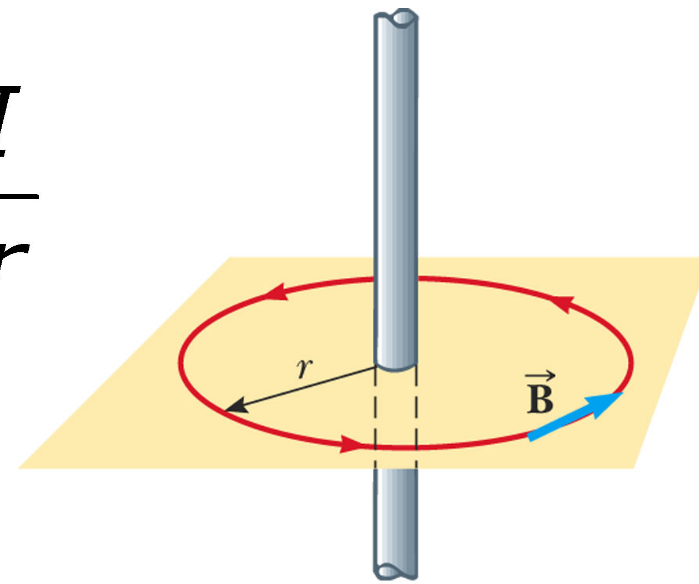
$$\Sigma B_{||} \Delta\ell = \mu_o I$$

Take advantage of symmetry. When B-field lines are circles, we choose a circular Amperian loop. B is already || to the circle at all points on the circle

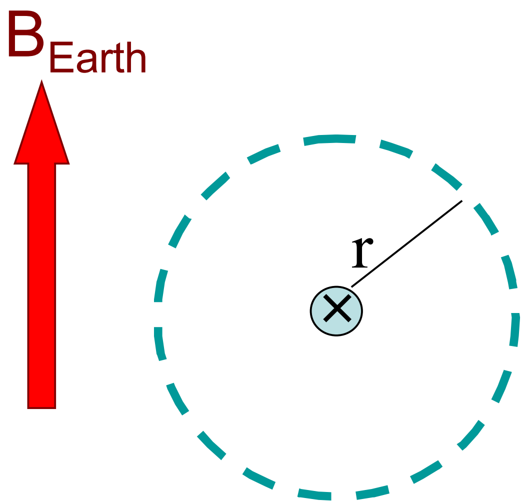
$\Sigma B_{||}\Delta\ell$ = $B_{||}$ * circumference

= $B_{||}$ * $2\pi r$ = $\mu_0 I$

Rearrange to get $$B = \frac{\mu_o I}{2\pi r}$$



Example: A wire carrying 5A of current travels vertically into the page. At what distance r will the B-field equal the Earth's B-field (which points northward) at the surface, 0.5 Gauss?



$$B = \frac{\mu_o I}{2\pi r}$$

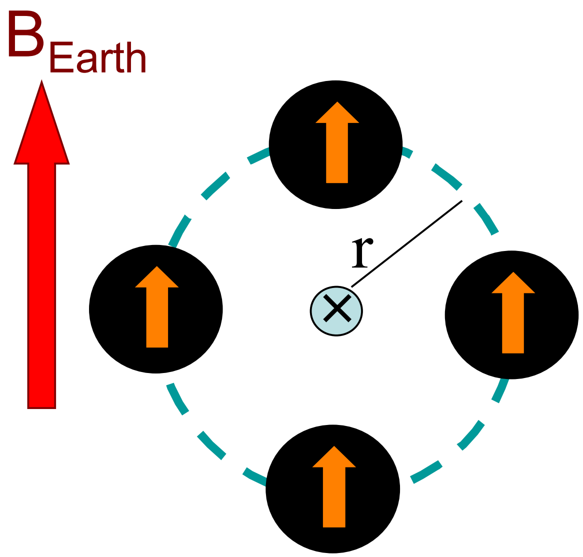
$r = \mu_0 I / 2\pi B$

$r = (4\pi \times 10^{-7}\ T\ m / A)(5A)/ (2\ \pi\ 0.5\times 10^{-4} T)$

$= (2 \times 10^{-7}\ T\ m / A)(5A) /\ \ 0.5\times 10^{-4} T$

$= 2 \times 10^{-2}$ m = 2 cm.

Example: A wire carrying 5A of current travels vertically into the page. At what distance r will the B-field equal the Earth's B-field (which points northward) at the surface, 0.5 Gauss? If 4 compasses are placed N,S,E,W of the wire at this radius, how will each compasses' needle be deflected?



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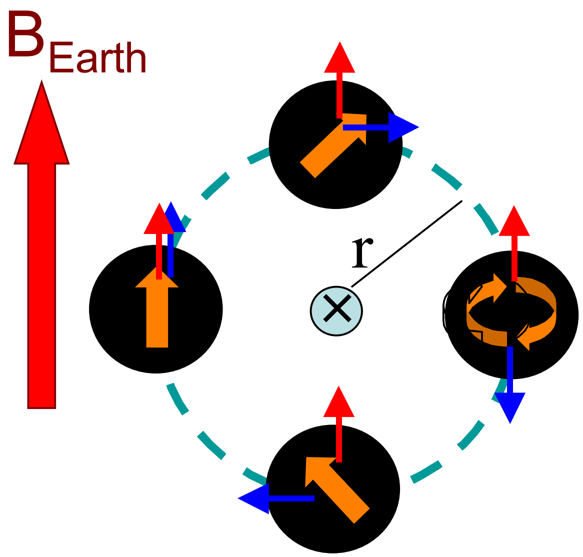

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$= (2 \times 10^{-7}\ T\ m / A)(5A) /\ \ 0.5\times10^{-4} T$

$= 2 \times 10^{-2}\ m = 2\ cm.$

The E compass will spin freely ($F_{B\_Earth} = F_{B\_wire}$).

The W compass will really point north, as it feels 2 * F_{B_Earth}

The N compass will point NE

The S compass will point NW

A power line 20m above the ground carries a current of 1000 A from E to W. Find the magnitude and direction of the B-field due to the wire at the ground below the line, and compare it to the Earth's B-field. Repeat for a wire 10m above ground.

At 20m:

$B = \mu_0 I / 2\pi r$

$= (4\pi \times 10^{-7}$ T m / A * 1000 A) / (2 π 20m)

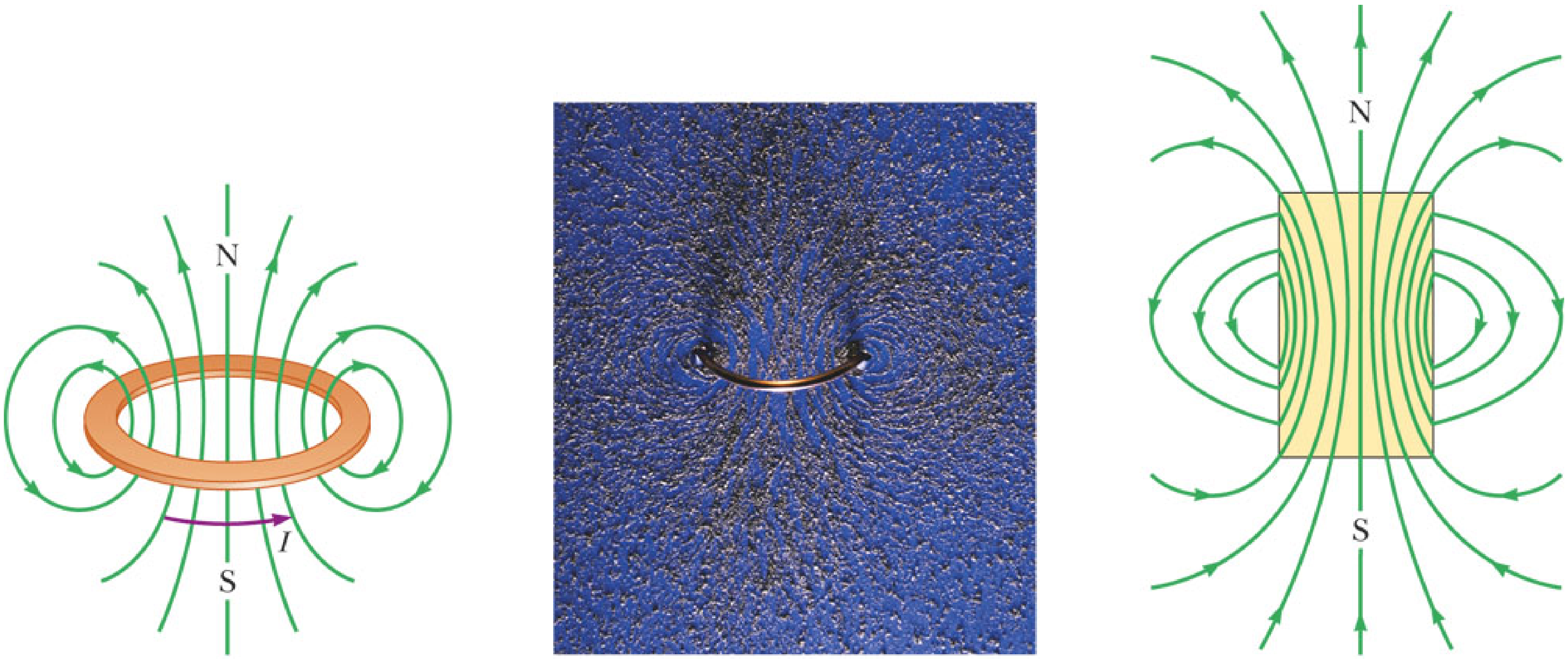
$= 1 \times 10^{-5}$ T -- about a factor of 3-5 smaller than the Earth's B field (0.3-0.5 Gauss).

At 10m: B(r=10) = 2 $\times$ B(r=20) = 2 $\times 10^{-5}$ T -- still just smaller than the Earth's B-field

Direction of B-field when you're below the wire: south

B-field at the center of a current-loop:

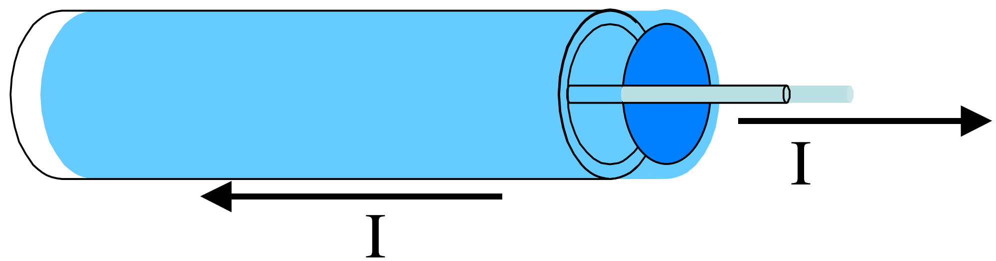
$$B_X = \frac{\mu_o I}{2R}$$



(a) (b) (c)

Example: A Co-axial cable

Inner conductor and outer conductor of radius R_{out} carrying currents in opposite directions.



What's B at $r > R_{out}$?

KEY: Remember superposition = vector addition: Total B-field from 2 sources = sum of B-fields from each source!

+I + –I = zero total current, so B-field = 0.

Example: B(r), inside wire: Ex. 22.7, p. 749

Use Ampere's Law to derive that inside a wire with a uniform current distribution, B(r) is proportional to r.

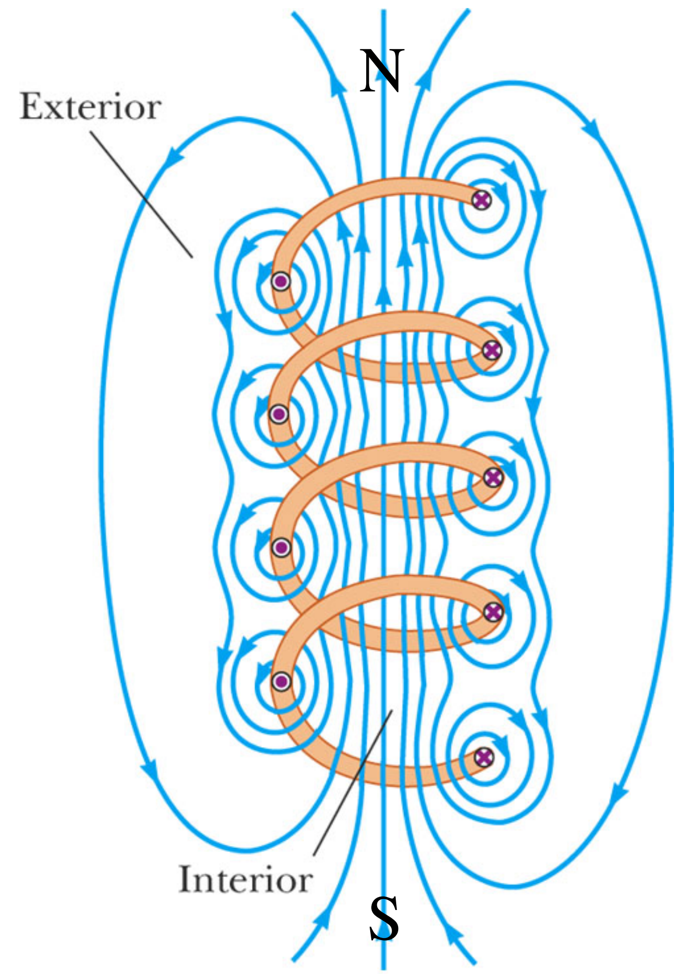
Recall the case of the electric field E(r) inside a wire with a uniform charge distribution: E(r) is also proportional to r.

Outside a long, straight wire, both E and B as proportional to 1/r.

22.10: Stack of current loops = solenoid

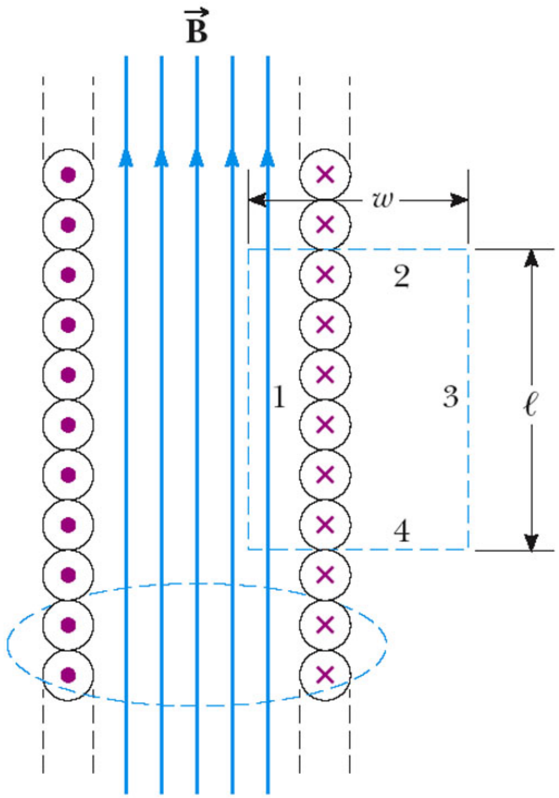
When the loop are spaced together tightly enough, the B-field inside is strong and rather uniform, and B-field outside is essentially negligible.

Commonly used in electromagnets, devices used to convert electrical current to magnetic field.



B-field in the center of a solenoid

Use Ampere's Law; choose a closed loop as follows:



Only segment 1 contributes: $B_{||}\Delta\ell = 0$ for other segments.

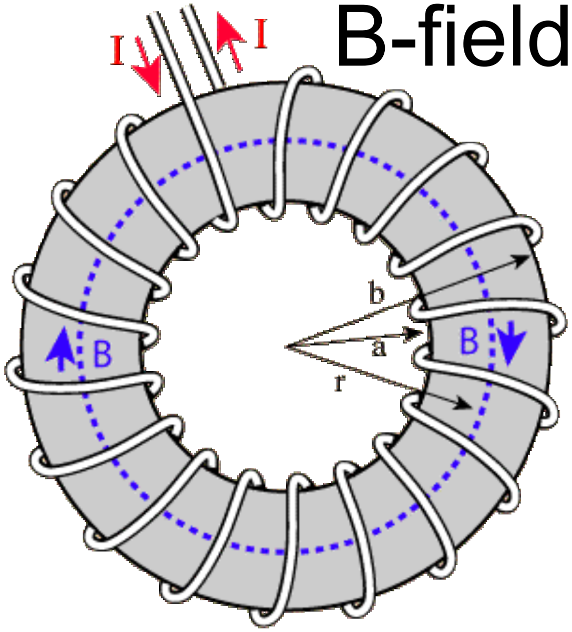
$BL = \mu_0(NI)$

$B = \mu_0 I\ (N/L) = \mu_0 In \quad (n=N/L)$

Example: An electromagnet consists of 100 turns of wire, and the length is 3.0 cm. The wire carries 20 Amps of current. What's the B-field at the center of the magnet?

$B = \mu_0 I \,(N/L) = 4\pi \times 10^{-7}$ T m / A * 20 A (100/0.03m) = 0.084 T

B-field of a toroid (ex. 22.8)



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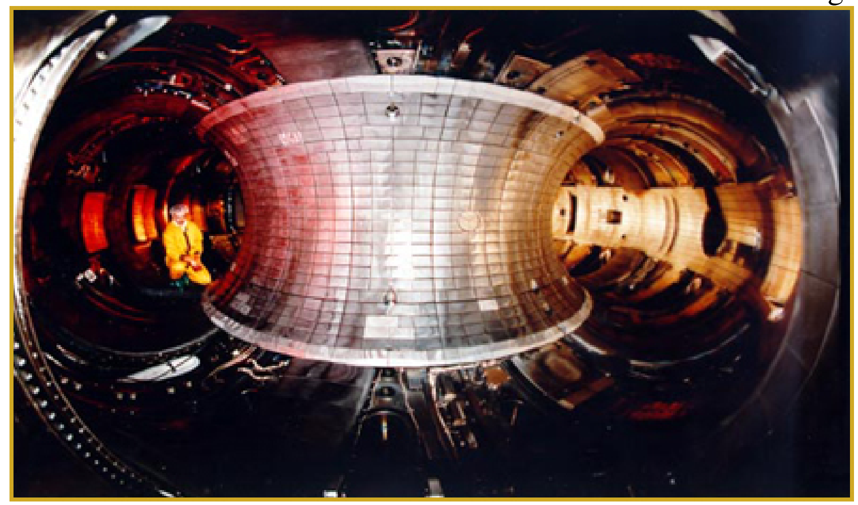


Tokamak: used for fusion energy research

$\Sigma B_{||}\Delta\ell = B\ 2\pi r$

enclosed current on blue line = μ_0 N I

B = μ_0 N I / 2πr

B-field higher towards inner radius (not perfectly uniform), but uniform along each radius

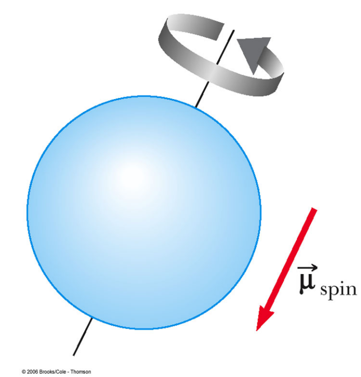
22.11: Magnetism in Matter / Magnetic Domains

Magnetic materials owe their properties to magnetic dipole moments of electrons in atoms

Classical model for electrons in atoms:

1.Orbital motion of electron: like a loop current (but B-field produced by 1 electron can be cancelled out by an oppositely revolving electron in the same atom)

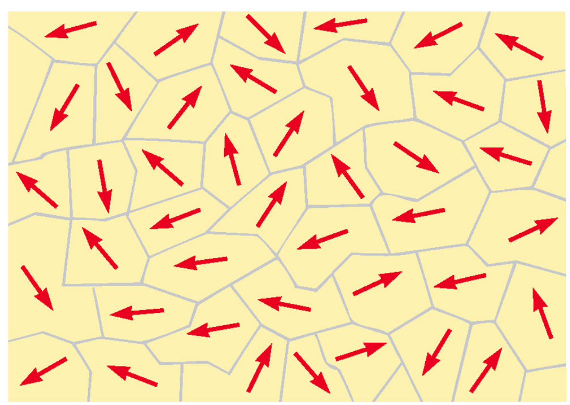
2. “spin” of individual electrons produces much stronger B-field: each electron itself acts like a magnetic dipole

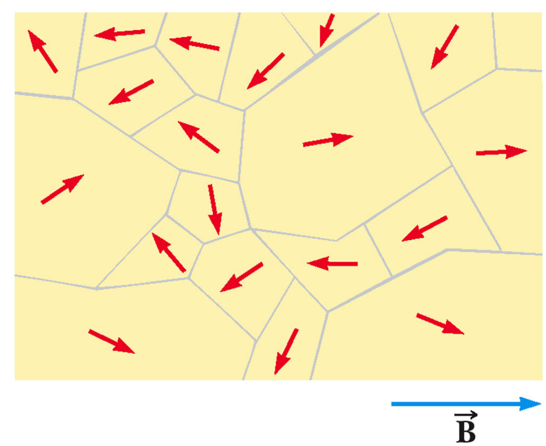


Magnetic Domains

Magnetic domains (10^{-4} - 10^{-1} cm): Each domain has a substantial fraction of atoms with magnetic moments coupled. They're separated by domain boundaries.

Ferromagnetic materials (Fe, Co, Ni): have these domains. Spins are randomly oriented, but when an external $\vec{B}$ is applied, domains tend to align with magnetic field; domain boundaries adjust accordingly.





Result: material produces its own **internal** $\vec{B}$

($\vec{B}_{net} = \vec{B}_{external} + \vec{B}_{internal}$)

Re-cap:

<u>Soft magnetic materials (e.g. Fe):</u> Easily magnetized in presence of external B, but doesn't retain magnetization for long. Used as cores for electromagnets.

When external B is turned off, thermal agitation returns dipoles to random orientations

<u>Hard magnetic materials</u> (e.g. metal alloys: Alnico (Aluminum, Nickel, Cobalt)): Harder to magnetize (requires higher $\vec{B}_{external}$) but retains the magnetization for a long time. Used as permanent magnets.