Ch. 21: Current, Resistance, Circuits

Current: How charges flow through circuits

Resistors: convert electrical energy into thermal/radiative energy

Electrical Energy & Power; Household Circuits

Time-Dependent Circuits
Current: Rate at which charge flows through an area A (cross-section of a wire)

$$I \equiv \frac{\Delta Q}{\Delta t}$$

Flow is assumed to be perpendicular to area.

**Units = Coul/sec = Amp.**

Remember: I is defined as the direction in which positive charges will travel (in metal, the charge carriers are actually electrons)
Potential difference sets up E-field to drive Current

$$V_1 - V_2 = \Delta V$$

Example: Terminals of a battery
Example:

A flashlight bulb carries a current of 0.1 A. Find the charge that passes through the bulb in 0.5 seconds:

\[ I = \frac{\Delta Q}{\Delta T} \rightarrow \Delta Q = I \times \Delta T = 0.1 \text{C/s} \times 0.5\text{s} = 0.05 \text{C} \]

How many electrons does this correspond to?
\[ \Delta Q = N \times e \]
\[ N = \frac{\Delta Q}{e} = \frac{0.05\text{C}}{(1.6 \times 10^{-19} \text{C/e}^-)} = 3.1 \times 10^{17} \text{ e}^-'s \]
Remember: I is defined as the direction in which positive charges will travel (in metal, the charge carriers are actually electrons)

From quick quiz 21.1:
Rank the currents from lowest to highest:
Remember: I is defined as the direction in which positive charges will travel (in metal, the charge carriers are actually electrons)

From quick quiz 21.1:
Rank the currents from lowest to highest:

Negative charges moving left are equivalent to positive charges moving right.

(a) Equivalent to 5 +’s moving right.
(b) 4 +’s moving left
(c) Equivalent to 4 +’s moving right
(d) Equivalent to 2 +’s moving right
Amp-hour

Unit of charge

charge = current \times time

Ex.: Ni-metal hydride battery: How much charge (in C) is equal to 2100 mAh?

Charge = (2100 \times 10^{-3} \ A) \ (1 \ hour) \\
= (2100 \times 10^{-3} \ C/s)(3600s) \\
= 7560 \ C.
Amp-hour

If one of these batteries is used to power a device which draws 0.15 Amps, how long will the battery last?

\[ I = \frac{\Delta Q}{\Delta T} \]

\[ \Delta T = \frac{\Delta Q}{I} = \frac{(2100 \times 10^{-3} \text{ Amp}\times\text{hr})}{0.15 \text{ Amps}} = 14 \text{ hours}. \]
Drift Velocity, $v_d$

Volume = $A \Delta x$

$n = \text{density of charge carriers} = \# \text{ of charge carriers per unit vol.}$

$N = \text{Total \# of charge carriers} = n \, A \, \Delta x$

Total charge in this volume: $\Delta Q = N \times \text{charge/crrier} = n \, A \, \Delta x \, q$

$\Delta x = v_d \Delta t$

$\Delta Q = nA \, v_d \Delta t \, q$

$I = \frac{\Delta Q}{\Delta t} = n \, A \, v_d \, q$
Drift Velocity, $v_d$

Electrons undergo repeated collisions and move randomly. Typical velocity for Cu is $2 \times 10^6$ m/s.

In the presence of an external field, the average motion is a slow drift.

Electric signal travels very fast -- almost at the speed of light: electrons interact and "push" other electrons in the conductor.
Example:

Find the drift velocity of electrons in a copper conductor whose diameter is 2 mm when the applied current is 0.5 A. The mass density of Cu is $\rho = 8.95\text{g/cm}^3$. Each Cu atom contributes 1 electron. One mole of Cu has a mass of 63.5 gm.

Soln: Need to calculate density of charge carriers (# of e$^-$’s/m$^3$)

How many moles per cm$^3$? $(8.95\text{gm/cm}^3)/(63.5\text{gm/mol}) = 0.14 \text{ mol/cm}^3$

Every mol contain $6\times10^{23}$ atoms.

Number of atoms per cm$^3$: $(0.14 \text{ mol/cm}^3)(6\times10^{23} \text{ atoms/mol}) = 8.4\times10^{22} \text{ atoms/cm}^3$

Density of charge carriers (given that 1e$^-$/atom) = $8.4\times10^{22} \text{ e}^-/\text{cm}^3$

$v_d = I/(nqA) = 0.5\text{A}/(8.4\times10^{28}\text{e}^-/\text{m}^3 \ 1.6\times10^{-19}\text{C} \ 3.14(.001\text{m})^2) = 1.2\times10^{-5} \text{ m/s} = 0.012 \text{ mm/s}$

If A = 50 Amps: $v_d$ would be 1.2 mm/s -- still a snail’s pace!
Current Density

\[ J = \frac{I}{A} = n q v_d \]

SI unit: Amps / m\(^2\)
Ammeter
Device used to measure current

All charge must pass through ammeter
21.2: Resistance & Ohm’s Law

Resistance of a conductor is defined as ratio of potential difference across it to the current that results:
Ohm’s Law: For many materials, $R$ remains constant over a wide range of applied $\Delta V$ or $I$.

$$R \equiv \frac{\Delta V}{I}$$

$\Delta V = IR$

Units of $R$: Ohms ($\Omega$)
Resistors

In a circuit: the resistance of the conducting wires is negligible, so $\Delta V = 0$ (no extra loss in potential) between points A & B.

But a resistor can cause a significant drop in $\Delta V$ (comparing $V$ before/after the resistor):
Resistors

Analogy: Waterfalls: sudden drop in gravitational potential energy

$\Delta PE$ converted to kinetic energy of water

$q\Delta V = \Delta PE$

electrical potential energy converted to thermal energy in resistor
Change in PE is
+qΔV (battery)
or –qΔV (resistor)

Points A and D are “grounded” -- Potential V = 0.
Points B and C are both at higher potential
Resistors

RESISTANCE regulates current and causes conversion of electrical potential energy to heat.

Common examples: heating elements in toasters, hair dryers, space heaters; light bulb filaments

carbon resistors
wire wound resistors
thin metal film resistors
Examples:

Consider a simple V-R circuit comprising a light bulb. Assume there is a 1.5-volt battery and the light bulb draws a current of 0.2 Amps. Find the R of the light bulb filament:

\[ R = \frac{\Delta V}{I} = \frac{1.5V}{0.2 \text{ A}} = 7.5 \ \Omega \]

A 120-Volt (rel. to ground) household circuit is connected to a lamp; the light bulb filament has \( R = 240 \ \Omega \). Find I.

\[ I = \frac{\Delta V}{R} = \frac{120V}{240\Omega} = 0.5 \ \text{A} \]
Resistance is determined by geometry & resistivity.

\[ R = \rho \frac{L}{A} \]

\( \rho \) = resistivity. units are \( \Omega \text{m} \)

- semi-conductors \{ \}
- insulators \{ \}

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity ( a ) (( \Omega \cdot \text{m} ))</th>
<th>Temperature Coefficient ( \alpha ) [( ^\circ \text{C}^{-1} )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>( 1.59 \times 10^{-8} )</td>
<td>( 3.8 \times 10^{-3} )</td>
</tr>
<tr>
<td>Copper</td>
<td>( 1.7 \times 10^{-8} )</td>
<td>( 3.9 \times 10^{-3} )</td>
</tr>
<tr>
<td>Gold</td>
<td>( 2.44 \times 10^{-8} )</td>
<td>( 3.4 \times 10^{-3} )</td>
</tr>
<tr>
<td>Aluminum</td>
<td>( 2.82 \times 10^{-8} )</td>
<td>( 3.9 \times 10^{-3} )</td>
</tr>
<tr>
<td>Tungsten</td>
<td>( 5.6 \times 10^{-8} )</td>
<td>( 4.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Iron</td>
<td>( 10 \times 10^{-8} )</td>
<td>( 5.0 \times 10^{-3} )</td>
</tr>
<tr>
<td>Platinum</td>
<td>( 11 \times 10^{-8} )</td>
<td>( 3.92 \times 10^{-3} )</td>
</tr>
<tr>
<td>Lead</td>
<td>( 22 \times 10^{-8} )</td>
<td>( 3.9 \times 10^{-3} )</td>
</tr>
<tr>
<td>Nichrome(^b)</td>
<td>( 1.50 \times 10^{-6} )</td>
<td>( 0.4 \times 10^{-3} )</td>
</tr>
<tr>
<td>Carbon</td>
<td>( 3.5 \times 10^{-5} )</td>
<td>( -0.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Germanium</td>
<td>( 0.46 )</td>
<td>( -48 \times 10^{-3} )</td>
</tr>
<tr>
<td>Silicon</td>
<td>( 640 )</td>
<td>( -75 \times 10^{-3} )</td>
</tr>
<tr>
<td>Glass</td>
<td>( 10^{10} ) to ( 10^{14} )</td>
<td></td>
</tr>
<tr>
<td>Hard rubber</td>
<td>( \sim 10^{13} )</td>
<td></td>
</tr>
<tr>
<td>Sulfur</td>
<td>( 10^{15} )</td>
<td></td>
</tr>
<tr>
<td>Quartz (fused)</td>
<td>( 75 \times 10^{16} )</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)All values are at 20\(^\circ\)C.

\(^b\)Nichrome is a nickel–chromium alloy commonly used in heating elements.
Resistance caused by charge carriers colliding with the lattice of the conductor. More collisions = more resistance

$L = \text{length}$

Double the length $\rightarrow$ double the resistance

(electrons must undergo twice as many collisions across the resistor)
\[ R = \rho \frac{L}{A} \]

\( A = \text{cross-section area} \)

Decrease Area: Resistance is raised since flow of charge carriers is constricted
Conductivity $\sigma$

$\sigma = 1/\rho$

$R = \rho L/A$

$R = L/(\sigma A)$

$V/I = L/(\sigma A)$

$V/L = I/(\sigma A)$

$E = J/\sigma$

$J = \sigma E$

Resistivity and conductivity are “microscopic” properties of the material.

Resistance is a macroscopic property of an object, and is a function of geometry and resistivity.
Some materials exhibit non-Ohmic resistance

Current-voltage curve for a semi-conducting diode: it’s non-linear

In this course, assume Ohmic resistance unless otherwise stated
Temperature dependence of resistance

At higher T, the charge carriers’ collisions with the lattice are more frequent.

$v_d$ becomes lower. So $I$ becomes lower.

And $R$ becomes larger for a given potential.
Temperature coefficient of resistivity

\[ \rho = \rho_0 [1 + \alpha (T - T_0)] \]

\[ R = R_0 [1 + \alpha (T - T_0)] \]

\( T_0 = \) reference temperature

\( \alpha = \) temperature coefficient of resistivity, units of \( (^\circ C)^{-1} \)

For Ag, Cu, Au, Al, W, Fe, Pt, Pb: values of \( \alpha \) are \( \sim 3-5 \times 10^{-3} (^\circ C)^{-1} \)
Example: A platinum resistance thermometer uses the change in R to measure temperature. Suppose $R_0 = 50 \ \Omega$ at $T_0=20 \ ^\circ\text{C}$.
\[ \alpha \text{ for Pt is } 3.92 \times 10^{-3} \ (\text{oC})^{-1} \text{ in this temperature range.} \]
What is R when $T = 50.0 \ ^\circ\text{C}$?
\[ R = R_0[1 + \alpha(T - T_0)] \]
\[ R = \ 50\Omega \ [1 + 3.92 \times 10^{-3} \ (\text{oC})^{-1} (30.0 \ ^\circ\text{C})] = 55.88 \ \Omega \]
Temperature coefficient of resistivity

Example: A platinum resistance thermometer has a resistance $R_0 = 50.0 \, \Omega$ at $T_0 = 20 \, ^\circ C$. $\alpha$ for Pt is $3.92 \times 10^{-3} \, (^\circ C)^{-1}$. The thermometer is immersed in a vessel containing melting tin, at which point $R$ increases to $91.6 \, \Omega$. What is the melting point of tin?

\[
R = R_0 \left[ 1 + \alpha (T - T_0) \right]
\]

\[
91.6 \, \Omega = 50 \, \Omega \left[ 1 + 3.92 \times 10^{-3} \, (^\circ C)^{-1} (T - 20^\circ C) \right]
\]

\[
1.83 = \left[ 1 + 3.92 \times 10^{-3} \, (^\circ C)^{-1} (T - 20^\circ C) \right]
\]

\[
0.83 = 3.92 \times 10^{-3} \, (^\circ C)^{-1} (T - 20^\circ C)
\]

\[
212^\circ C = T - 20^\circ C
\]

\[
T = 232 \, ^\circ C
\]