Chapter 2

Motion in One Dimension

SOLUTIONS TO PROBLEMS

Section 2.1 Average Velocity

P2.3 (a) 
\[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{10 \text{ m}}{2 \text{ s}} = 5 \text{ m/s} \]

(b) 
\[ \bar{v} = \frac{5 \text{ m}}{4 \text{ s}} = 1.2 \text{ m/s} \]

(c) 
\[ \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5 \text{ m} - 10 \text{ m}}{4 \text{ s} - 2 \text{ s}} = -2.5 \text{ m/s} \]

(d) 
\[ \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5 \text{ m} - 5 \text{ m}}{7 \text{ s} - 4 \text{ s}} = -3.3 \text{ m/s} \]

(e) 
\[ \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = 0 \text{ m/s} \]

*P2.4 (a) Let \( d \) represent the distance between A and B. Let \( t_1 \) be the time for which the walker has the higher speed in \( 5.00 \text{ m/s} = \frac{d}{t_1} \). Let \( t_2 \) represent the longer time for the return trip in \( -3.00 \text{ m/s} = - \frac{d}{t_2} \). Then the times are \( t_1 = \frac{d}{(5.00 \text{ m/s})} \) and \( t_2 = \frac{d}{(3.00 \text{ m/s})} \). The average speed is:

\[ \bar{v} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d + d}{d \cdot \left( \frac{1}{5.00 \text{ m/s}} + \frac{1}{3.00 \text{ m/s}} \right)} = \frac{2d}{\left( \frac{15.0 \text{ m}^2/\text{s}^2}{(5.00 \text{ m/s})} + \frac{15.0 \text{ m}^2/\text{s}^2}{(3.00 \text{ m/s})} \right)} \]

\[ \bar{v} = \frac{2(15.0 \text{ m}^2/\text{s}^2)}{8.00 \text{ m/s}} = 3.75 \text{ m/s} \]

(b) She starts and finishes at the same point A. With total displacement = 0, average velocity = 0.
Section 2.2 Instantaneous Velocity

P2.5 (a) at \( t_i = 1.5 \) s, \( x_i = 8.0 \) m (Point A)  
at \( t_f = 4.0 \) s, \( x_f = 2.0 \) m (Point B)  

\[
\mathbf{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0) \text{ m}}{(4 - 1.5) \text{ s}} = -\frac{6.0 \text{ m}}{2.5 \text{ s}} = -2.4 \text{ m/s}
\]

(b) The slope of the tangent line can be found from Points C and D. \( (t_C = 1.0 \) s, \( x_C = 9.5 \) m) and \( (t_D = 3.5 \) s, \( x_D = 0 \))  

\[
v = \frac{-3.8 \text{ m/s}}{s}.
\]

c) The velocity is zero when \( x \) is a minimum. This is at \( t = \frac{4 \text{ s}}{s} \).

P2.8 (a) \[
v = \frac{(5 - 0) \text{ m}}{(1 - 0) \text{ s}} = \frac{5 \text{ m/s}}{s}
\]

(b) \[
v = \frac{(5 - 10) \text{ m}}{(4 - 2) \text{ s}} = \frac{-2.5 \text{ m/s}}{s}
\]

c) \[
v = \frac{(5 \text{ m} - 5 \text{ m})}{(5 \text{ s} - 4 \text{ s})} = \frac{0}{s}
\]

d) \[
v = \frac{0 - (-5 \text{ m})}{(8 \text{ s} - 7 \text{ s})} = \frac{+5 \text{ m/s}}{s}
\]

Section 2.3 Analysis Models—The Particle Under Constant Velocity

P2.9 Once it resumes the race, the hare will run for a time of  

\[
t = \frac{x_f - x_i}{v_x} = \frac{1000 \text{ m} - 800 \text{ m}}{8 \text{ m/s}} = 25 \text{ s}.
\]

In this time, the tortoise can crawl a distance  

\[
x_f - x_i = (0.2 \text{ m/s})(25 \text{ s}) = \frac{5.00 \text{ m}}{s}.
\]


Section 2.4 Acceleration

P2.12 (a) At \( t = 2.00 \text{ s} \), \( x = \left[ 3.00(2.00)^2 - 2.00(2.00) + 3.00 \right] \text{ m} = 11.0 \text{ m} \).

At \( t = 3.00 \text{ s} \), \( x = \left[ 3.00(9.00)^2 - 2.00(3.00) + 3.00 \right] \text{ m} = 24.0 \text{ m} \)

so

\[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{24.0 \text{ m} - 11.0 \text{ m}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{13.0 \text{ m/s}}. \]

(b) At all times the instantaneous velocity is

\[ v = \frac{d}{dt}(3.00t^2 - 2.00t + 3.00) = (6.00t - 2.00) \text{ m/s} \]

At \( t = 2.00 \text{ s} \), \( v = \left[ 6.00(2.00) - 2.00 \right] \text{ m/s} = 10.0 \text{ m/s} \).

At \( t = 3.00 \text{ s} \), \( v = \left[ 6.00(3.00) - 2.00 \right] \text{ m/s} = 16.0 \text{ m/s} \).

(c) \[ \ddot{a} = \frac{\Delta v}{\Delta t} = \frac{16.0 \text{ m/s} - 10.0 \text{ m/s}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{6.00 \text{ m/s}^2} \]

(d) At all times \( a = \frac{d}{dt}(6.00t - 2.00) = \boxed{6.00 \text{ m/s}^2} \). (This includes both \( t = 2.00 \text{ s} \) and \( t = 3.00 \text{ s} \)).

P2.13 \( x = 2.00 + 3.00t - t^2 \), \( v = \frac{dx}{dt} = 3.00 - 2.00t \), \( a = \frac{dv}{dt} = -2.00 \)

At \( t = 3.00 \text{ s} \):

(a) \( x = (2.00 + 9.00 - 9.00) \text{ m} = \boxed{2.00 \text{ m}} \)

(b) \( v = (3.00 - 6.00) \text{ m/s} = \boxed{-3.00 \text{ m/s}} \)

(c) \( a = \boxed{-2.00 \text{ m/s}^2} \)

Section 2.6 The Particle Under Constant Acceleration

P2.21 (a) \( v_i = 100 \text{ m/s}, a = -5.00 \text{ m/s}^2 \), \( v_f = v_i + at \) so \( 0 = 100 - 5t \), \( v_f^2 = v_i^2 + 2a(x_f - x_i) \) so \( 0 = (100)^2 - 2(5.00)(x_f - 0) \). Thus \( x_f = 1000 \text{ m} \) and \( t = \boxed{20.0 \text{ s}} \).

(b) At this acceleration the plane would overshoot the runway: \boxed{No}.
P2.22  (a) Compare the position equation \( x = 2.00 + 3.00t - 4.00t^2 \) to the general form

\[
x_f = x_i + v_i t + \frac{1}{2} a t^2
\]

to recognize that \( x_i = 2.00 \text{ m} \), \( v_i = 3.00 \text{ m/s} \), and \( a = -8.00 \text{ m/s}^2 \). The velocity equation, \( v_f = v_i + at \), is then

\[
v_f = 3.00 \text{ m/s} - (8.00 \text{ m/s}^2) t.
\]

The particle changes direction when \( v_f = 0 \), which occurs at \( t = \frac{3}{8} \text{ s} \). The position at this time is:

\[
x = 2.00 \text{ m} + (3.00 \text{ m/s})\left(\frac{3}{8} \text{ s}\right) - (4.00 \text{ m/s}^2)\left(\frac{3}{8} \text{ s}\right)^2 = 2.56 \text{ m}.
\]

(b) From \( x_f = x_i + v_i t + \frac{1}{2} a t^2 \), observe that when \( x_f = x_i \), the time is given by \( t = -\frac{2v_i}{a} \). Thus, when the particle returns to its initial position, the time is

\[
t = -\frac{2(3.00 \text{ m/s})}{-8.00 \text{ m/s}^2} = \frac{3}{4} \text{ s}
\]

and the velocity is \( v_f = 3.00 \text{ m/s} - (8.00 \text{ m/s}^2)\left(\frac{3}{4} \text{ s}\right) = -3.00 \text{ m/s} \).
P2.25  (a) The time it takes the truck to reach 20.0 m/s is found from \( v_f = v_i + at \). Solving for \( t \) yields

\[
t = \frac{v_f - v_i}{a} = \frac{20.0 \text{ m/s} - 0 \text{ m/s}}{2.00 \text{ m/s}^2} = 10.0 \text{ s}.
\]

The total time is thus

\[
10.0 \text{ s} + 20.0 \text{ s} + 5.00 \text{ s} = 35.0 \text{ s}.
\]

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

\[
x_1 = vt = (0 + 20.0)(10.0) = 100 \text{ m}.
\]

With \( a \) being 0 for this interval, the distance traveled during the next 20.0 s is

\[
x_2 = v_i t + \frac{1}{2} a t^2 = (20.0)(20.0) + 0 = 400 \text{ m}.
\]

The distance traveled in the last 5.00 s is

\[
x_3 = \bar{v} t = \left( \frac{20.0 + 0}{2} \right)(5.00) = 50.0 \text{ m}.
\]

The total distance \( x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550 \text{ m} \), and the average velocity is given by

\[
\bar{v} = \frac{x}{t} = \frac{550}{35.0} = 15.7 \text{ m/s}.
\]

Section 2.7 Freely Falling Objects

P2.29  (a) \( v_f = v_i - gt : v_f = 0 \) when \( t = 3.00 \text{ s}, \ g = 9.80 \text{ m/s}^2 \). Therefore,

\[
v_i = gt = (9.80 \text{ m/s}^2)(3.00 \text{ s}) = 29.4 \text{ m/s}.
\]

(b) \( y_f - y_i = \frac{1}{2} (v_f + v_i) t \)

\[
y_f - y_i = \frac{1}{2} (29.4 \text{ m/s})(3.00 \text{ s}) = 44.1 \text{ m}
\]

P2.31  (a) \( y_f - y_i = v_i t + \frac{1}{2} a t^2 : 4.00 = (1.50)v_i - (4.90)(1.50)^2 \) and \( v_i = 10.0 \text{ m/s upward} \).

(b) \( v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s} \)

\[
v_f = 4.68 \text{ m/s downward}
\]
P2.32  We have \( y_f = -\frac{1}{2}gt^2 + v_i t + y_i \)

\[
0 = -\left( 4.90 \text{ m/s}^2 \right)t^2 - (8.00 \text{ m/s})t + 30.0 \text{ m}.
\]

Solving for \( t \),

\[
t = \frac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}.
\]

Using only the positive value for \( t \), we find that \( t = 1.79 \text{ s} \).

*P2.34  (a)  Consider the upward flight of the arrow.

\[
v_{yi}^2 = v_{yi}^2 + 2a_y (y_f - y_i)
\]

\[
0 = (100 \text{ m/s})^2 + 2\left( -9.8 \text{ m/s}^2 \right)\Delta y
\]

\[
\Delta y = \frac{10000 \text{ m}^2/\text{s}^2}{19.6 \text{ m/s}^2} = 510 \text{ m}
\]

(b)  Consider the whole flight of the arrow.

\[
y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2
\]

\[
0 = 0 + (100 \text{ m/s})t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2
\]

The root \( t = 0 \) refers to the starting point. The time of flight is given by

\[
t = \frac{100 \text{ m/s}}{4.9 \text{ m/s}^2} = 20.4 \text{ s}.
\]

Additional Problems

P2.39  (a)  \( a = \frac{v_f - v_i}{t} = -\frac{632(5.280)}{3.600} \frac{\text{in}}{1.40} = -662 \text{ ft/s}^2 = -202 \text{ m/s}^2 = -20.6 \text{ g}
\]

(b)  \( x_f = v_i t + \frac{1}{2} at^2 = (632)\left( \frac{5.280}{3.600} \right)(1.40) - \frac{1}{2}(662)(1.40)^2 = 649 \text{ ft} = 198 \text{ m} \)
P2.49  (a)  Let $x$ be the distance traveled at acceleration $a$ until maximum speed $v$ is reached. If this is achieved in time $t_1$ we can use the following three equations:

$$x = \frac{1}{2} (v + v_i) t_1, \quad 100 - x = v (10.2 - t_1) \quad \text{and} \quad v = v_i + at_1.$$ 

The first two give

$$100 = \left(10.2 - \frac{1}{2} t_1 \right) v = \left(10.2 - \frac{1}{2} t_1 \right) at_1$$

$$a = \frac{200}{(20.4 - t_1) t_1}.$$ 

For Maggie: $a = \frac{200}{(18.4)(2.00)} = \boxed{5.43 \text{ m/s}^2}$

For Judy: $a = \frac{200}{(17.4)(3.00)} = \boxed{3.83 \text{ m/s}^2}$

(b)  $v = at_1$

Maggie: $v = (5.43)(2.00) = \boxed{10.9 \text{ m/s}}$

Judy: $v = (3.83)(3.00) = \boxed{11.5 \text{ m/s}}$

(c)  At the six-second mark

$$x = \frac{1}{2} at_1^2 + v (6.00 - t_1)$$

Maggie: $x = \frac{1}{2} (5.43)(2.00)^2 + (10.9)(4.00) = 54.3 \text{ m}$

Judy: $x = \frac{1}{2} (3.83)(3.00)^2 + (11.5)(3.00) = 51.7 \text{ m}$

Maggie is ahead by $\boxed{2.62 \text{ m}}$.

P2.55  (a)  \[ d = \frac{1}{2} (9.80) t_1^2 \]

\[ t_1 + t_2 = 2.40 \quad \Rightarrow \quad 336t_2 = 4.90 (2.40 - t_2)^2 \]

\[ 4.90t_2^2 - 359.5t_2 + 28.22 = 0 \quad \Rightarrow \quad t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80} \]

\[ t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.076 \text{ s} \quad \text{so} \quad d = 336t_2 = \boxed{26.4 \text{ m}} \]

(b)  Ignoring the sound travel time, \[ d = \frac{1}{2} (9.80)(2.40)^2 = 28.2 \text{ m} \], an error of $\boxed{6.82\%}$.