"If you neglect to recharge a battery, it dies. And if you run full speed ahead without stopping for water, you lose momentum to finish the race."

--Oprah Winfrey
Static Equilibrium

Example
An 8.00m, 200N uniform ladder rests against a smooth wall. The coefficient of static friction between the ladder and the ground is 0.600, and the ladder makes a 50.0° angle with the ground. How far up the ladder can an 800N person climb before the ladder begins to slip?

Answer
First, you must define a coordinate system. Choose x to the right as positive and up as the positive y-direction.
Static Equilibrium

**Answer**

Next, draw an extended force diagram:

- $F_{\text{normal, wall on ladder}}$
- $F_{\text{normal, floor on ladder}}$
- $F_{\text{gravity, Earth on ladder}}$
- $F_{\text{friction, floor on ladder}}$
- $F_{\text{contact, person on ladder}}$

Since the wall is smooth it has no friction.

If the ladder is on the verge of slipping, then the force due to friction from the floor will be a maximum:

$$F_{\text{friction}} = \mu_s F_N$$
Answer

Try summing the forces in the y-direction:

$$\sum F_y = 0$$

$$\sum F_y = F_N - F_g - F_{\text{person}} = 0$$

$$F_N = F_g + F_{\text{person}}$$

$$F_N = 200\text{N} + 800\text{N} = 1,000\text{N}$$

Thus, we can go back and calculate the frictional force:

$$F_{\text{friction}} = \mu_s F_N$$

$$F_{\text{friction}} = (0.600)(1,000\text{N}) = 600\text{N}$$

Try summing the forces in the x-direction:

$$\sum F_x = 0$$

$$\sum F_x = F_{\text{friction}} - F_{\text{wall}} = 0$$

$$F_{\text{wall}} = F_{\text{friction}} = 600\text{N}$$
Static Equilibrium

**Answer**

Next, we can turn to calculating the net torque.

Choose the pivot point at the bottom of the ladder (this choice eliminates $F_N$ and $F_{friction}$).

- $r_{grav} = (0.5)L = 4\text{m}$.
- $r_{wall} = L = 8\text{m}$.
- $r_{person} = d$.

Calculate the individual torques for the three remaining forces.
**Static Equilibrium**

**Answer**

\[
\tau_{\text{wall}} = (r_{\text{wall}})(F_{\text{wall}})\sin\theta
\]

\[
\tau_{\text{wall}} = (8\text{m})(600\text{N})\sin50^\circ
\]

\[
\tau_{\text{wall}} = +3,667\text{N} \cdot \text{m}
\]

\[
\text{--- plus sign comes from counterclockwise rotation}
\]

\[
\tau_{\text{grav}} = (r_{\text{grav}})(F_{\text{grav}})\sin\theta
\]

\[
\tau_{\text{grav}} = (4\text{m})(200\text{N})\sin40^\circ
\]

\[
\tau_{\text{grav}} = -514\text{N} \cdot \text{m}
\]

\[
\text{--- minus sign comes from clockwise rotation}
\]

\[
\tau_{\text{person}} = (r_{\text{person}})(F_{\text{person}})\sin\theta
\]

\[
\tau_{\text{person}} = (d)(800\text{N})\sin40^\circ
\]

\[
\tau_{\text{person}} = -514\text{N}(d)
\]

\[
\text{--- minus sign comes from clockwise rotation}
\]
Static Equilibrium

Answer

Next, we can sum the torques and set them equal to zero since it is in equilibrium.

\[ \sum \tau = \tau_{\text{wall}} + \tau_{\text{grav}} + \tau_{\text{person}} = 0 \]

\[ \sum \tau = 3,667 \text{N} \cdot \text{m} - 514 \text{N} \cdot \text{m} - 514 \text{N}(d) = 0 \]

\[ 514 \text{N}(d) = 3,667 \text{N} \cdot \text{m} - 514 \text{N} \cdot \text{m} = 3,163 \text{N} \cdot \text{m} \]

\[ d = \frac{3,163 \text{N} \cdot \text{m}}{514 \text{N}} = 6.15 \text{m} \]

This is the distance that the person climbed up the ladder.
Newton’s Laws (Rotationally)

- We can relate Newton’s Laws with rotational motion with the following (if rotational inertia is constant):

\[ \sum \vec{t} = I \vec{\alpha} \]

- This is very similar to \( \Sigma \vec{F} = ma \):

- \( \Sigma \vec{t} \) is like a rotational force.

- \( I \) is like a rotational mass (inertia)

- \( \alpha \) is angular acceleration.

- This means that a net torque on a rigid object will lead to an angular acceleration.
Newton’s Laws (Rotationally)

- We can also turn to Newton’s 3rd Law rotationally:

\[ \vec{\tau}_{1on2} = -\vec{\tau}_{2on1} \]

- This means that if I exert a torque on an object, then it will exert the same torque right back at me but opposite in direction.

- This demonstrates the vector nature of torques and angular motion.

- This stool is an excellent example.
Work

- Just like a force can perform work over a distance, torque can perform work over an angle.

- For a constant torque: \[ W = \tau(\Delta \theta) \]

- For a variable torque: \[ W = \int_{\theta_i}^{\theta_f} \tau \, d\theta \]

- By the work-energy theorem, we can say that: \[ W = \Delta KE \]

- Making something spin around an axis is another place to put energy.
Rotational Kinetic Energy

So we can define a new type of kinetic energy; rotational kinetic energy, $KE_{rot}$:

$$KE_{rot} = \frac{1}{2}I\omega^2$$

Rotational kinetic energy is similar to linear kinetic energy (just switch from linear variables to rotational variables).

The units of rotational kinetic energy are still Joules.

Rotational kinetic energy is just a measure of how much energy is going into rotating an object.
Conceptual Question

A solid disk and a hoop are rolled down an inclined plane (without slipping). Both have the same mass and the same radius. Which one will reach the bottom of the incline first?

A) The solid disk.
B) The hoop.
C) They will reach the bottom at the same time.
Rotational Kinetic Energy

Example
A solid disk and a hoop are rolled down an inclined plane (without slipping). Both have a mass of 1.0kg and a radius of 50cm. They are both originally placed at a height of 1.5m. What is the ratio of their velocities ($v_{\text{disk}}/v_{\text{hoop}}$) at the bottom of the inclined plane?

Answer
Choose up as the positive y-direction ($y = 0$ at bottom of the ramp).
We also know that: $I_{\text{disk}} = (1/2)mr^2$ \hspace{1cm} $I_{\text{hoop}} = mr^2$
Rotational Kinetic Energy

**Answer**

- Use conservation of energy.

- At the top of the inclined plane, there is no KE such that:
  \[ E_{top} = KE + PE = 0 + PE \]
  \[ E_{top} = mgh = (1.0\text{kg})(9.8\text{ m/s}^2)(1.5\text{m}) \]
  \[ E_{top} = 14.7\text{J} \]

- At the bottom of the inclined plane, there is no PE such that:
  \[ E_{bot} = KE + PE = KE + 0 = KE_{trans} + KE_{rot} \]
  \[ E_{bot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \]
Rotational Kinetic Energy

Answer

Since $v = r\omega$ (no slipping), we can write:

$$E_{\text{bot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 = \frac{1}{2}v^2\left(m + \frac{I}{r^2}\right)$$

Due to conservation of energy we can say:

$$E_{\text{top}} = E_{\text{bot}}$$

$$14.7J = \frac{1}{2}v^2\left(m + \frac{I}{r^2}\right)$$

$$v^2 = \frac{2(14.7J)}{\left(m + \frac{I}{r^2}\right)} = \frac{(29.4J)}{\left(m + \frac{I}{r^2}\right)}$$

This equation is true for either shape.
Rotational Kinetic Energy

Answer

For the solid disk, $I = \frac{1}{2}mr^2$

So, for the disk we can say that:

$$v^2 = \frac{(29.4\, \text{J})}{\left( m + \frac{1}{2} \frac{mr^2}{r^2} \right)} = \frac{(29.4\, \text{J})}{\left( m + \frac{1}{2} m \right)} = \frac{3}{2}(1\, \text{kg})$$

$$v^2 = 19.6\, \text{m}^2/\text{s}^2 \quad v_{disk} = 4.4\, \text{m/s}$$

This is the disk’s velocity at the bottom of the incline.
Rotational Kinetic Energy

Answer

For the hoop, \( I = mr^2 \)

So, for the hoop we can say that:

\[
v^2 = \frac{(29.4 \text{J})}{m + \frac{(mr^2)}{r^2}} = \frac{(29.4 \text{J})}{(m + m)} = \frac{(29.4 \text{J})}{2(1\text{kg})}
\]

\[
v^2 = 14.7 \text{m}^2/\text{s}^2 \quad v_{\text{hoop}} = 3.8 \text{m/s}
\]

Ratio is:

\[
\frac{v_{\text{disk}}}{v_{\text{hoop}}} = \frac{4.4 \text{m/s}}{3.8 \text{m/s}} = 1.2
\]

This is the hoop’s velocity at the bottom of the incline.
Angular Momentum

We are aware of linear momentum. There is a rotational equivalent known as angular momentum.

Angular momentum, \( L \), is given by:

\[
L = \vec{r} \times \vec{p}
\]

It is a measure of how perpendicular \( p \) and \( r \) are.

The units for angular momentum are: \( \text{kg}(\text{m}^2/\text{s}) \).

Take the time derivative of angular momentum to find:

\[
\frac{dL}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})
\]

Apply the product rule to get:

\[
\frac{dL}{dt} = m \left( \frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt} \right)
\]
Angular Momentum

\[
\frac{d\vec{L}}{dt} = m(\vec{v} \times \vec{v} + \vec{r} \times \vec{a})
\]

\[
\frac{d\vec{L}}{dt} = m(\vec{r} \times \vec{a}) = (\vec{r} \times m\vec{a})
\]

\[
\frac{d\vec{L}}{dt} = (\vec{r} \times \sum \vec{F}) = \sum \vec{\tau}
\]

- If the net external torque on an object is zero, then angular momentum is conserved.

- Angular momentum creates an “axis of stability” which takes some effort to remove.

- This axis is caused by the angular momentum which would prefer to be conserved.
Angular Momentum

- The “real” way to define net torque is:

\[ \sum \vec{\tau} = \frac{d\vec{L}}{dt} \]

\[ L = \int I\alpha \, dt = \int I \frac{d\omega}{dt} \, dt \]

\[ L = I\omega \]

- This is another equation to find the magnitude of angular momentum.

- If you decrease your moment of inertia, then your angular velocity will increase.
Angular Momentum

Example

A merry-go-round (m = 100kg, r = 2.00m) spins with an angular velocity of 2.50(rad/s). A monkey (m = 25.0kg) hanging from a nearby tree, drops straight down onto the merry-go-round at a point 0.500m from the edge. What is the new angular velocity of the merry-go-round?

Answer

We can assume that the merry-go-round rotates in the positive direction (ccw).
Angular Momentum

Answer

Use conservation of angular momentum; there is no net external torque.

Before the monkey jumps on, \( L \) is:

\[
L_i = I_i \omega_i = I_{\text{disk}} \omega_i = \left( \frac{1}{2} m_d r_d^2 \right) \omega_i
\]

\[
L_i = (0.5)100\text{kg}(2.00\text{m})^2(2.50\text{ rad/s}) = 500\text{kg m}^2/\text{s}
\]

After the monkey jumps on (at \( r = 1.50\text{m} \)), \( L \) is:

\[
L_f = I_f \omega_f = (I_{\text{disk}} + I_{\text{monkey}}) \omega_f = \left( \frac{1}{2} m_d r_d^2 + m_m r_m^2 \right) \omega_f
\]

\[
L_f = \left[(0.5)100\text{kg}(2.00\text{m})^2 + 25.0\text{kg}(1.50\text{m})^2\right] \omega_f
\]
Angular Momentum

Answer

\[ L_f = (256 \text{kg} \cdot \text{m}^2) \omega_f \]

By conservation of angular momentum:

\[ L_i = L_f \]

\[ 500 \text{kg m}^2/\text{s} = (256 \text{kg} \cdot \text{m}^2) \omega_f \]

\[ \omega_f = \frac{500 \text{kg m}^2/\text{s}}{256 \text{kg} \cdot \text{m}^2} = 1.95 \text{ rad/s} \]

As the monkey jumps on the merry-go-round, the moment of inertia of the system increases.

The angular velocity will decrease due to conservation of angular momentum.
For Next Time (FNT)

- Finish the Homework for Chapter 10.

- Start reading Chapter 15.