PHYSICS 140A — MIDTERM 2 — FALL QUARTER 2010

PROF. HIRSCH — November 8th

Formulas:

First law $\Delta U = Q + W$; $C_V = \frac{\partial U}{\partial T})_V$; $H = U + PV$; $C_P = \frac{\partial H}{\partial T})_P$

$k = 1.381\times 10^{-23} J/K$, $N_A = 6.02\times 10^{23}$, $R = 8.31 \mathrm{J/(mol^o}K)$

Ideal gas: $PV = NkT$ $U = N\frac{f}{2}kT$ $W = -P\Delta V$, $W = -\int PdV$; $C_P = C_V + Nk$

adiabatic process: $PV^\gamma = const.$, $\gamma = (f+2)/f$

$S = k\ln\Omega$; $\frac{1}{T} = \frac{\partial S}{\partial U})_V$; $dS = \frac{Q}{T} = \frac{C_V dT}{T}$ (constant volume)

Stirling's approximation: $n! = n^n e^{-n}\sqrt{2\pi n}$

Ideal gas (monoatomic) : $\Omega_N = C_N V^N U^{3N/2}/N!$

Einstein solid: $\Omega(N,q) = \frac{(q+N-1)!}{q!(N-1)!}$ $\sim (\frac{eq}{N})^N$ for q >> N

Two-state system: $\Omega = \frac{N!}{N_\uparrow ! N_\downarrow !}$

Paramagnetism: $\mathrm{M = N}\mu \tanh(\mu\mathrm{B/kT})$, $U = -MB$, $M = \mu(N_\uparrow - N_\downarrow)$

$dU = TdS - PdV$; $Q = TdS$, $W = -PdV$ (quasistatic)

$U(S,V)$; $F(T,V) = U - TS$; $H(S,P) = U + PV$; $G(T,P) = U - TS + PV$

$C_V = T\frac{\partial S}{\partial T})_V$; $C_P = T\frac{\partial S}{\partial T})_P$; $\beta = \frac{1}{V}\frac{\partial V}{\partial T})_P$; $\kappa_{T,S} = -\frac{1}{V}\frac{\partial V}{\partial P})_{T,S}$; $\mu = -T\frac{\partial S}{\partial N})_{U,V} = \frac{\partial U}{\partial N})_{S,V}$

$\mu = -kT \ln(\frac{V}{N}(\frac{2\pi m \mathrm{kT}}{h^2}))$ Ideal gas (monoatomic)

Heat engines: $e = \frac{W}{Q_h} \le 1 - \frac{T_c}{T_h}$; Refrigerators: $COP = \frac{Q_c}{W} \le \frac{T_c}{T_h - T_c}$

Maxwell relations: $\frac{\partial T}{\partial V})_S = -\frac{\partial P}{\partial S})_V$; $\frac{\partial T}{\partial P})_S = \frac{\partial V}{\partial S})_P$; $\frac{\partial S}{\partial V})_T = \frac{\partial P}{\partial T})_V$; $\frac{\partial S}{\partial P})_T = -\frac{\partial V}{\partial T})_P$

Van der Waals: $(P + a\frac{N^2}{V^2})(V - Nb) = NkT$; Virial: $PV = NKT(1 + \frac{N}{V}B_2(T) + \frac{N^2}{V^2}B_3(T) + ...)$

Boltzmann distribution: $P(E_i) = Ce^{-E_i/kT}$; $C = 1/\sum_i e^{-E_i/kT} \equiv 1/Z$; $P(E) = C\Omega(E)e^{-E/kT}$

Justify all your answers to all (3) problems

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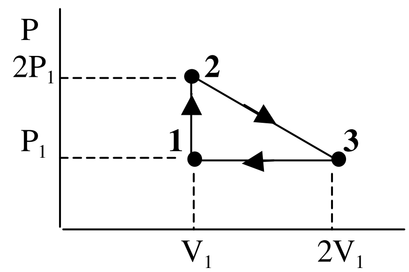
Problem 1

Consider a heat engine that uses a monoatomic ideal gas undergoing the cycle through points 1, 2 and 3 shown in the diagram below:



The temperature at point 1 is $T_1=P_1V_1/Nk$, with N the number of atoms, the energy is $U_1=(3/2)NkT_1$.

(a) Find T_2 and T_3 in terms of T_1 .
(b) Find the work done by this engine in one cycle in terms of T_1 (and N).
(c) Find the heat absorbed in the step 1-2 in terms of T_1 (and N).
(d) Find the heat absorbed in the step 2-3 in terms of T_1 (and N).
(e) Find the efficiency of this engine, defined as: e = (work performed) / (heat absorbed) per cycle.
(f) Find the highest and lowest temperatures in this cycle and the efficiency of a Carnot engine operating between those temperatures.

Problem 2

A system of N particles with energy U and volume V has entropy given by

$$S(U,V) = Nk\ \ln(U/N) + Nk\ \ln(V/N)$$

(a) Find an expression for U(S,V)
(b) From the thermodynamic identity (dU=TdS-PdV) and the result in (a), derive expressions for T(S,V) and P(S,V).
(c) From the results in (b), find expressions for $\partial T/\partial V)_S$ and $\partial P/\partial S)_V$ both as functions of variables S and V, and from them verify one of the Maxwell relations.
(d) By eliminating S from the expressions for T(S,V) and P(S,V) found in (b), find a relation between T, P and V.
(e) Find an expression for the enthalpy H, <u>as function of S and V.</u>
(f) Comparing the results in (e) and (a), find a relation between U, P and V.

Problem 3

A Van der Waals gas has equation of state

$$P = \frac{NkT}{V} - a\frac{N^2}{V^2}$$

with a a constant. Its energy as function of temperature and volume is given by

$$U(T,V) = 2NkT - a\frac{N^2}{V}$$

Find an expression that relates T and V in a process where the entropy S is constant, by:

(a) From the thermodynamic identity $dS = \frac{1}{T}dU + \frac{P}{T}dV$, find an expression for dS as function of T and V (and dT and dV).

(b) Integrate the expression found in (a) along a line of constant S.

(c) For a process at constant S where the initial temperature is T_i and the final temperature is $4T_i$, what is the final volume V_f if the initial volume is V_i?

Justify all your answers to all problems