PHYSICS 110A : CLASSICAL MECHANICS

1. Introduction to Dynamics
   motion of a mechanical system
   equations of motion : Newton’s second law
   ordinary differential equations (ODEs)
   dynamical systems
   simple examples

2. Systems of Particles
   kinetic, potential, and interaction potential energies
   forces; Newton’s third law
   momentum conservation
   torque and angular momentum
   kinetic energy and the work-energy theorem

3. Motion in $d = 1$: Two-Dimensional Phase Flows
   $(x, v)$ phase space
   dynamical system $\frac{d}{dt} \{ x, v \} = \{ a(x, v) \}$
   two-dimensional phase flows
   examples: harmonic oscillator and pendulum
   fixed points in two-dimensional phase space; separatrices

4. Solution of the Equations of One-Dimensional Motion
   Potential energy $U(x)$
   Conservation of energy
   sketching phase flows from $U(x)$
   solution by quadratures
   turning points; period of orbit

5. Linear Oscillations
   Taylor’s theory and the ubiquity of harmonic motion
   the damped harmonic oscillator: $\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = 0$
     reduction to algebraic equation
     generalization to all autonomous homogeneous linear ODEs
   solution to the damped harmonic oscillator: underdamped and overdamped behavior

6. Forced Linear Oscillations
   $\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = f(t)$
   solution for harmonic forcing $f(t) = A \cos(\Omega t)$
   presence of homogeneous solution: transients
   amplitude resonance and phase lag; Q factor

7. Green’s functions for autonomous linear ODEs
   Fourier transform
   physical meaning of $G(t - t')$; causality
   response to a pulse

8. MIDTERM EXAMINATION #1
9. **Calculus of Variations I**
   Snell’s law for refraction at an interface
   continuum limit of many interfaces
   functionals
   variational calculus: extremizing $\int dx L(y, y', x)$
   preview: Newton’s second law from $L = T - U$

10. **Calculus of Variations II**
    Examples
    surfaces of revolution
    geodesics
    brachistochrone
    generalization to several dependent and independent variables
    Constrained Extremization
    Lagrange undetermined multipliers in calculus: review
    systems with integral constraints
    hanging rope of fixed length
    holonomic constraints

11. **Lagrangian Dynamics**
    generalized coordinates
    action functional
    equations of motion: Newton’s second law
    examples: spring, pendulum, etc.
    double pendulum: Lagrangian and equations of motion
    Lagrangian for a charged particle interacting with an electromagnetic field
    Lorentz force law

12. **Noether’s Theorem and Conservation Laws**
    continuous symmetries
    “one-parameter family of diffeomorphisms” $q_i \rightarrow h_i^\lambda(q_1, \ldots, q_N)$
    Noether’s theorem and the conserved “charge” $Q = \sum_i \frac{\partial L}{\partial q_i} \frac{\partial h_i^\lambda}{\partial \lambda} \bigg|_{\lambda=0}$
    linear and angular momentum

13. **Constrained Dynamical Systems**
    undetermined multipliers as forces of constraints
    simple pendulum with $r = l$ or $x^2 + y^2 = l^2$ constraint
    Examples

14. MIDTERM EXAMINATION #2
15. The Two-Body Central Force Problem

- CM and relative coordinates
- angular momentum conservation and Kepler’s law \( \dot{A} = \text{const.} \)
- energy conservation
- the effective potential
  - radial equation of motion for the relative coordinate
  - the effective potential and its interpretation
- phase curves
- solution for \( r(t) \) and \( \phi(t) \) by quadratures

16. The Shape of the Orbit

- equation for \( r(\phi) \), the geometric shape of the orbit
  - \( s = 1/r \) substitution
- examples
  - almost circular orbits: bound versus closed motion, precession

17. Coupled Oscillations I: The Double Pendulum

- review: Lagrangian for the double pendulum
- equations of motion
- linearization
- solution of two coupled linear equations
- normal modes

18. Coupled Oscillations II: General Theory

- harmonic potentials
- \( T \) and \( V \) matrices
- normal modes
- the mathematical problem: simultaneous diagonalization of \( T \) and \( V \)

19. Coupled Oscillations III: The Recipe

- eigenvalues: \( \det(\omega^2 T - V) = 0 \)
- eigenvectors: \( (\omega^2 T_{ij} - V_{ij})a^{(s)}_j = 0 \)
- normalization: \( a^{(s)}_i T_{ij} a^{(s')}_j = \delta_{ss'} \)
- modal matrix: \( A_{js} = a^{(s)}_j \)
- examples

- **COMPREHENSIVE FINAL EXAMINATION**