Physics 110A: Problem Set #4

`Reading: MT chapters 6 and 7; lecture notes pp. 115-185.`

[1] Show that the geodesics on a cylinder are helices.

[2] Maximize the volume of a cylinder subject to the constraint

$$2\pi R + \frac{L^2}{a} = b$$

where R is the radius and L the height.

[3] A cylinder of radius a rolls without slipping on the lower half of a larger cylinder of radius $R > a$. Find the Lagrangian and the equations of motion. Solve for the frequency of small oscillations.

[4] A particle of mass m rests on a smooth plane. The angle of inclination of the plane increases linearly according to $\alpha(t) = \omega t$, causing the particle to move down the plane. Determine the motion of the particle

[5] A simple pendulum of length ℓ and bob mass m is attached to a massless support which accelerates horizontally at a constant rate a. Determine the equations of motion and the period of small oscillations. Is your result in conformity with the Principle of Equivalence (uniform acceleration equals gravitational field)? Repeat your analysis when the acceleration is vertical.

[6] A pendulum is constructed by attaching a mass to a string of length ℓ which is fixed to the uppermost point of a vertically oriented disk of radius R ($R < \ell/\pi$). Obtain the pendulum's equation of motion and find the frequency of small oscillations.

[7] Express the Lagrangian of a simple pendulum in terms of the Cartesian variables $x = \ell\cos\theta$ and $y = \ell\sin\theta$. Solve this problem two ways. First, use the holonomic constraint $x^2 + y^2 = \ell^2$ to eliminate y in favor of x. Find the equation of motion for x. Second, enforce the constraint using a Lagrange multiplier.

[8] The point of support of a pendulum of length ℓ and mass m_2 is itself a mass m_1 which can move freely along a horizontal line in the plane of the pendulum's motion. Choose an appropriate set of generalized coordinates and write the Lagrangian for this system. Find the equations of motion. Linearize the equations and solve for the frequencies of small oscillations. Comment on your solution.