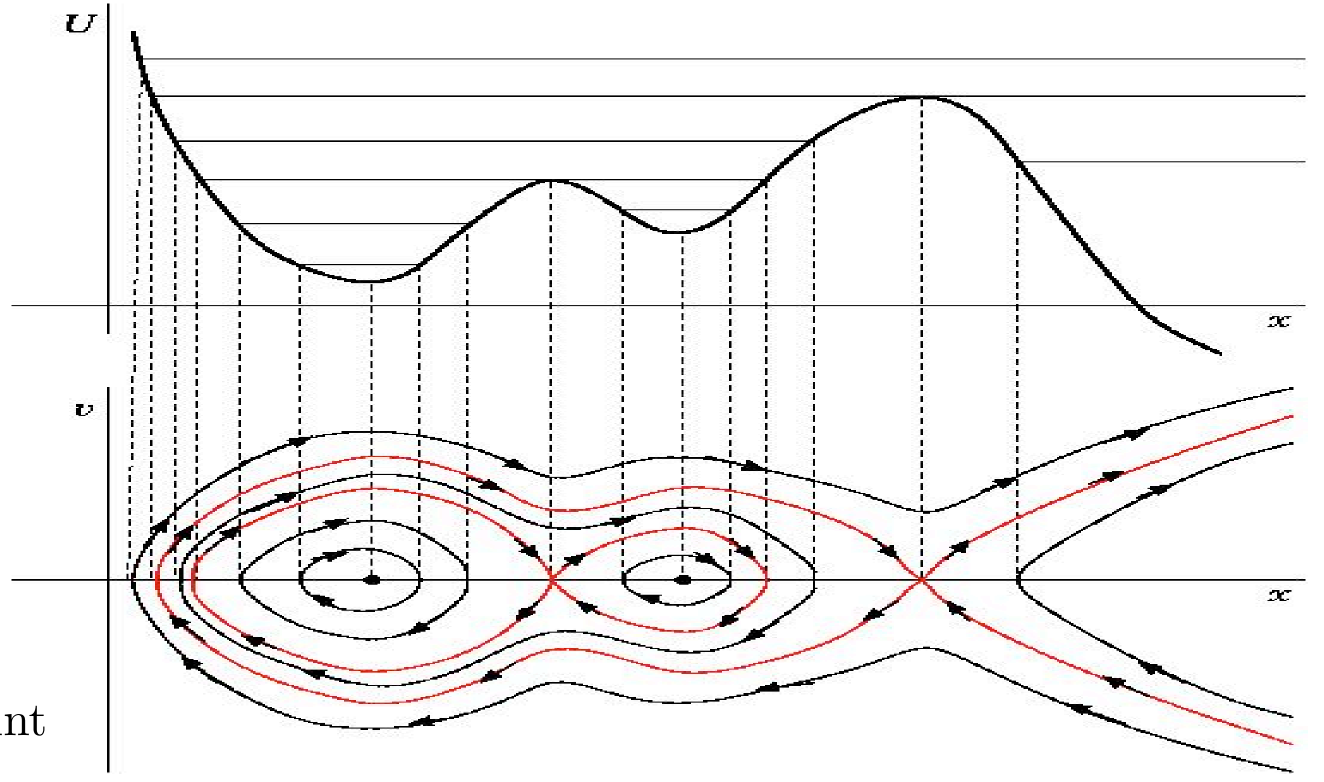
Mechanics in d=1



$$F = ma \implies m\,\ddot{x} = -U'(x)$$

$$E = \tfrac{1}{2}m\dot{x}^2 + U(x) \text{ is conserved}$$

$$\frac{dx}{dt} = \pm\sqrt{\frac{2}{m}\Big(E - U(x)\Big)}$$

$$U(x) = E \implies x(E) \text{ a turning point}$$

2 turning points $x_\pm(E) \;\Rightarrow\;$ motion bounded

1 turning point $x_*(E) \;\Rightarrow\;$ motion unbounded

period of bounded motion:

$$T(E) = \sqrt{2m}\int\limits_{x_-(E)}^{x_+(E)} \frac{dx}{\sqrt{E-U(x)}} \;=\; m\,\frac{d\mathcal{A}}{dE}$$

$$\mathcal{A}(E) = \oint\limits_E v\,dx \;=\; \text{phase space area}$$

$$N = 2 \text{ system:}\quad \frac{d}{dt}\begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} v \\ -U'(x)/m \end{pmatrix}$$

fixed points $\Rightarrow\; v = 0$ and $U'(x) = 0$

phase curves: flow is to right for $v > 0$ and to left for $v < 0$

local minima of $U(x) \implies$ centers

local maxima of $U(x) \implies$ saddles

red curves through saddles are *separatrices*

(topology of phase space motion changes across separatrix)

Taylor's theorem: $f(x+a) = f(x) + f'(x)\,a + \frac{1}{2}f''(x)\,a^2 + \ldots = \sum\limits_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(x)\,a^n$