PHYSICS 110A : CLASSICAL MECHANICS
PROBLEM SET #4

[1] Consider the mechanical thingy in Fig. 1. Two masses m are connected to the central axis and to a third mass M by massless rigid rods of length ℓ, as shown. The mass M slides frictionlessly along the axis. There is one degree of freedom, which is the angle θ, also shown. The thingy is rotated about its symmetry axis with angular velocity ω.

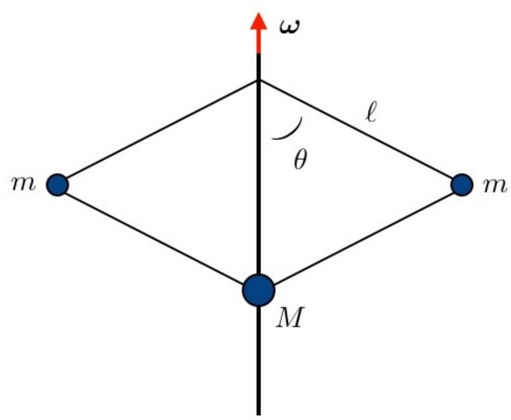

Figure 1: The thingy.

(a) Find the Lagrangian for the thingy.

(b) Find the equation of motion.

(c) Show that there is one stable equilibrium when ω is small. Find the equilibrium point and find the frequency of small oscillations about this equilibrium.

(d) Show that for $\omega > \omega_{\rm c}$ the stable equilibrium shifts to a nonzero value of θ. Find the location of the new equilibrium and compute the corresponding frequency of small oscillations.

(e) Identify any and all conserved quantities.

[2] A mechanical system is described by the Lagrangian

$$L = \tfrac{1}{2}m\big(\dot{x}^2 + \dot{y}^2 + \dot{x}^2\big) + \tfrac{1}{2}m\big(\dot{\rho}^2 + \rho^2\dot{\phi}^2\big) - V(x + ky + lz, y + a\phi) \ , \qquad (1)$$

where k, l, and a are constants.

(a) How many independent one-parameter families of transformations leave L invariant?

(b) For each continuous symmetry, identify the associated conserved quantity.

(c) Are there any other conserved quantities? Find an expression for any other conserved quantities which may exist.

[3] If a particle of mass m is projected vertically upwards, the Lagrangian is

$$L = \tfrac{1}{2}m\dot{z}^2 - \frac{GMm}{R+z} \tag{2}$$

where M and R are the mass and radius of the earth, respectively, and z is the height of the particle above the surface.

(a) Sketch the potential and the phase curves.

(b) Identify the separatrix.

(c) Find the Hamiltonian.

[4] A particle has the Lagrangian

$$L = -mc^2\sqrt{1 - \frac{\dot{x}^2}{c^2}} - U(x)\ . \tag{3}$$

(a) Find the equations of motion.

(b) Find any and all conserved quantities.

(c) Find an expression for the period of bound motion.

[5] A simple pendulum of length ℓ and mass m is attached to a massless support which moves with constant acceleration a at an angle α with respect to the horizontal.

(a) Find the Lagrangian.

(b) Find the equations of motion.

(c) Solve for the frequency of small oscillations.

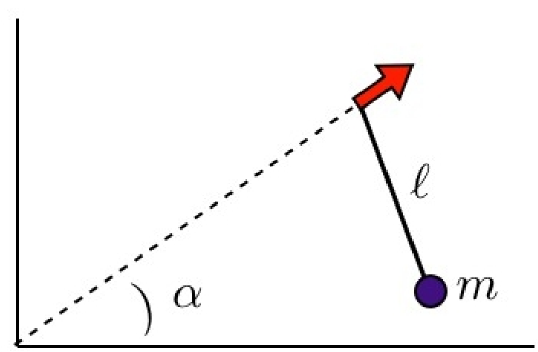

Figure 2: A pendulum whose accelerating point of support moves at angle α.