

Formulas:

Relativistic energy - momentum relation $E = \sqrt{m^2 c^4 + p^2 c^2}$; $c = 3 \times 10^8 \text{ m/s}$

Electron rest mass : $m_e = 0.511 \text{ MeV}/c^2$; Proton : $m_p = 938.26 \text{ MeV}/c^2$; Neutron : $m_n = 939.55 \text{ MeV}/c^2$

Planck's law : $u(\lambda) = n(\lambda) \bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda} \frac{1}{e^{hc/\lambda k_B T} - 1}$

Energy in a mode/oscillator : $E_f = nhf$; probability $P(E) \propto e^{-E/k_B T}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda) d\lambda$

Wien's displacement law : $\lambda_m T = \frac{hc}{4.96 k_B}$

Photons : $E = pc$; $E = hf$; $p = h/\lambda$; $f = c/\lambda$

Photoelectric effect : $eV_0 = (\frac{1}{2} m v^2)_{\max} = hf - \phi$, $\phi \equiv$ work function

Compton scattering : $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$

Rutherford scattering: $b = \frac{kq_1 q_2}{m_\alpha v^2} \cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Constants : $hc = 12,400 \text{ eV}\cdot\text{\AA}$; $\hbar c = 1,973 \text{ eV}\cdot\text{\AA}$; $k_B = 1/11,600 \text{ eV/K}$; $ke^2 = 14.4 \text{ eV}\cdot\text{\AA}$

Electrostatics : $F = \frac{kq_1 q_2}{r^2}$ (force) ; $U = q_0 V$ (potential energy) ; $V = \frac{kq}{r}$ (potential)

Hydrogen spectrum: $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3\text{\AA}}$

Bohr atom: $E_n = -\frac{ke^2 Z}{2r_n} = -\frac{Z^2 E_0}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{mk^2 e^4}{2\hbar^2} = 13.6 \text{ eV}$; $E_n = E_{kin} + E_{pot}$, $E_{kin} = -E_{pot}/2 = -E_n$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529\text{\AA}$; $L = mvr = n\hbar$ angular momentum

X - ray spectra: $f^{1/2} = A_n(Z - b)$; K : $b = 1$, L : $b = 7.4$

de Broglie : $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Wave function $\Psi(x,t) = |\Psi(x,t)| e^{i\theta(x,t)}$; $P(x,t) dx = |\Psi(x,t)|^2 dx =$ probability

Schrodinger equation : $-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi(x,t) = i\hbar \frac{\partial \Psi}{\partial t}$; $\Psi(x,t) = \psi(x) e^{-i\frac{E}{\hbar}t}$

Time - independent Schrodinger equation : $-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx \psi^* \psi = 1$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$; $x_{op} = x$, $p_{op} = \frac{\hbar}{i} \frac{\partial}{\partial x}$; $\langle A \rangle = \int_{-\infty}^{\infty} dx \psi^* A_{op} \psi$

Eigenvalues and eigenfunctions: $\hat{A}_{op} \Psi = a \Psi$ (a is a constant) ; uncertainty: $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$

Harmonic oscillator: $\Psi_n(x) = C_n H_n(x) e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n + \frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$; $\Delta n = \pm 1$

Step potential: $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - V)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T \sim e^{-2\alpha\Delta x}$; $T \sim e^{-2\int_a^b \alpha(x) dx}$; $\alpha(x) = \sqrt{\frac{2m[V(x) - E]}{\hbar^2}}$

3D square well: $\Psi(x,y,z) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $E = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2})$

Spherically symmetric potential: $\Psi_{n,\ell,m}(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$; $Y_{\ell m}(\theta,\phi) = f_{\ell m}(\theta)e^{im\phi}$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$; $L_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$; $L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m}$; $L_z = m\hbar$

Radial probability density: $P(r) = r^2 |R_{n\ell}(r)|^2$; Energy: $E_n = -13.6eV \frac{Z^2}{n^2}$

Ground state of hydrogen and hydrogen-like ions: $\Psi_{1,0,0} = \frac{1}{\pi^{1/2}}(\frac{Z}{a_0})^{3/2} e^{-Zr/a_0}$

Orbital magnetic moment: $\vec{\mu} = \frac{-e}{2m_e} \vec{L}$; $\mu_z = -\mu_B m_l$; $\mu_B = \frac{e\hbar}{2m_e} = 5.79 \times 10^{-5} eV/T$

Spin 1/2: $s = \frac{1}{2}$, $|S| = \sqrt{s(s+1)}\hbar$; $S_z = m_s \hbar$; $m_s = \pm 1/2$; $\vec{\mu}_s = \frac{-e}{2m_e} g\vec{S}$

Total angular momentum: $\vec{J} = \vec{L} + \vec{S}$; $|J| = \sqrt{j(j+1)}\hbar$; $|l-s| \leq j \leq l+s$; $-j \leq m_j \leq j$

Orbital + spin mag moment: $\vec{\mu} = \frac{-e}{2m}(\vec{L} + g\vec{S})$; Energy in mag. field: $U = -\vec{\mu} \cdot \vec{B}$

Two particles: $\Psi(x_1, x_2) = +/- \Psi(x_2, x_1)$; symmetric/antisymmetric

Screening in multielectron atoms: $Z \rightarrow Z_{eff}$, $1 < Z_{eff} < Z$

Orbital ordering:

1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 6d ~ 5f

Justify all your answers to all problems