PHYSICS 4E QUIZ 2 SPRING QUARTER 2009
PROF. HIRSCH APRIL 24

Formulas:

Relativistic energy - momentum relation $E = \sqrt{m^2c^4 + p^2c^2}$; $c = 3\times 10^8 m/s$

Electron rest mass : $m_e = 0.511\ MeV/c^2$;Proton : $m_p = 938.26\ MeV/c^2$;Neutron : $m_n = 939.55\ MeV/c^2$

Planck's law : $u(\lambda) = n(\lambda)\bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda}\frac{1}{e^{hc/\lambda k_BT} - 1}$

Energy in a mode/oscillator : $E_f = nhf$; probability $P(E) \propto e^{-E/k_BT}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67\times 10^{-8} W/m^2K^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda)d\lambda$

Wien's displacement law : $\lambda_m T = \frac{hc}{4.96k_B}$

Photons : $E = pc$; $E = hf$; $p = h/\lambda$; $f = c/\lambda$

Photoelectric effect : $eV_0 = (\frac{1}{2}mv^2)_{\max} = hf - \phi$, $\phi \equiv$ work function

Compton scattering : $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

Rutherford scattering: $b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Constants : $hc = 12{,}400\ eV\,A$; $\hbar c = 1{,}973\ eV\,A$; $k_B = 1/11{,}600\ \ eV/K$; $ke^2 = 14.4 eVA$

Electrostatics : $F = \frac{kq_1q_2}{r^2}$ (force) ; $U = q_0V$ (potential energy) ; $V = \frac{kq}{r}$ (potential)

Hydrogen spectrum : $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097\times 10^7\ m^{-1} = \frac{1}{911.3A}$

Bohr atom : $E_n = -\frac{ke^2Z}{2r_n} = -\frac{Z^2E_0}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{mk^2e^4}{2\hbar^2} = 13.6eV$; $E_n = E_{kin} + E_{pot}, E_{kin} = -E_{pot}/2 = -E_n$

$hf = E_i - E_f$; $r_n = r_0n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529A$; $L = mvr = n\hbar$ angular momentum

X - ray spectra : $f^{1/2} = A_n(Z - b)$; K : $b = 1$, L : $b = 7.4$

de Broglie : $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x\Delta p \sim \hbar$; $\Delta t\Delta E \sim \hbar$

Wave function $\Psi(x,t) = |\Psi(x,t)|e^{i\theta(x,t)}$; $P(x,t)\,dx = |\Psi(x,t)|^2\,dx$ = probability

Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well : $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2n^2}{2mL^2}$

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Justify all your answers to all problems

Problem 1 (10 pts)
An electron is described by the wavepacket
$$\psi(x,t) = \int dk e^{i(kx-\omega(k)t)} a(k)$$
with a(k)=A for k in the interval (k_1,k_2) and a(k)=0 outside that interval. k_1=99A^{-1}, k_2=101A^{-1} and A is a constant.
(a) Find an expression for $\psi(x,t=0)$. What is the most probable position for this electron at t=0?
(b) Estimate the uncertainty in the position of this electron at t=0, in A.
(c) Estimate the most probable position for this electron at t=1s.
(d) Estimate the group velocity of this electron, expressed as v/c.

Problem 2 (10 pts)
An electron is confined to move in a one-dimensional potential of the form
$$V(x) = V_0 \frac{x^2}{x_0^2}$$
where V_0=5eV and x_0=0.01 A and x is the position coordinate.
(a) Assume this electron is known for certain to be within a distance of 0.01A from the origin (x=0). Give a rough estimate of (i) its potential energy and (ii) its kinetic energy, both in eV.
(b) Assume now the electron is in a state that minimizes its total energy (kinetic plus potential). Estimate the uncertainty in its position and its potential and kinetic energies, in eV.
Hint: Use Heisenberg's uncertainty principle. Use $\hbar^2/m_e = 7.62\ eV\,A^2$

Problem 3 (10 pts)
(a) An electron is in the ground state of an infinite square well of length L. How much more likely is it to be found at the center of the well (x=L/2) than at x=L/4?
(b) An electron in an infinite square well is in a state of energy 1000eV. Give a lower bound for the length of this well, in A. Can you also give an upper bound?

Justify all your answers to all problems