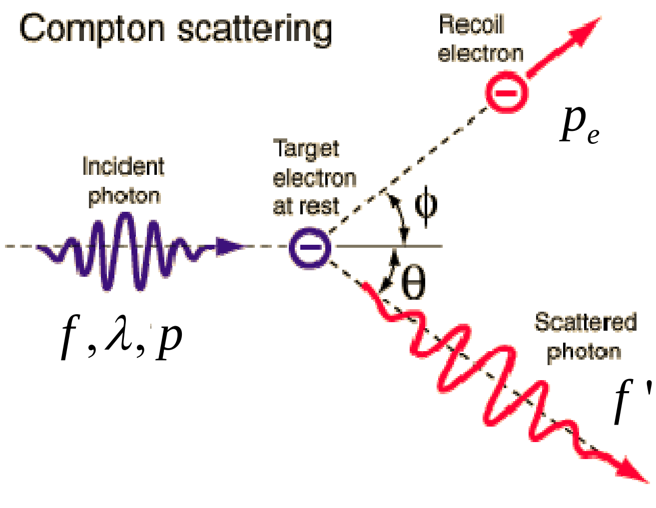
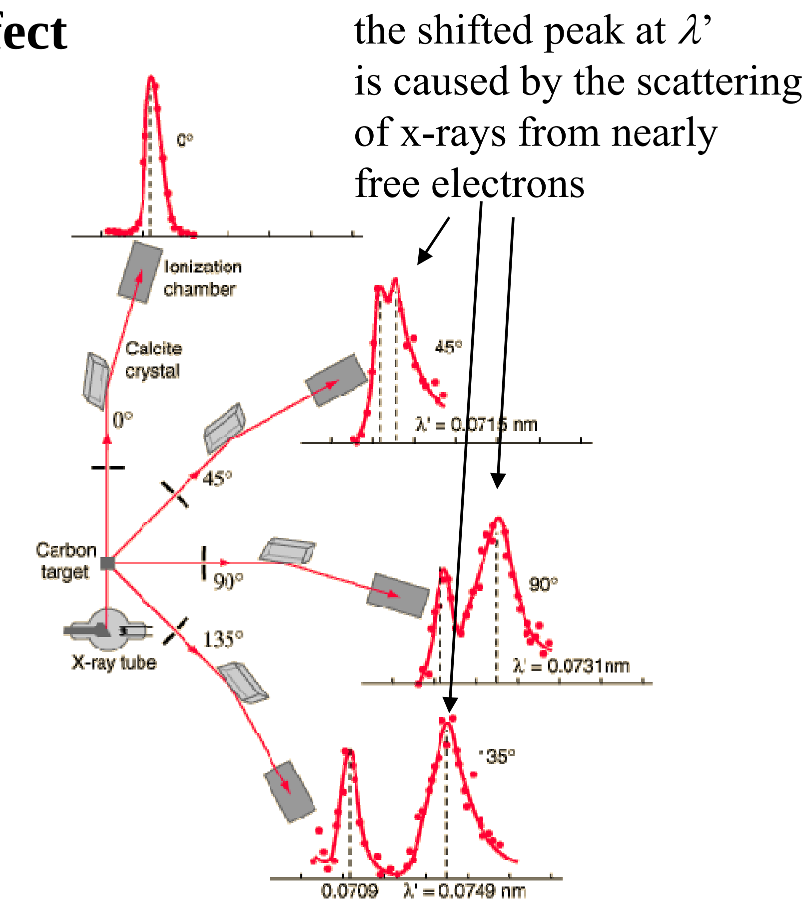



# Compton effect




conservation of energy:

$$E + E_e = E' + E_e'$$

$$hf + m_e c^2 = hf' + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

conservation of momentum:

$$p = p'\cos\theta + p_e\cos\phi$$

$$p'\sin\theta = p_e\sin\phi$$

$$p = \frac{hf}{c}$$

$$p_e^2 = (p')^2 + p^2 - 2pp'\cos\theta =$$

$$= \left(\frac{hf'}{c}\right)^2 + \left(\frac{hf}{c}\right)^2 - \frac{2h^2 ff'}{c^2}\cos\theta$$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

the Compton wavelength

$$\frac{h}{m_e c} = 0.00243 \text{ nm}$$

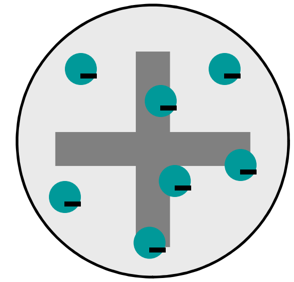
Why x-rays? $\theta = 90^0$

$$\frac{\Delta\lambda}{\lambda} = \frac{0.00243 \text{ nm}}{0.00106 \text{ nm}} = 2.29$$ ← x-rays from cobalt $\lambda$=0.00106 nm

$$\frac{\Delta\lambda}{\lambda} = \frac{0.00243 \text{ nm}}{546.1 \text{ nm}} = 4.45\times 10^{-6}$$ ← green light from mercury lamp, $\lambda$=546.1 nm

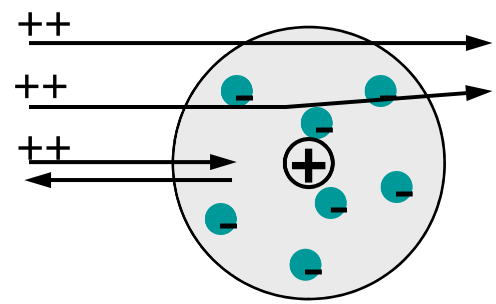
Lenard's experiments: Electrons are easily transmitted through thin metals → porosity of atoms

search for a model which explains porosity



**Thomson "plum-pudding" model, 1898:**
The atom is a homogeneous sphere of uniformly distributed mass and positive charge in which were embedded, like raisins in a plum pudding, negatively charged electrons

**Ernest Rutherford, Hans Geiger, Ernest Marsden, 1909-1914:**
Probed the distribution of mass within the atom by measuring the angular distribution of a beam of collimated $\alpha$-particles with speeds of $\sim 10^7$ m/s after scattering from metal foils




**Discovered that some of $\alpha$-particles scatter back**

**most of the mass and all of the positive charge lie in a minute central nucleus of the atom**

**Ernest Rutherford:**
"It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

$\alpha$-particles $He^{2+}$ are positively charged nuclei of helium atom

At low kinetic energies $K_\alpha$ collisions are governed by Coulomb's law

$$F = k\frac{(2e)(Ze)}{r^2}$$

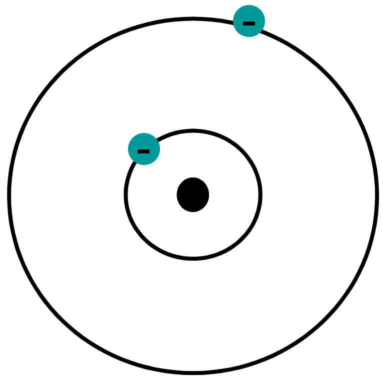
where $k = 1/4\pi\varepsilon_0$
$= 8.988\times 10^9\ \mathrm{N\cdot m^2/C^2}$

With increasing $K_\alpha$ $\alpha$-particles begin to penetrate the nucleus and collisions are no longer governed by Coulomb's law. It takes place when

$$K_\alpha = k\frac{(2e)(Ze)}{d_n}$$

This allows measuring nucleus radius $d_n \sim 10^{-14}$ m

**Rutherford:** Electrons must revolve around the nucleus in order to avoid falling to it




In classical physics, a charged particle in an orbit will radiate energy, because it is accelerating. Thus a classical planetary electron would continuously radiate away its energy, moving to progressively lower orbits.

**Niels Bohr, 1913:**
applied Plank's ideas of quantized energy levels to orbiting atom electrons

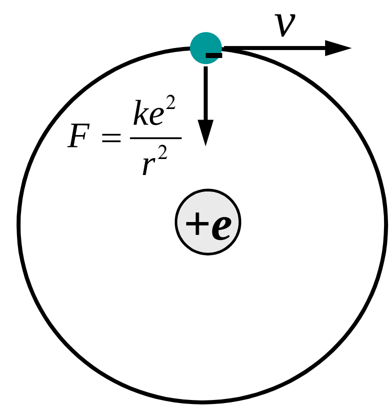
**1st Bohr's postulate:** only those electron orbits occur for which the angular momentum of the electron is $nh/2\pi$, where $n$ is an integer and $h$ is Planck's constant.

$$m_e vr = n\frac{h}{2\pi} = n\hbar$$
$$n = 1, 2, 3, \ldots$$

**2nd Bohr's postulate:** an electron remaining in one of the allowed orbital states does not radiate energy – electron orbits are stable; radiation is emitted only when an electron goes from a state of higher energy ($n_i$) to a state of lower energy ($n_f$), where $n_i > n_f$.; the energy of the quantum of radiation emitted, $\Delta E = hf$, is equal to the difference in energy of the two states.

$$E_i - E_f = hf$$

# Allowed orbit radii and energy levels




$$\frac{ke^2}{r^2} = \frac{m_e v^2}{r}$$ ← Coulomb attraction force creates centripetal acceleration of the electron

$$K = \frac{m_e v^2}{2} = \frac{ke^2}{2r}$$

$$E = K + U = \frac{m_e v^2}{2} - \frac{ke^2}{r} = -\frac{ke^2}{2r}$$

$$m_e v r = n\hbar$$

$$\frac{m_e v^2}{2} = \frac{ke^2}{2r}$$

$$E_n = -\frac{ke^2}{2a_0}\frac{1}{n^2}, \quad n = 1,2,3,\ldots$$

$$E_n = -\frac{13.6}{n^2}\ \text{eV}$$

$$r_n = n^2 \frac{\hbar^2}{m_e k e^2}, \quad n = 1,2,3,\ldots$$

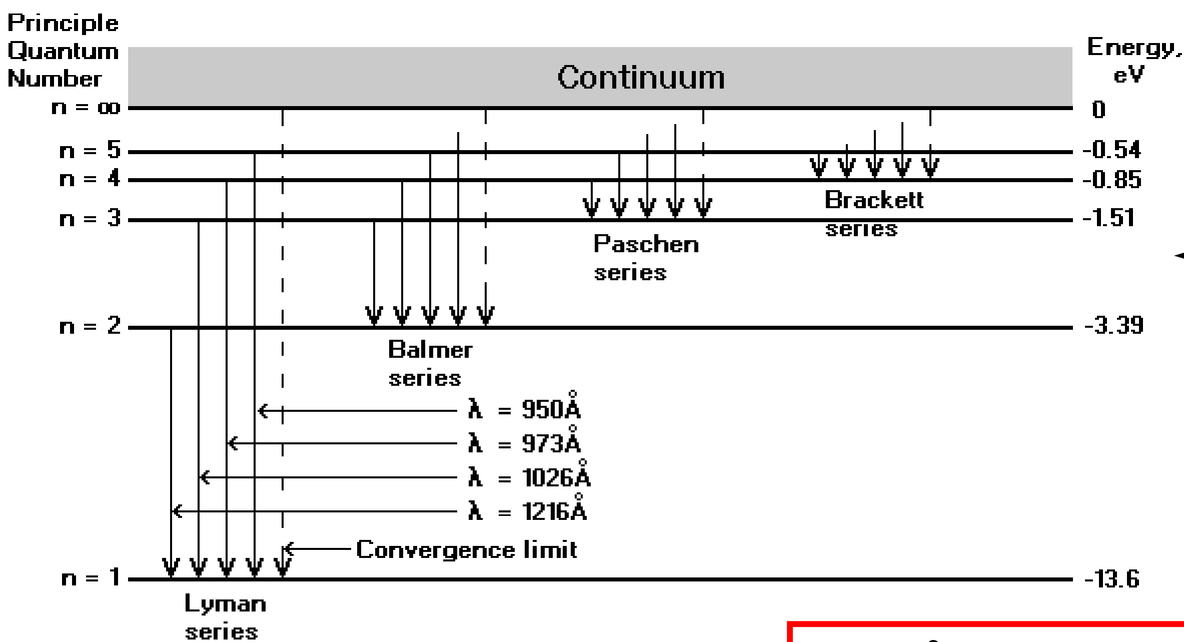
$$a_0 = \frac{\hbar^2}{m_e k e^2} = 0.0529\ \text{nm}$$ ← the Bohr radius

$$r_n = n^2 a_0$$

# Energy of emitted photons

$$f = \frac{E_i - E_f}{h} = \frac{ke^2}{2a_0 h}\left(\frac{1}{n_f{}^2} - \frac{1}{n_i{}^2}\right)$$

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{ke^2}{2a_0 hc}\left(\frac{1}{n_f{}^2} - \frac{1}{n_i{}^2}\right)$$




Energy level diagram of the hydrogen atom

for a single electron orbiting a nucleus of charge $+Ze$

$$r_n = n^2 a_0 / Z$$

$$E_n = -\frac{ke^2}{2a_0}\frac{Z^2}{n^2}, \quad n = 1, 2, 3, \ldots$$

**James Franck and Gustav Hertz, 1913:**
Direct experimental demonstration of discrete energy levels in atoms

# EXPERIMENT #5
# The Franck-Hertz Experiment:
# Electron Collisions with Mercury

## GOALS

### Physics

Measure the energy difference between the ground state and the first excited state in mercury atoms.

### Error

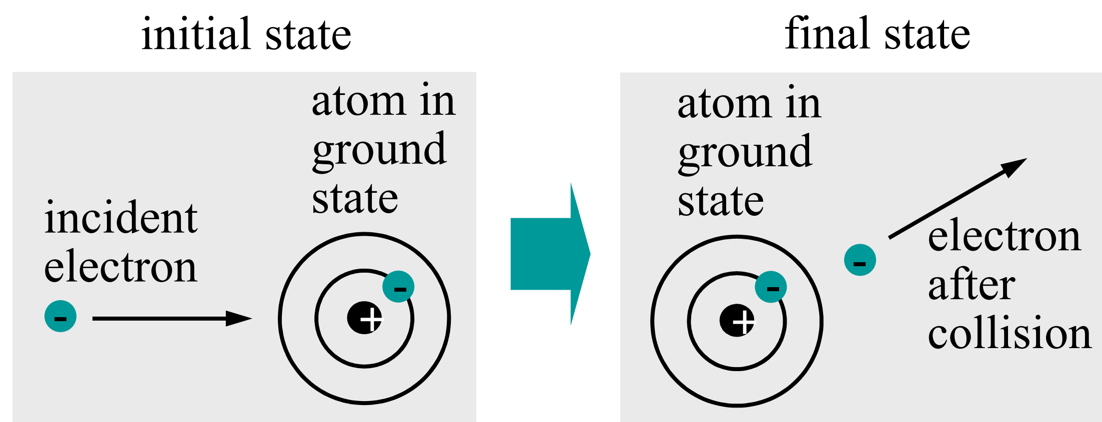
Calculate whether your mean value for the energy difference is in reasonable agreement with one of the principal lines in the mercury spectrum.

### References

Serway, Moses, Moyer §4.5

**Principle of the experiment:**
A beam of electrons collides with the gas of atoms.

If the energy of the electrons in the beam is less than the energy separation of the ground state from the first excited state, then no energy is absorbed and the collisions are elastic.

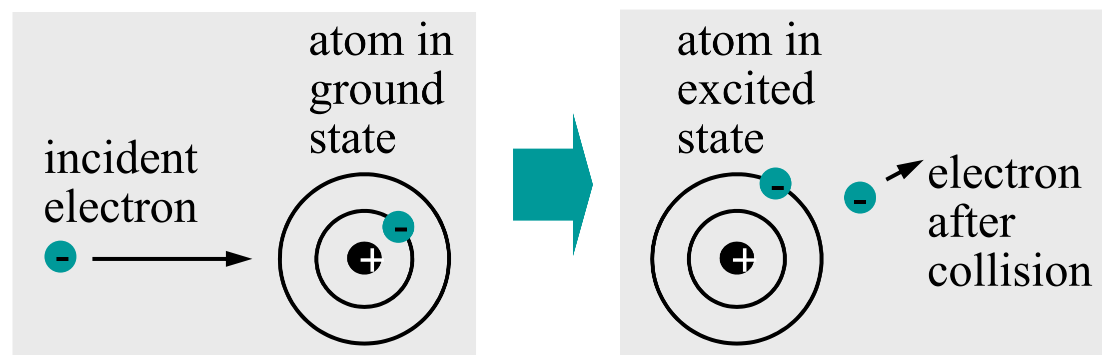



elastic collision ↔ kinetic energy is conserved

$$K_{e-initial} + K_{atom-initial} = K_{e-final} + K_{atom-final} \rightarrow K_{e-initial} \approx K_{e-final}$$

loss of electron kinetic energy is very small $\frac{\Delta K_e}{K_e} \sim \frac{m_e}{M} << 1$ e.g, for scattering back $\frac{\Delta K_e}{K_e} = \frac{4M}{m_e\left(1+M/m_e\right)^2} \approx \frac{4m_e}{M}$

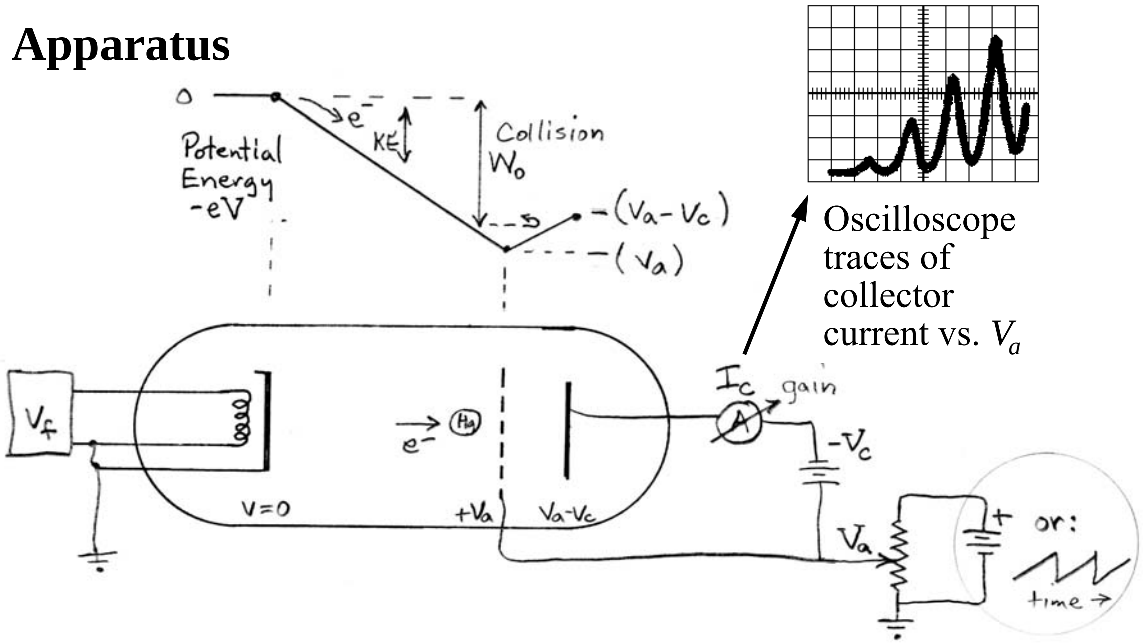
If the beam energy is equal to or greater than the separation of the lowest states, then energy is absorbed equal to the energy separation of the states (*hf*) and the collision is inelastic.




inelastic collision ↔ kinetic energy is not conserved

$$K_{e-initial} + K_{atom-initial} = K_{e-final} + K_{atom-final} + hf \rightarrow K_{e-initial} \approx K_{e-final} + hf$$

## Apparatus



Oscilloscope traces of collector current vs. $V_a$

The sealed tube with a small amount of mercury which is vaporized by heating the tube.

The tube contains a cathode which is heated to emit electrons,

a perforated anode maintained at a positive accelerating voltage $V_a$ with respect to the cathode,

and a collector electrode held at a small retarding voltage $V_c$.

Electrons emitted by the cathode are accelerated by the electric field as they fall "down" the potential hill toward the anode. When $eV_a < \Delta E$, only elastic collisions can occur between the electrons and the mercury atoms. The energetic electrons will pass through the perforated anode and will be able to go up the small potential hill to reach the collector.

When $eV_a$ approaches $\Delta E$ the collector current begins the drop because some electrons make inelastic collisions with mercury atoms, lose all their kinetic energy, and therefore cannot overcome the final potential barrier $e(V_a$-$V_c)$.

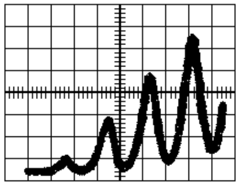
The current rises as $V_a$ is increased further, since electrons can now make one inelastic collision then accelerate again and get enough energy to reach the collector.

Successive minima in collector current with a spacing of $\Delta V = \Delta E/e$ occur when an electron gains enough energy to make $N$ inelastic collisions, with no energy remaining to overcome the final barrier.

**CAUTION**
The outside of the cabinet containing the Franck-Hertz tube gets HOT when the furnace power is ON. Connections should be done while the furnace power is OFF. Turn OFF the furnace power when you complete your experiment.

**THE EXPERIMENT**

● Set the temperature control on the side of the furnace.
It takes 10-15 minutes for the furnace to reach "equilibrium" from a cold start.
Switch on the control unit. Set the $V_a$ switch to “ramp".
Set the filament heater and retarding potential at about mid-position.


Oscilloscope traces of collector current vs. $V_a$

● Produce a Franck-Hertz signal on the oscilloscope screen.
Note that the first minimum appears at a voltage larger than the voltage between the succeeding equally spaced minima. This results from the fact that the electrons must surmount the reverse bias voltage as well as a "contact potential" that exists between the cathode and anode.
Produce paper versions of $I$ vs. $V_a$ and include a sample x-y graph in your notebook.

● Compute the corresponding average spacing and its uncertainty. What are some important sources of uncertainty in these results?

● Determine the best value and uncertainty for the energy and wavelength corresponding to the difference between the ground state and first excited state for mercury.
Is it in reasonable agreement with one of the principal lines in the mercury spectrum?