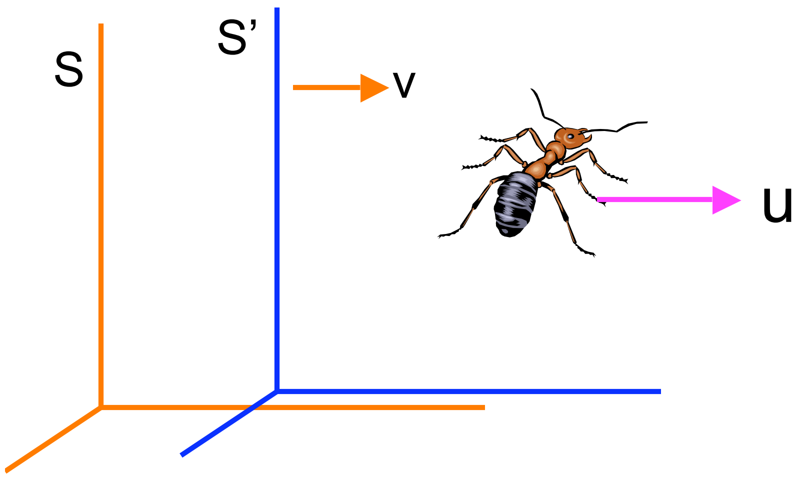
Physics 2D Lecture Slides
Lecture 5

April 6, 2009

Lorentz Velocity Transformation Rule

S and S' are measuring ant's speed u along x, y, z axes



In S' frame, $u_{x'} = \dfrac{x_2' - x_1'}{t_2' - t_1'} = \dfrac{dx'}{dt'}$

$dx' = \gamma(dx - vdt), \quad dt' = \gamma\left(dt - \dfrac{v}{c^2}dx\right)$

$$u_{x'} = \frac{dx - vdt}{dt - \frac{v}{c^2}dx}, \text{ divide by dt}$$

$$u_{x'} = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

For v << c, $u_{x'} = u_x - v$

(Galilean Trans. Restored)

Velocity Transformation Perpendicular to S-S' motion

$$dy' = dy, \quad dt' = \gamma(dt - \frac{v}{c^2}dx)$$

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{\gamma(dt - \frac{v}{c^2}dx)}$$

divide by dt on RHS

$$u'_y = \frac{u_y}{\gamma(1 - \frac{v}{c^2}u_x)}$$

There is a change in velocity in the direction $\perp$ to S-S' motion !

Similarly Z component of Ant's velocity transforms as

$$u'_z = \frac{u_z}{\gamma(1 - \frac{v}{c^2}u_x)}$$

Inverse Lorentz Velocity Transformation

Inverse Velocity Transform:

$$u_x = \frac{u_{x'} + v}{1 + \frac{v u_{x'}}{c^2}}$$

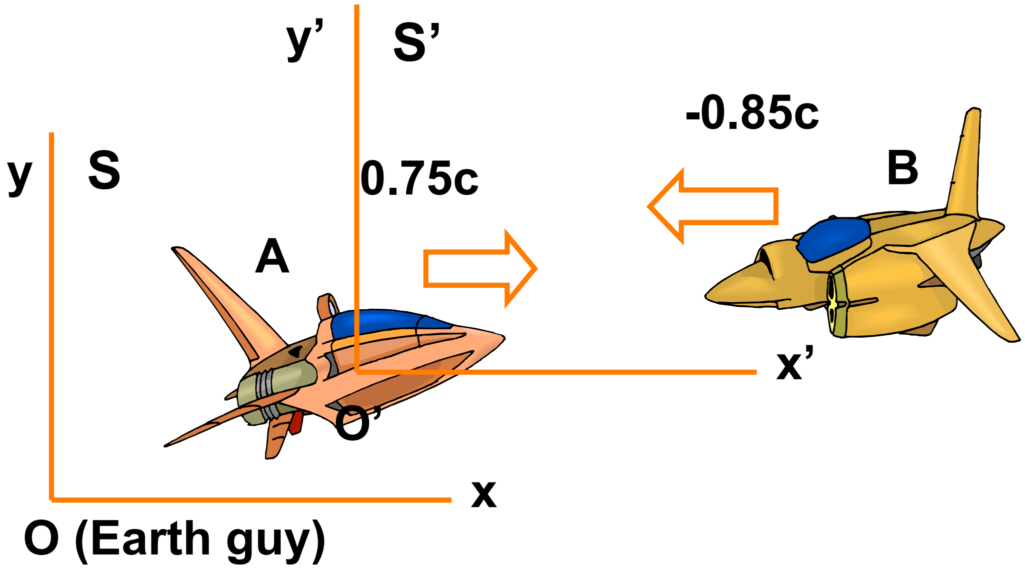
$$u_y = \frac{u_y'}{\gamma(1 + \frac{v}{c^2} u_x')}$$

$$u_z = \frac{u_z'}{\gamma(1 + \frac{v}{c^2} u_x')}$$

As usual, replace

$v \Rightarrow -v$

Does Lorentz Transform "work" For Topgun ?



Two rockets A &B travel in opposite directions

An observer on earth (S) measures speeds = 0.75c And 0.85c for A & B respectively

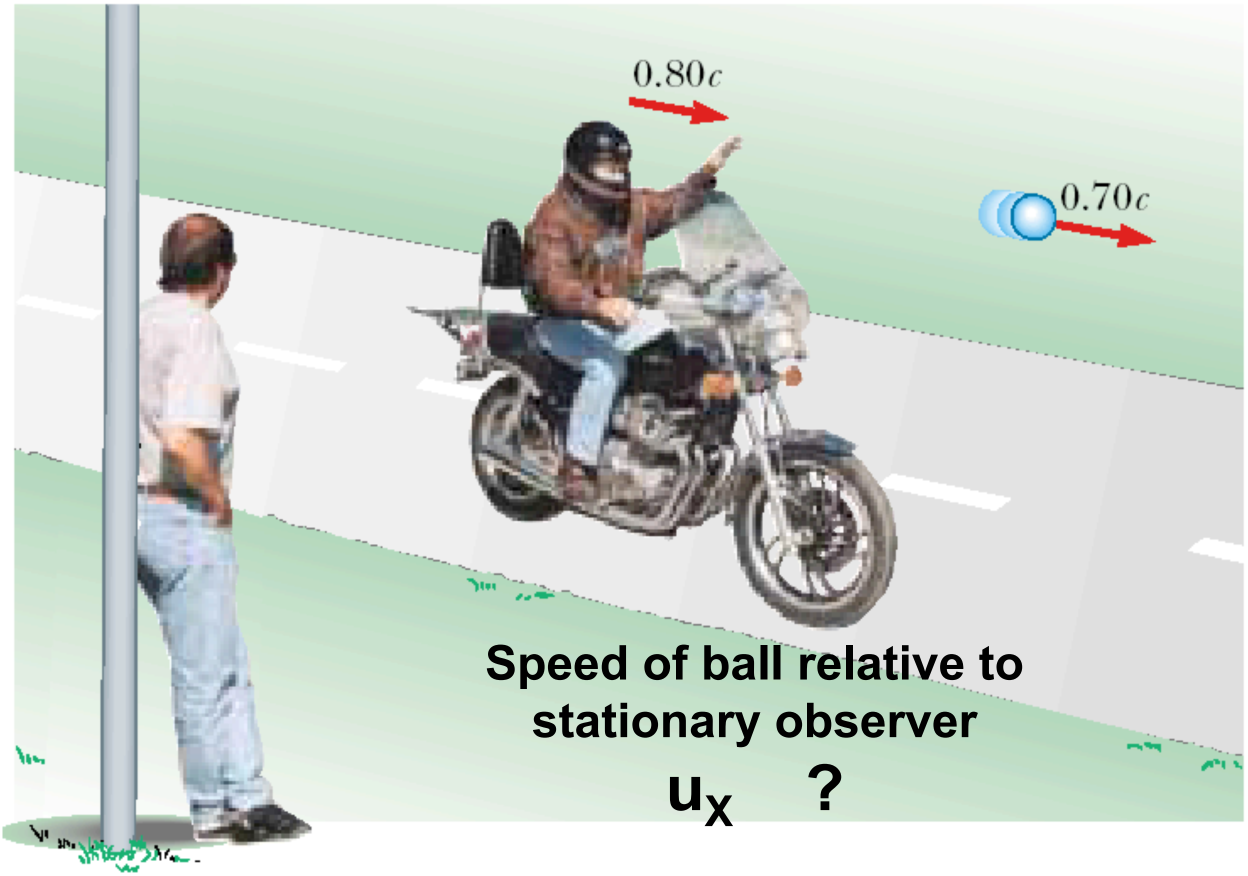
What does A measure as B's speed?

Place an imaginary S' frame on Rocket A $\Rightarrow$ v = 0.75c relative to Earth Observer S

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$

Consistent with Special Theory of Relativity

Example of Inverse velocity Transform



Speed of ball relative to stationary observer

u_x ?

Biker moves with speed = 0.8c past stationary observer

Throws a ball forward with speed = 0.7c

What does stationary observer see as velocity of ball ?

Place S' frame on biker
Biker sees ball speed

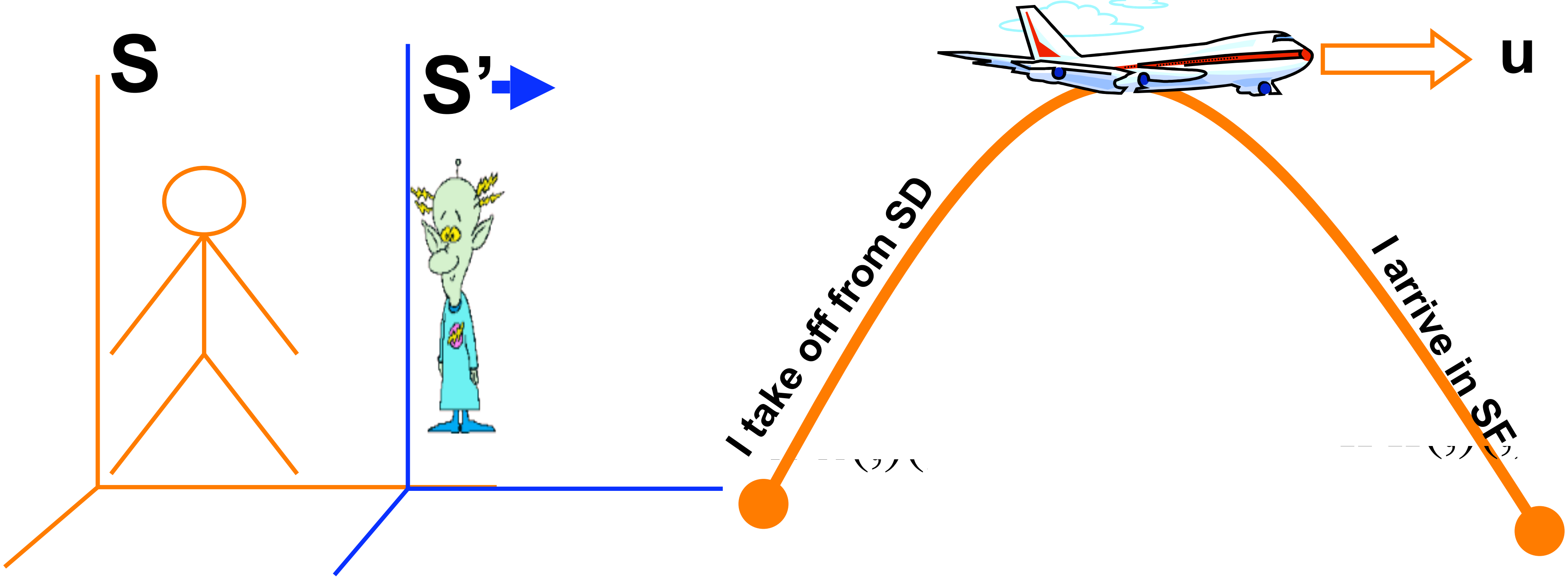
$u_{x'} = 0.7c$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = \boxed{0.96c}$$

Hollywood Yarns Of Time Travel !

Terminator : Can you be **seen** to be born before your mother?

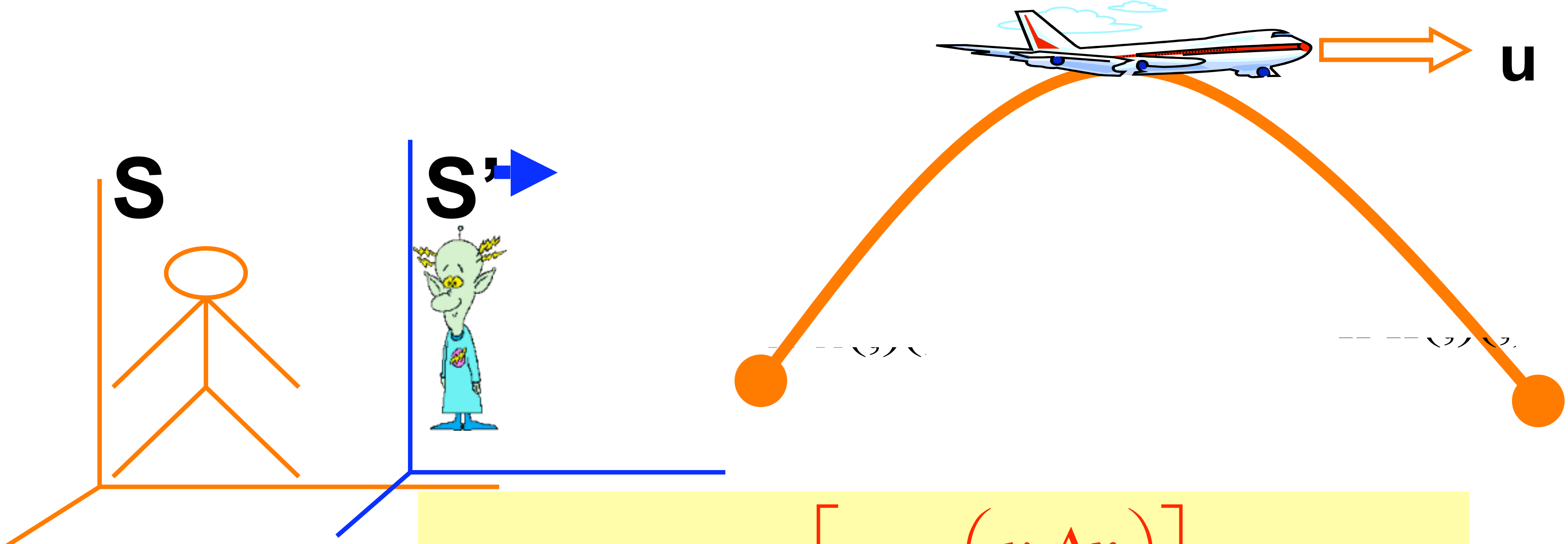
A frame of Ref where sequence of events is REVERSED **?!!**



$$\Delta t' = t_2^{'} - t_1^{'} = \gamma\left[\Delta t - \left(\frac{v\,\Delta x}{c^2}\right)\right]$$

Reversing sequence of events $\Rightarrow \Delta t' < 0$

I Can't be seen to arrive in SF before I take off from SD

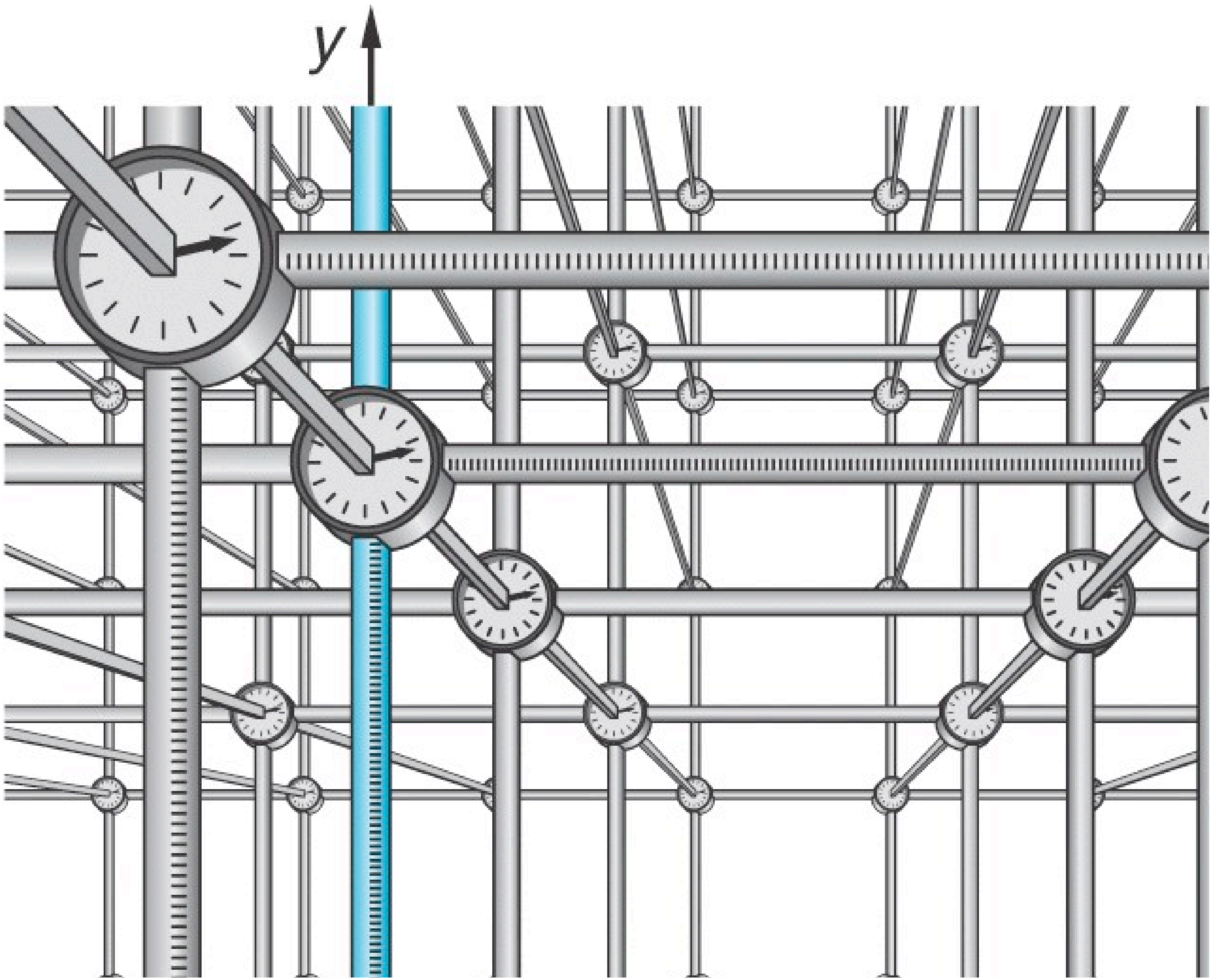


$$\Delta t' = t_2' - t_1' = \gamma\left[\Delta t - \left(\frac{v\,\Delta x}{c^2}\right)\right]$$

For what value of v can $\Delta t' < 0$

$$\Delta t' < 0 \Rightarrow \Delta t < \frac{v\,\Delta x}{c^2} \quad \Rightarrow 1 < \frac{v\,\Delta x}{c^2\Delta t} = \frac{v\,\mathrm{u}}{c^2}$$

$$\Rightarrow \frac{\mathrm{v}}{\mathrm{c}} > \frac{c}{u} \qquad \Rightarrow v > c \text{ : Not allowed}$$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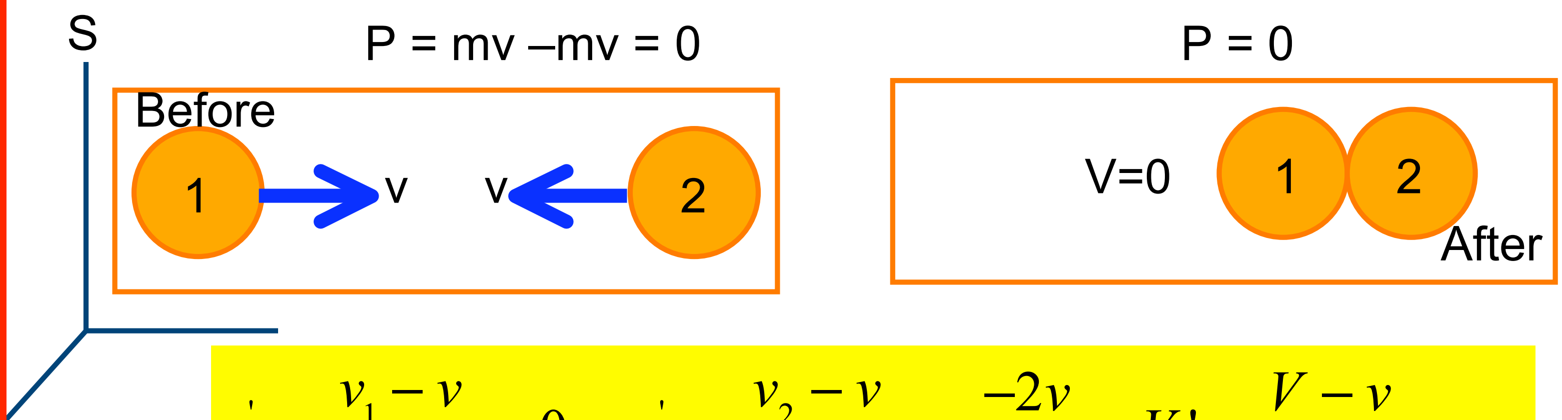
Relativistic Momentum and Revised Newton's Laws

Need to generalize the laws of Mechanics & Newton to confirm to Lorentz Transform and the Special theory of relativity: Example :

Watching an Inelastic Collision between two putty balls

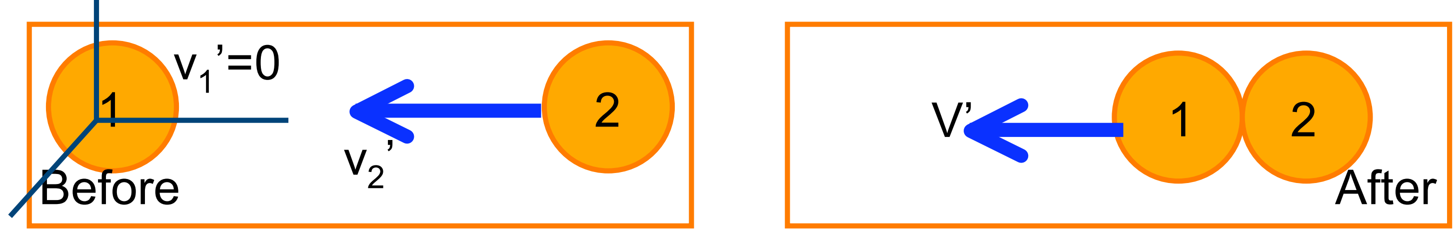


$$v_1' = \frac{v_1 - v}{1 - \frac{v_1 v}{c^2}} = 0, \quad v_2' = \frac{v_2 - v}{1 - \frac{v_1 v}{c^2}} = \frac{-2v}{1 + \frac{v^2}{c^2}}, \quad V' = \frac{V - v}{1 - \frac{Vv}{c^2}} = -v$$

$$p'_{before} = mv_1' + mv_2' = \frac{-2mv}{1 + \frac{v^2}{c^2}}, \quad p'_{after} = 2mV' = -2mv$$

$$\mathrm{p}'_{\mathrm{before}} \neq \mathrm{p}'_{\mathrm{after}}$$

S'



Definition (without proof) of Relativistic Momentum

$$\mathbf{p} = \frac{m\mathbf{u}}{\sqrt{1 - u^2 / c^2}}$$

With the new definition relativistic momentum is conserved in all frames of references : Do the exercise

New Concept

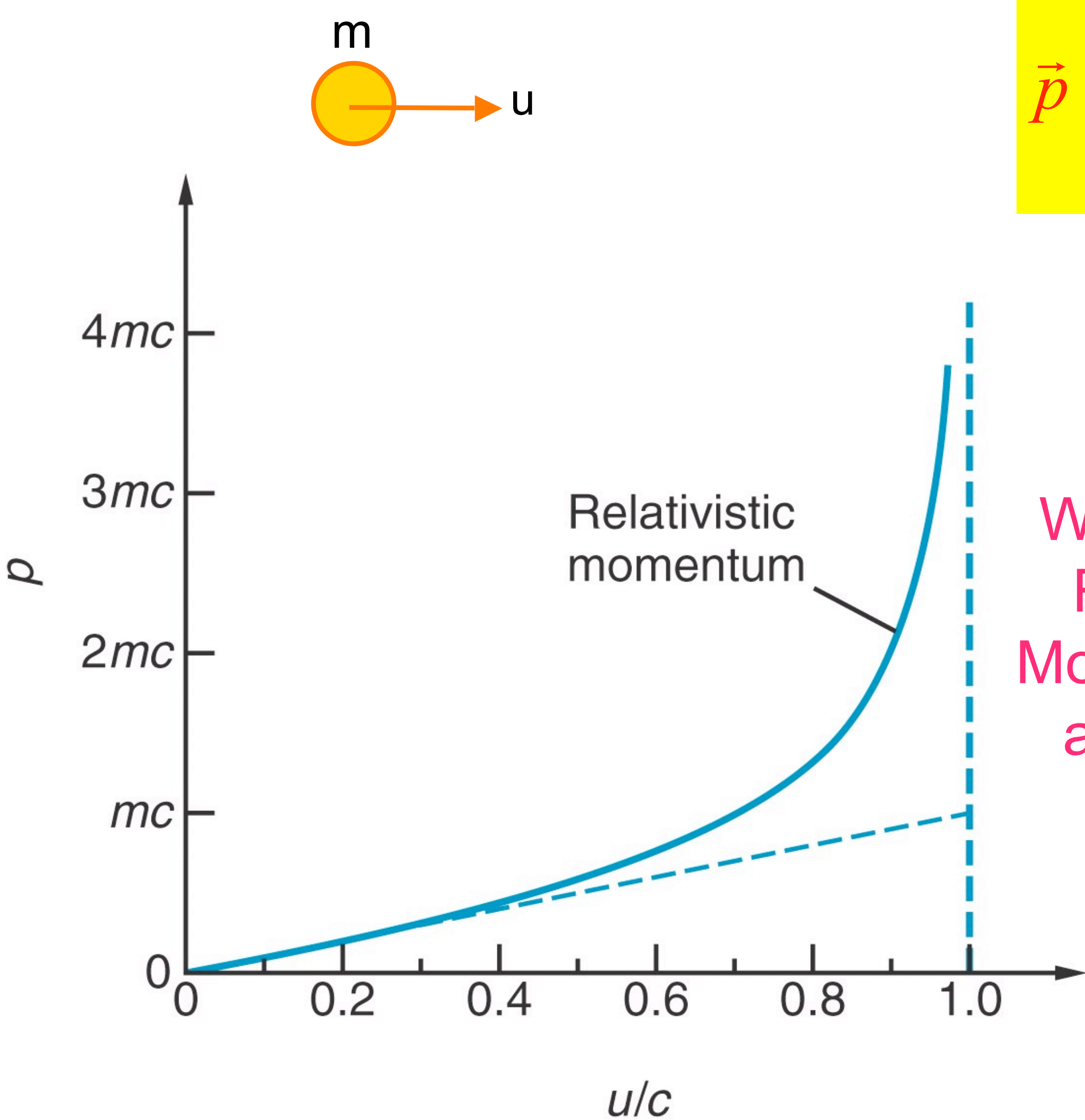
Rest Mass is mass of object in frame in which it at rest.

$$\gamma = 1 / \sqrt{1 - u^2 / c^2}$$

u is speed of object, NOT reference frame.

Nature of Relativistic Momentum

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$



With the new definition of Relativistic momentum Momentum is conserved in all frames of references

Relativistic Force & Acceleration

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

Relativistic
Force
And
Acceleration

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-(u/c)^2}}\right) \quad use \quad \frac{d}{dt} = \frac{du}{dt}\frac{d}{du}$$

$$F = \left[\frac{m}{\sqrt{1-(u/c)^2}} + \frac{mu}{\left(1-(u/c)^2\right)^{3/2}} \times (\frac{-1}{2})(\frac{-2u}{c^2})\right]\frac{du}{dt}$$

$$F = \left[\frac{mc^2 - mu^2 + mu^2}{c^2\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt}$$

$$F = \left[\frac{m}{\left(1-(u/c)^2\right)^{3/2}}\right]\frac{du}{dt} \quad : \text{Relativistic Force}$$

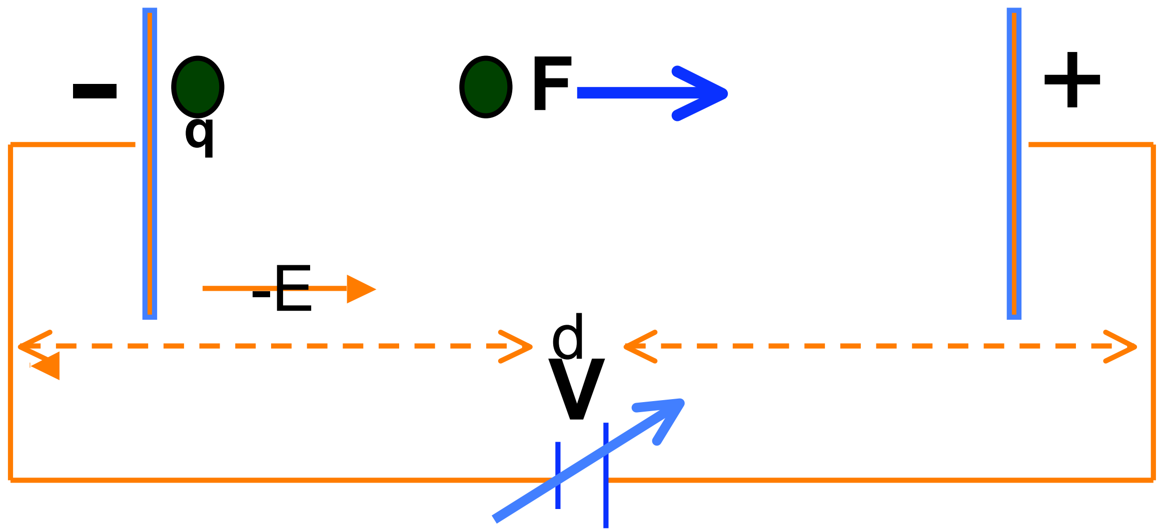
Since Acceleration $\vec{a} = \frac{d\vec{u}}{dt}$,[rate of change of velocity]

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m}\left[1-(u/c)^2\right]^{3/2}$$

Reason why you cant quite get up to the speed of light no matter how hard you try!

Note: As $u/c \to 1$, $\vec{a} \to 0$!!!!
Its harder to accelerate when you get closer to speed of light

A Linear Particle Accelerator



Parallel Plates

E= V/d

F= eE

Charged particle q moves in straight line in a uniform electric field $\vec{E}$ with speed $\vec{u}$ accelarates under force $\vec{F}=q\vec{E}$

$$|\vec{a}| = \left|\frac{d\vec{u}}{dt}\right| = \left|\frac{\vec{F}}{m}\right|\left(1-\frac{u^2}{c^2}\right)^{3/2} = \left|\frac{q\vec{E}}{m}\right|\left(1-\frac{u^2}{c^2}\right)^{3/2}$$

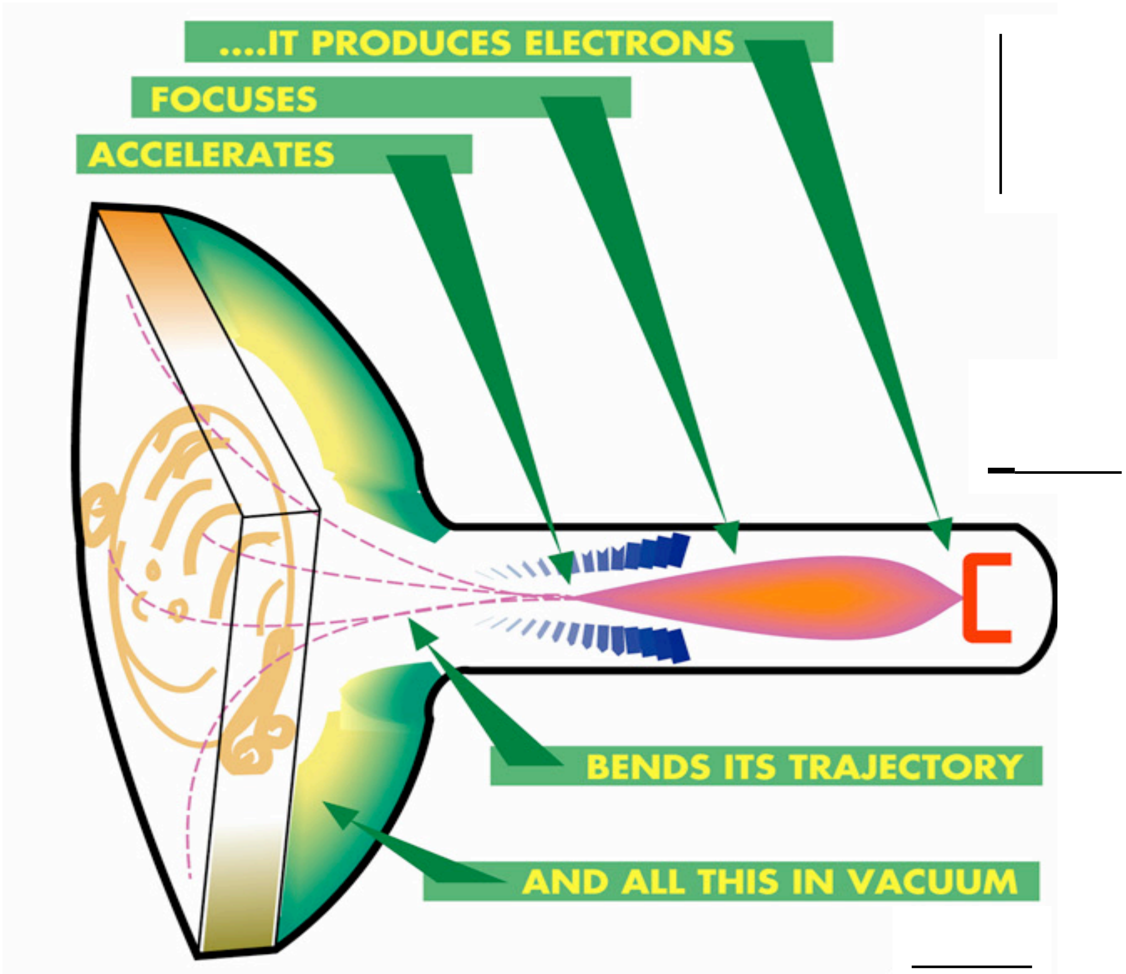
larger the potential difference V across plates, larger the force on particle

Under force, work is done on the particle, it gains Kinetic energy

<u>New Unit of Energy</u>

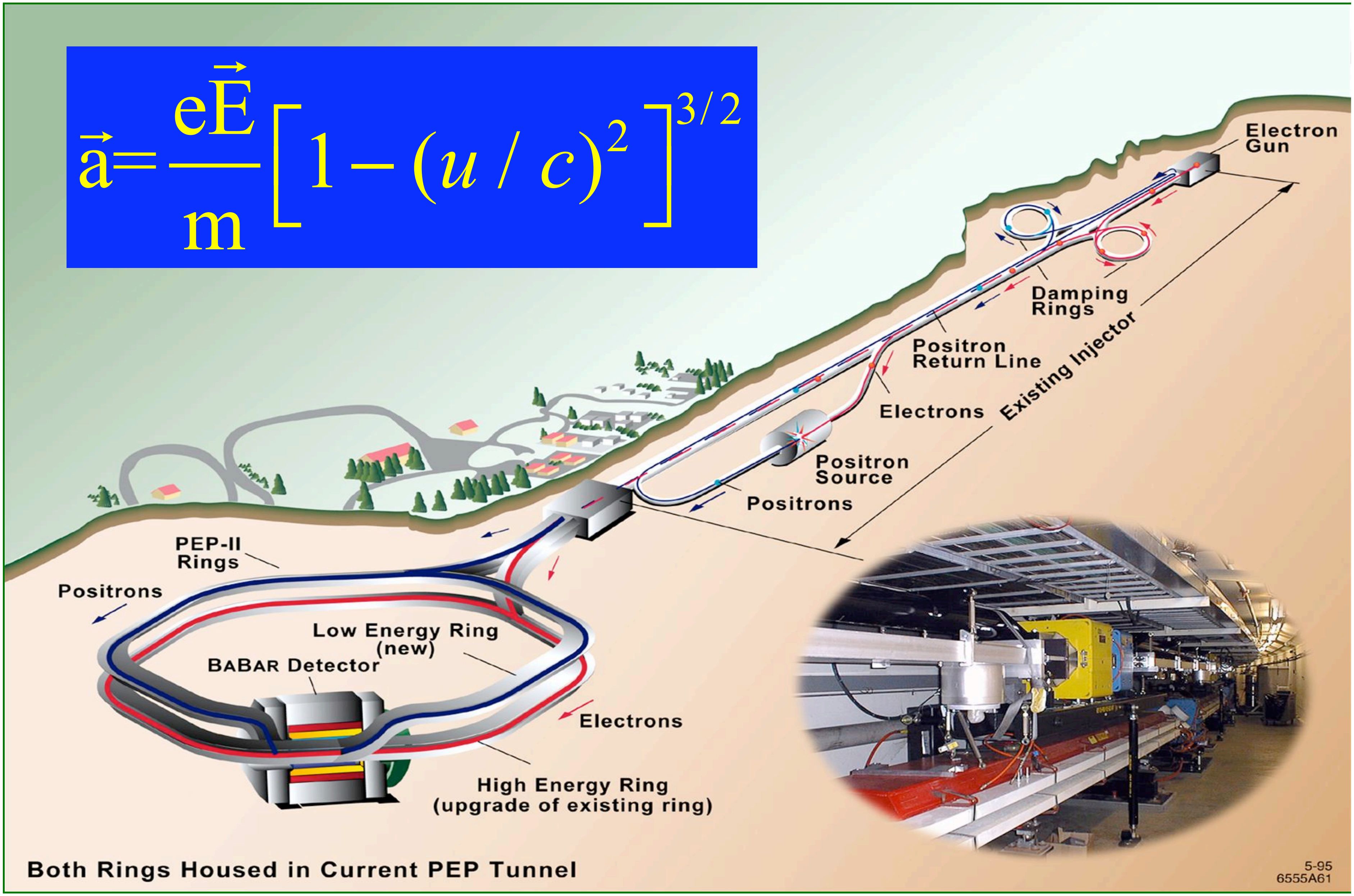
1 eV = 1.6x10^{-19} Joules
1 MeV = 1.6x10^{-13} Joules
1 GeV = 1.6x10-10 Joules

Your Television (the CRT type) is a Small Particle Accelerator !



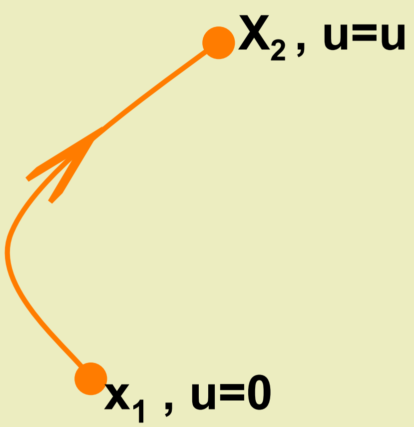
Linear Particle Accelerator : 50 GigaVolts Accelating Potential

$$\vec{a} = \frac{e\vec{E}}{m}\left[1 - (u/c)^2\right]^{3/2}$$



PEP-II accelerator schematic and tunnel view

Relativistic Work Done & Change in Energy



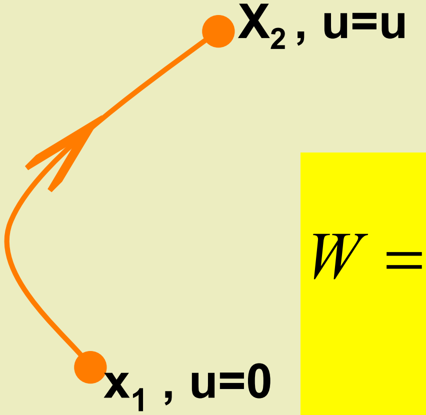
$$W = \int_{x_1}^{x_2} \vec{F}.d\vec{x} = \int_{x_1}^{x_2} \frac{d\vec{p}}{dt}.d\vec{x}$$

$$p = \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}} \quad \therefore \quad \frac{d\vec{p}}{dt} = \frac{m\frac{du}{dt}}{\left[1 - \frac{u^2}{c^2}\right]^{3/2}},$$

substitute in W,

$$\therefore \quad W = \int_0^u \frac{m\frac{du}{dt}udt}{\left[1 - \frac{u^2}{c^2}\right]^{3/2}} \quad \text{(change in var x} \rightarrow \text{u)}$$

Relativistic Work Done & Change in Energy



$$W = \int_0^u \frac{mudu}{\left[1 - \frac{u^2}{c^2}\right]^{3/2}} = \frac{mc^2}{\left[1 - \frac{u^2}{c^2}\right]^{1/2}} - mc^2$$

$$= \gamma mc^2 - mc^2$$

Work done is change in energy (KE in this case)

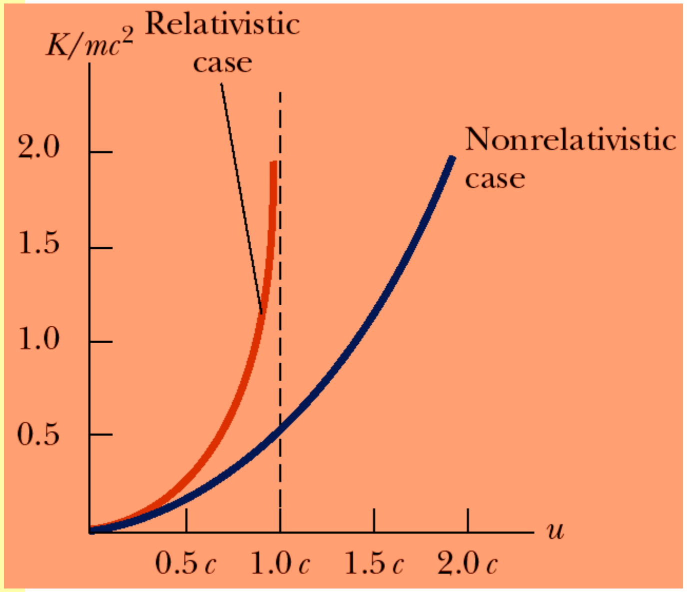
$\boxed{K = \gamma mc^2 - mc^2}$ or Total Energy $\boxed{E = \gamma mc^2 = K + mc^2}$

But Professor… Why Can't ANYTHING go faster than light ?

$$K = \frac{mc^2}{\left[1 - \frac{u^2}{c^2}\right]^{1/2}} - mc^2 \Rightarrow \left(K + mc^2\right)^2 = \left(\frac{mc^2}{\left[1 - \frac{u^2}{c^2}\right]^{1/2}}\right)^2$$

$$\Rightarrow \left[1 - \frac{u^2}{c^2}\right] = m^2c^4\left[K + mc^2\right]^{-2}$$

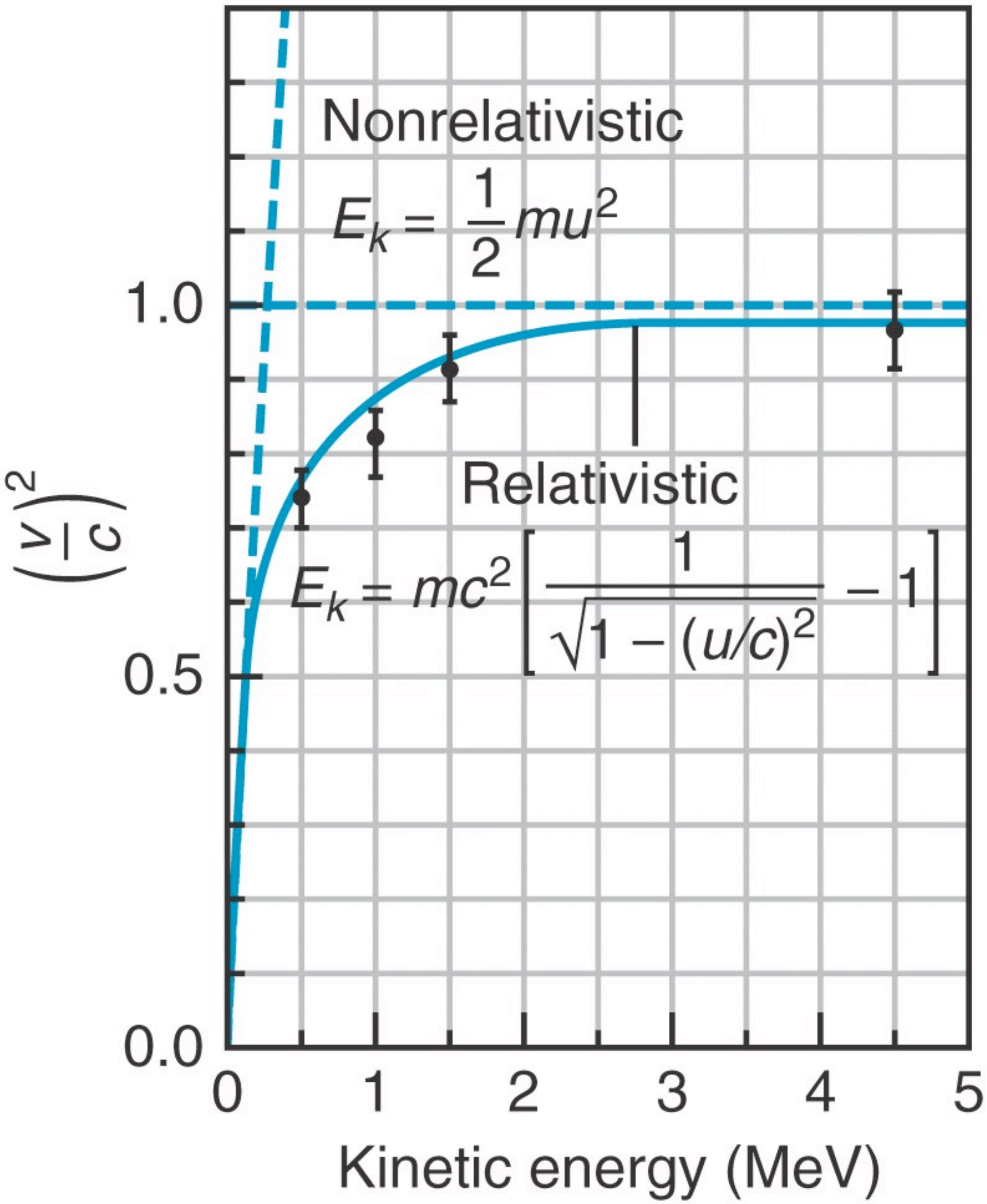
$$\Rightarrow u = c\sqrt{1 - (\frac{K}{mc^2} + 1)^{-2}}$$ (Parabolic in u Vs $\frac{K}{mc^2}$)



$\boxed{\text{As K } \rightarrow \infty \text{ , u } \rightarrow \text{ c}}$

$\boxed{\text{Non-relativistic case: K} = \frac{1}{2}mu^2 \Rightarrow u = \sqrt{\frac{2K}{m}}}$

Relativistic Kinetic Energy Vs Velocity



A Digression: How to Handle Large/Small Numbers

- Example: consider very energetic particle with very large Energy E

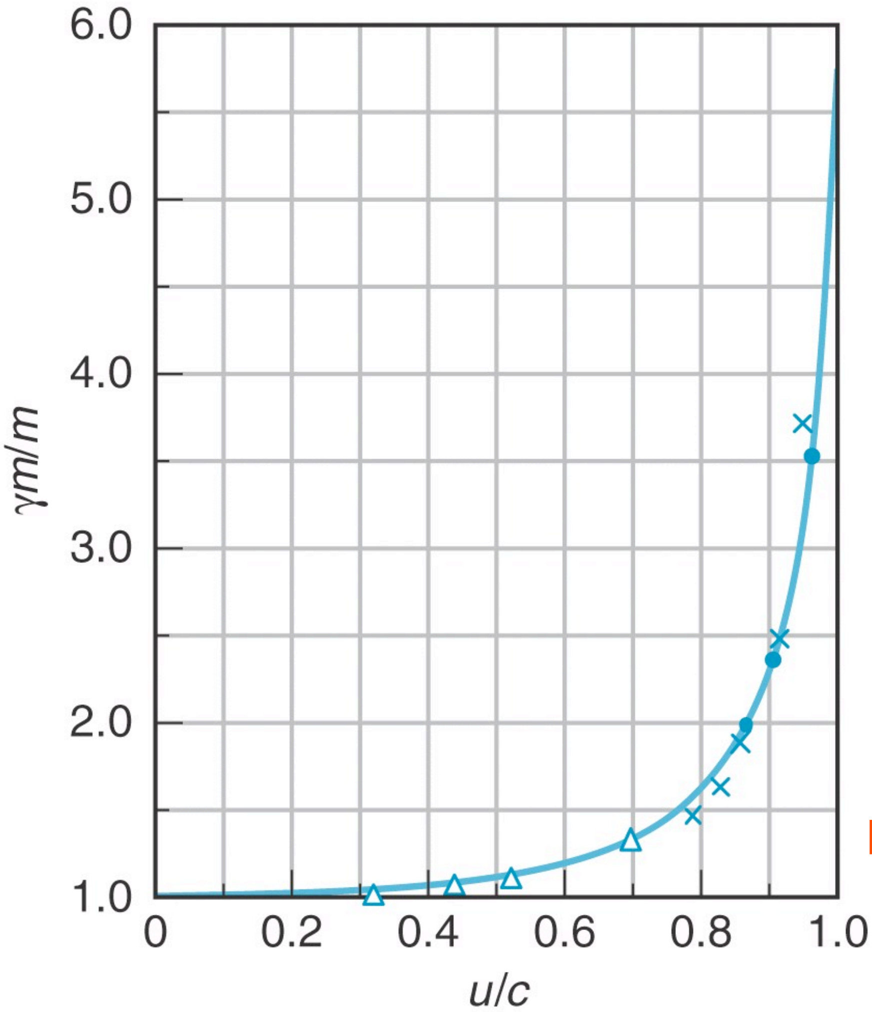
$$\gamma = \frac{E}{mc^2} = \frac{mc^2 + K}{mc^2} = 1 + \frac{K}{mc^2}$$

- Lets Say $\gamma = 3 \times 10^{11}$, Now calculate u from → $\frac{u}{c} = \left[1 - \frac{1}{\gamma^2}\right]^{1/2}$
- Try this on your el-cheapo calculator, you will get u/c =1, u=c due to limited precision.
- In fact u≅c but not exactly!, try to get this analytically

In Quizzes, you are Expected to perform Such simple approximations

When Electron Goes Fast it Gets “Fat”

Total Energy $\boxed{E = \gamma mc^2 = K + mc^2}$



New Concept

Rest Mass = particle mass when its at rest

Relativistic Kinetic Energy & Newtonian Physics

Relativistic KE $K = \gamma mc^2 - mc^2$

$$\left[\begin{array}{l}\text{Remember Binomial Theorem} \\ \text{for } x \ll 1;\ (1+x)^n = (1 + \dfrac{nx}{1!} + \dfrac{n(n-1)}{2!}x^2 + \text{smaller terms})\end{array}\right]$$

$$\therefore \text{ When } u \ll c, \left[1 - \frac{u^2}{c^2}\right]^{-\frac{1}{2}} \cong 1 + \frac{1}{2}\frac{u^2}{c^2} + \text{...smaller terms}$$

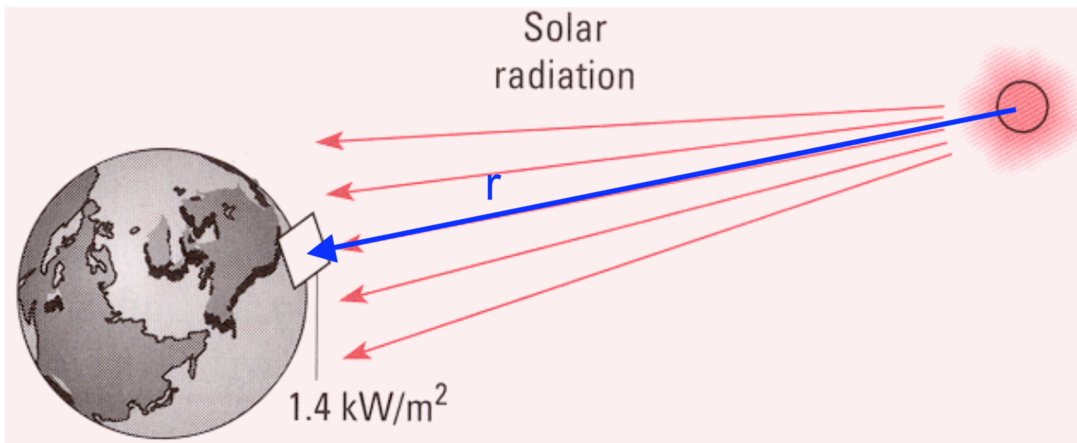
$$\text{so } K \cong mc^2[1 + \frac{1}{2}\frac{u^2}{c^2}] - mc^2 = \frac{1}{2}mu^2 \quad \text{(classical form recovered)}$$

Total Energy of a Particle $\boxed{E = \gamma mc^2 = KE + mc^2}$

For a particle at rest, u = 0 $\Rightarrow$ $\boxed{\text{Total Energy } E = mc^2}$

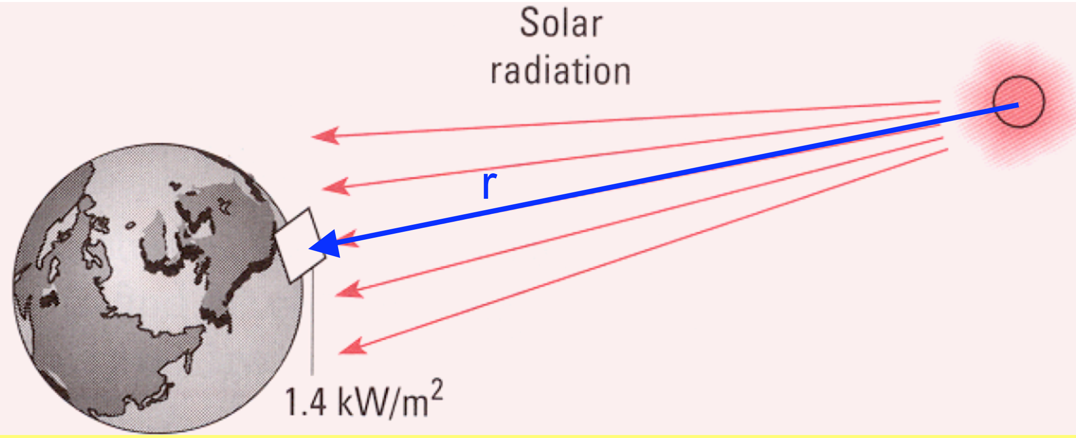
$E=mc^2 \Rightarrow$ Sunshine Won't Be Forever !

Q: Solar Energy reaches earth at rate of 1.4kW per square meter of surface perpendicular to the direction of the sun. by how much does the mass of sun decrease per second owing to energy loss? The mean radius of the Earth's orbit is 1.5×10^{11}m.



- Surface area of a sphere of radius r is $A = 4\pi r^2$
- Total Power radiated by Sun = power received by a sphere whose radius is equal to earth's orbit radius

$E = mc^2 \Rightarrow$ Sunshine Won't Be Forever !



Total Power radiated by Sun = power received by a sphere with radius equal to earth-sun orbit radius(r in figure)

$$P^{sun}_{lost} = \frac{P^{Earth}_{incident}}{A} A_{earth-sun} = \frac{P^{Earth}_{incident}}{A} 4\pi r^2_{earth-sun} = (1.4\times10^3 W/m^2)(4\pi)(1.5\times10^{11})^2$$

$$P^{sun}_{lost} = 4.0\times10^{26} W$$

So Sun loses E = 4.0×10^{26} J of rest energy per second

Its mass decreases by $m = \frac{E}{c^2} = \frac{4.0\times10^{26}\,\text{J}}{(3.0\times10^8)^2} = 4.4\times10^9\, kg$ per sec!!

If the Sun's Mass = $2.0 \times 10^{30} kg$ So how long with the Sun last ?

One day the sun will be gone and the solar system will not be a hospitable place for life

Relationship between P and E

$$\boxed{E = \gamma mc^2} \Rightarrow E^2 = \gamma^2 m^2 c^4$$

$$\boxed{p = \gamma mu} \Rightarrow \boxed{p^2c^2 = \gamma^2 m^2 u^2 c^2}$$

$$\Rightarrow E^2 - p^2c^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^2 (c^2 - u^2)$$

$$= \frac{m^2c^2}{1 - \frac{u^2}{c^2}}(c^2 - u^2) = \frac{m^2c^4}{c^2 - u^2}(c^2 - u^2) = m^2c^4$$

$$\boxed{E^2 = p^2c^2 + (mc^2)^2}$$ ....important relation

For particles with zero rest mass like photon (EM waves)

$\boxed{E = pc \text{ or } p = \frac{E}{c}}$ (light has momentum!)

Relativistic Invariance : $\boxed{E^2 - p^2c^2 = m^2c^4}$: In all Ref Frames

Rest Mass is a "finger print" of the particle

Mass Can "Morph" into Energy & Vice Verca

- Unlike in Newtonian mechanics
- In relativistic physics : Mass and Energy are the same thing
- New word/concept : MassEnergy , just like SpaceTime
- It is the mass-energy that is always conserved in every reaction : Before & After a reaction has happened
- Like squeezing a balloon : Squeeze here, it grows elsewhere
  - **If you "squeeze" mass, it becomes (kinetic) energy & vice verca !**
    - **CONVERSION FACTOR = C^2**
    - **This exchange rate never changes !**