ActivPhysics can help with these problems:

Activities 15.1, 15.2

Section 35-2: Reflection

Problem

1. Through what angle should you rotate a mirror in order that a reflected ray rotate through 30°?

Solution

Since \( \theta_1 = \theta'_1 \) for specular reflection, (Equation 35-1) a reflected ray is deviated by \( \phi = 180° - 2\theta_1 \) from the incident direction. If rotating the mirror changes \( \theta_1 \) by \( \Delta \theta_1 \), then the reflected ray is deviated by \( \Delta \phi = 2\Delta \theta_1 \) or twice this amount. Thus, if \( \Delta \phi = 30° \), \( 2\Delta \theta_1 = 15° \).

Problem

2. The mirrors in Fig. 35-33 make a 60° angle. A light ray enters parallel to the symmetry axis, as shown. (a) How many reflections does it make? (b) Where and in what direction does it exit the mirror system?

Solution

The first reflected ray leaves the upper mirror at a grazing angle of 30°, and therefore strikes the lower mirror normally. It is then reflected twice more in retracing its path in the opposite direction.

Section 35-3: Refraction

Problem

8. In which substance in Table 35-1 does the speed of light have the value \( 2.292 \times 10^8 \) m/s?

Solution

Since the speed of light in a medium is \( v = c/n \), \( n = 3 \times 10^8/2.292 \times 10^8 = 1.309 \). This matches ice in Table 35-1.
Problem
20. The prism in Fig. 35-36 has \( n = 1.52 \), \( \alpha = 60^\circ \), and is surrounded by air. A light beam is incident at \( \theta_1 = 37^\circ \). Find the angle \( \delta \) through which the beam is deflected.

Solution
A more general case of refraction through the same type of prism is treated in Problem 55. For the data in this problem, the other angles and the deflection are:

- \( \phi_2 = \sin^{-1}(\sin 37^\circ / 1.52) = 23.3^\circ \)
- \( \phi_1 = \sin^{-1}(1.52 \sin 36.7^\circ) = 65.2^\circ \), and \( \delta = \phi_1 - 23^\circ = 42.2^\circ \).

Section 35-4: Total Internal Reflection

Problem
21. Find the critical angle for total internal reflection in (a) ice, (b) polystyrene, and (c) rutile. Assume the surrounding medium is air.

Solution
For \( n_{air} \approx 1 \), the critical angle for total internal reflection in a medium of refractive index \( n \) is \( \theta_c = \sin^{-1}(1/n) \). (Air is medium-2 in Equation 35-5.) From Table 35-1, \( n = 1.309 \) (ice), 1.49 (polystyrene), and 2.62 (rutile), so \( \theta_c = \sin^{-1}(1/1.309) = 49.8^\circ \), 42.2° and 22.4°, respectively, for these media.

Problem
27. What is the minimum refractive index for which total internal reflection will occur as shown in Fig. 35-15a? Assume the surrounding medium is air and that the prism is an isosceles right triangle.

Solution
Figure 35-15a shows a 45° right-triangle prism with critical angle less than 45°. Thus, \( \theta_c = \sin^{-1}(1/n) < 45^\circ \), or \( n > 1/\sin 45^\circ = \sqrt{2} \). (We used \( n_2 = 1 \) for air, and \( n_1 = n \) for the prism, in Equation 35-5.)

Problem
33. A scuba diver sets off a camera flash a distance \( h \) below the surface of water with refractive index \( n \). Show that light emerges from the water surface through a circle of diameter \( 2h/\sqrt{n^2 - 1} \).

Solution
Light from the flash will strike the water surface at the critical angle for a distance \( r = h \tan \theta_c \) from a point directly over the flash. Therefore, the diameter of the circle through which light emerges is \( 2r = 2h \tan \theta_c \). But \( \sin \theta_c = 1/n \) (Equation 33-5 at the water-air interface), and \( \tan^2 \theta_c = (\csc^2 \theta_c - 1)^{-1} \) (a trigonometric identity), so we can write \( 2r = 2h/\sqrt{n^2 - 1} \).

Problem
36. White light propagating in air is incident at 45° on the equilateral prism of Fig. 35-38. Find the angular dispersion \( \gamma \) of the outgoing beam, if the prism has refractive indices \( n_{red} = 1.582 \), \( n_{violet} = 1.633 \).

Solution
This geometry for refraction through a prism is treated in Problem 55. The difference in the directions of the exiting red and violet light is the dispersion. For the given values of \( \alpha = 60^\circ \), \( \theta_1 = 45^\circ \), \( n_r = 1.582 \), and \( n_v = 1.633 \), we find: \( \theta_2 = 26.5^\circ \), \( \phi_2 = 33.5^\circ \), \( \phi_1 = 60.7^\circ \), and \( \delta = 45.7^\circ \), while \( \theta_2 = 26.6^\circ \), \( \phi_2 = 34.3^\circ \), \( \phi_1 = 67.1^\circ \), and \( \delta = 52.1^\circ \). The angular dispersion is \( \gamma = \delta_v - \delta_r \) (or \( \phi_1 \cdot \phi_1 - \phi_2 \cdot \phi_2 \)) = 6.41°. (Note: We did not need to calculate the deflection, but did so because this is the customary parameter in prism optics.)
Problem
37. Two of the prominent spectral lines—discrete wavelengths of light—emitted by glowing hydrogen are hydrogen-α at 656.3 nm and hydrogen-β at 486.1 nm. Light from glowing hydrogen passes through a prism like that of Fig. 35-22, then falls on a screen 1.0 m from the prism. How far apart will these two spectral lines be? Use Fig. 35-20 for the refractive index.

Solution
The angular dispersion of $H_\alpha$ and $H_\beta$ light in the prism of Fig. 35-22 can be found from the analysis in Example 35-6. For normal incidence on the prism, rays emerge with refraction angles of $\sin^{-1}(n \sin 40^\circ)$. From Fig. 35-20, we estimate that $n_\alpha = 1.517$ and $n_\beta = 1.528$, so the angular dispersion is $\gamma = 79.2^\circ - 77.2^\circ = 1.98^\circ$. We can assume that the size of the prism is small compared to the distance, $r$, to the screen, so the separation on the screen corresponding to $\gamma$ is $\Delta x = \gamma \cdot r = (1.98^\circ)(\pi/180^\circ)(1 \text{ m}) = 3.45 \text{ cm}$.

Section 35-6: Reflection and Polarization

Problem
39. Find the polarizing angle for diamond when light is incident from air.

Solution
Equation 35-6 gives the polarizing angle, for light in air reflected from diamond; $\theta_p = \tan^{-1}(2.419/1) = 67.5^\circ$. 