Waves
1.3

- Superposition principle
- Interference
- Beats
- Fourier analysis

Superposition principle

The displacement due to two waves that pass through the same point in space is the algebraic sum of displacements of the two waves.

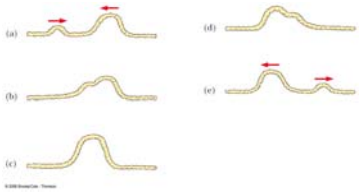
The combination of waves is called interference.

Constructive interference – the waves reinforce each other

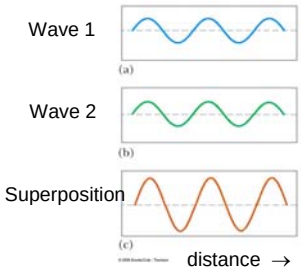
Destructive interference – the waves cancel each other.

Superposition Principle

- When two waves overlap in space the displacement of the wave is the sum of the individual displacements.

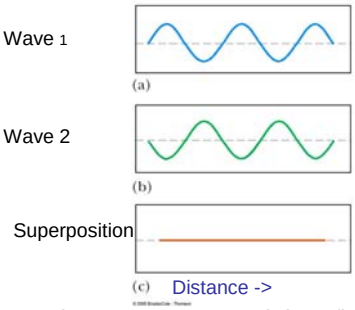


Constructive Interference



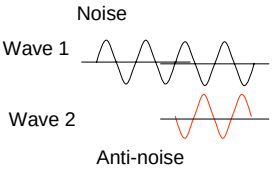
The two waves have the same phase

Destructive Interference



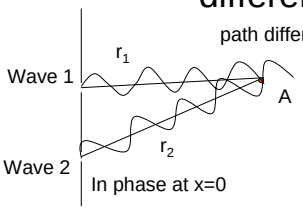
The two waves are out of phase (by 180^o , or π)

Noise canceling headphones



Interference due to path difference

path difference $=\delta = r_2 - r_1$



In phase at x=0

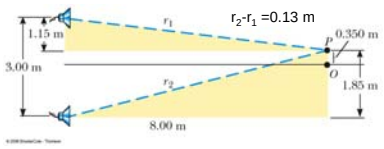
Superposition of waves at A shows interference due to path differences

Condition for constructive interference $\delta = m\lambda$

Condition for destructive interference $\delta = (m + \frac{1}{2})\lambda$

where m is any integer m = 0 ± 1, ± 2,….

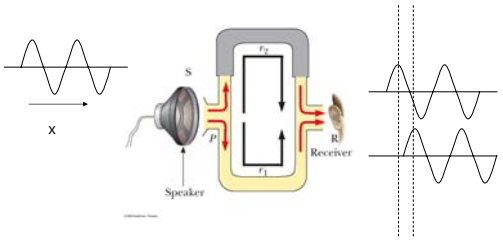
Interference of sound waves.



As the listener moves from position O to position P he hears the first minimum in sound intensity. Find the frequency of the oscillation. v_{sound} =340 m/s

Interference of sound waves

Phase shift due to path differences



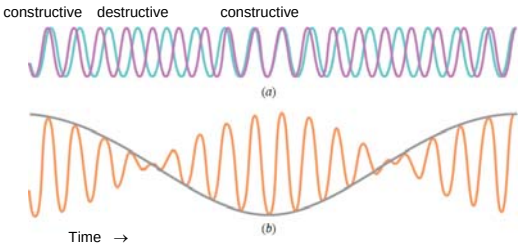
When $r_2 - r_1 = m\lambda$ Constructive Interference

When $r_2 - r_1 = (m+½)\lambda$ Destructive Interference

m is any integer

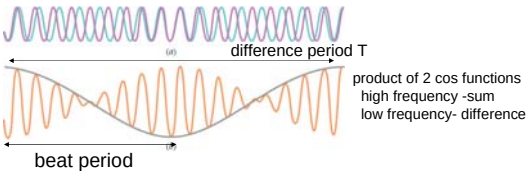
Beats

The two waves moving in the same direction have different frequencies and wavelengths



The superposition shows maxima and minima in displacement amplitude.

Beat Frequency



product of 2 cos functions
high frequency -sum
low frequency- difference

$$y(t) = A\cos\omega_1 t + A\cos\omega_2 t$$

$$\cos\omega_1 t + \cos\omega_2 t = 2\cos\frac{(\omega_1 - \omega_2)t}{2}\cos\frac{(\omega_1 + \omega_2)t}{2}$$

beat frequency = $f_1 - f_2$

tuning musical instruments

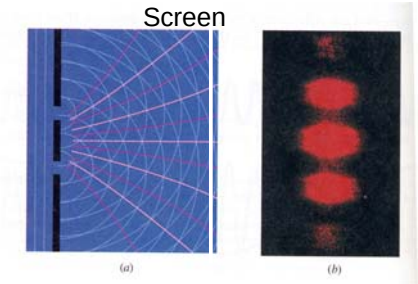
- two instruments play the same note.
- the beat frequency tell how different the frequencies are.
- one player adjusts his instrument to minimize the beat frequency.
- when the beat frequency => zero the instruments are in tune.

Interference in 2 dimensions

White-maxima
Black-minima
Blue- zero

Water Waves show constructive and destructive interference

Interference in light waves



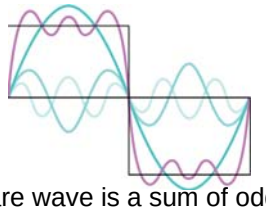
wave fronts in 2 dimensions (white lines=maxima)

Intensity profile for light waves (red=high intensity)

Fourier Analysis

Any periodic wave can be built up of simple harmonic waves

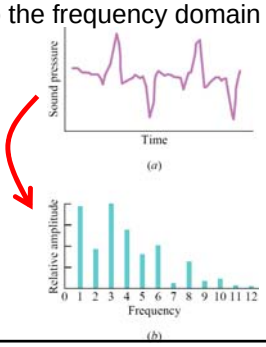
Example- Square wave



Square wave is a sum of odd harmonics of sine waves $f_1 = f_0$, $f_3 = 3f_0$, $f_5 = 5\,f_0$ .....

Fourier Analysis

Transforms the signal from the time domain to the frequency domain



This is done mathematically using a Fourier Transform

Superposition is responsible for many properties of waves.

Color is due to the superposition of light of different wavelengths.

Sound texture is due to the superposition of sounds with different frequencies.