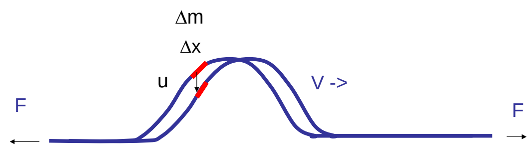
Waves
1.2

- Transverse waves on a string
  - speed
  - power
- Intensity
- Wave equation

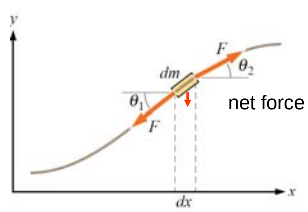
Speed of the transverse wave on a string.



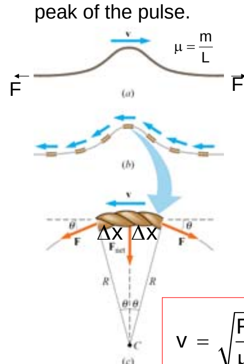
$$\mu = \frac{\Delta m}{\Delta x}$$ mass density

$$v = \sqrt{\frac{F}{\mu}}$$ speed of transverse wave on a string depends on the tension on the string and the mass density

Force in the y direction depends on θ



Derive the wave speed from the mass element at the peak of the pulse.



a) Pulse moving along the string

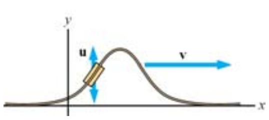
b) In coordinate system moving with the pulse. The string is moving.

c) Apply Newton's law to the mass at the top of the pulse. For small angles θ

$$F_{net} = 2F\sin\theta \approx 2F\theta = 2F\frac{\Delta x}{R}$$

$$ma = 2\mu\Delta x\frac{v^2}{R}$$

$$v = \sqrt{\frac{F}{\mu}}$$



dimensional check

$$v = \sqrt{\frac{F}{\mu}} \qquad \sqrt{\frac{\text{N}}{\text{kg/m}}} = \sqrt{\frac{\text{kg}\cdot\text{m/s}^2}{\text{kg/m}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}}$$

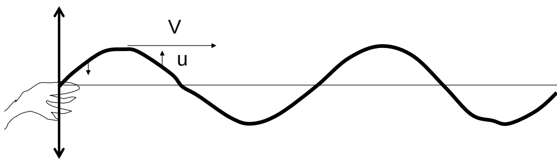
Increasing tension increases speed

Increasing mass/length decreases speed.

A 3.1 kg mass hangs from a 2.7 m long string whose total mass is 0.62 g. What is the speed of the transverse wave on the string?

How much mass would have to be added to increase the speed by a factor of 2?

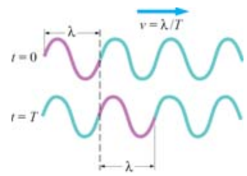
Power carried by harmonic wave



Displacement is in the vertical , Y direction, Displacement speed u in the Y direction

Wave carries Kinetic and Potential energy due to the displacement.

Average power P_{av}



average power is the energy in one wavelength / period

$$P_{av} = \frac{E(\lambda)}{T}$$

For a harmonic oscillator the average KE is equal to the average PE so the total energy is 2xKE =mu^2

$$P_{av} = \frac{2\overline{KE}}{T} = \frac{2(\frac{1}{2}m\overline{u^2})}{T} = \frac{\mu\lambda\overline{u^2}}{T} = \mu v\overline{u^2}$$

Instantaneous power varies with time.

$$y = A\cos(kx - \omega t)$$

$$u = \frac{\partial y}{\partial t} = \omega A\sin(kx - \omega t)$$

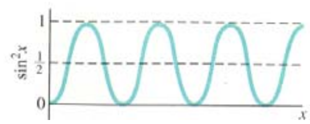
$$u^2 = \omega^2 A^2 \sin^2(kx - \omega t)$$

$$P = \mu v u^2$$

$$P = \mu v \omega^2 A^2 \sin^2(kx - \omega t)$$

Average Power

the average value of $\sin^2(kx-\omega t)$ over one cycle is 1/2



$$P_{av} = \frac{1}{2}\mu\omega^2 A^2 v$$

$$P_{av} = \frac{1}{2}Fk\omega A^2$$

since

$$\omega = kv$$

$$v^2 = \frac{F}{\mu}$$

The power depends on A squared for many systems

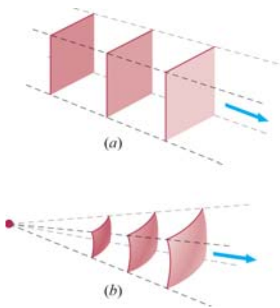
38. A steel wire with linear density of 5.0 g/m is under 450 N tension. What is the maximum power that can be carried by transverse waves if the wave amplitude is not to exceed 10% of the wavelength?

Intensity

- Intensity is defined as the power per area

$$I = \frac{P}{Area} \qquad (Watt/m^2)$$

For different geometries the intensity can vary with distance from the source.



Wave propagates in one direction- Plane wave
Intensity is constant with distance

Wave propagates radially
Spherical wave
Intensity varies as $1/r^2$

$$I = \frac{P}{4\pi r^2}$$

Question

You are reading a book using light from a single light bulb. At a distance of 2 m from the bulb the intensity is too low. To increase the intensity by a factor of 2 you need to go to a distance of_____ m from the bulb.

a) 1.0 m
b) 0.5 m
c) 1.4 m
d) 2.0 m

The wave equation

- Goal:
  - We will evaluate Newton's law F = m a for the system of a wave on a string.
  - We will specifically look at dF, the change in F as we go from x to x+dx
- Result:
  - We will find a universal equation that applies to all wave phenomena. In fact, it defines wave phenomena. All physical systems that follow this equation will display waves.

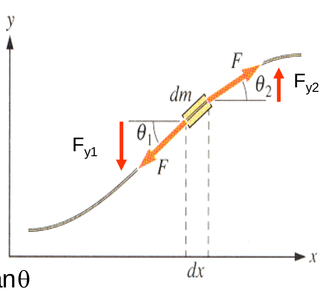
F = m a

- F = restoration force in the string
- a = acceleration in y direction
- F = m a is Newton's law applied to the "oscillators" in the string that "swing" vertically on the string as the wave passes by!

Consider the net force acting on the segment of string

$$dF_y = F_{y2} - F_{y1}$$
$$dF_y = F(\tan\theta_2 - \tan\theta_1)$$

Net force in Y direction



small angle approximation $\sin\theta = \tan\theta$

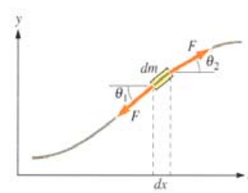
Tanθ is a derivative

$$\tan\theta = \frac{\partial y}{\partial x}$$

$$\tan\theta_2 - \tan\theta_1 = \left.\frac{\partial y}{\partial x}\right|_{x+dx} - \left.\frac{\partial y}{\partial x}\right|_{x}$$

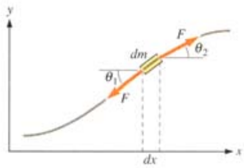
$$= \frac{\partial^2 y}{\partial x^2} dx$$

$$\Rightarrow dF = F\frac{\partial^2 y}{\partial x^2} dx$$



Newtons Law

dF=dm a



$$F \frac{\partial^2 y}{\partial x^2} dx = \mu \frac{\partial^2 y}{\partial t^2} dx$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F} \frac{\partial^2 y}{\partial t^2}$$

Since $v = \sqrt{\frac{F}{\mu}}$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Wave Equation

Significance of wave equation

- All physical phenomena that lead to a relationship described by the wave equation will exhibit waves !!!
- Examples:
  - Sound
  - Electromagnetism
  - General Relativity (i.e. Gravity)

y(x,t) = A cos(kx-ωt)

- We can show explicitly that this general form obeys the wave equation.
- As a homework excercise show that this function satisfies the wave equation.