7.2 Fluids

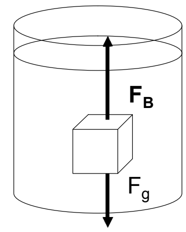
Buoyancy

Fluid dynamics I. Principles

Archimedes Principle

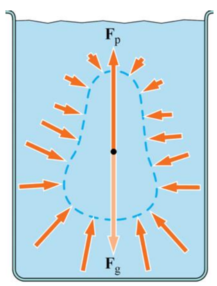
When an object is placed in a fluid the fluid exerts a buoyant force on the object that tends to oppose the gravitational force.

The buoyant force is equal to the weight of the fluid displaced



Pressure force on fluid at equilibrium

The force of pressure on the surface of the fluid section (in dotted lines) F_p is equal to the weight of the fluid F_g



$F_P = F_g$ =Weight of fluid

net force is zero

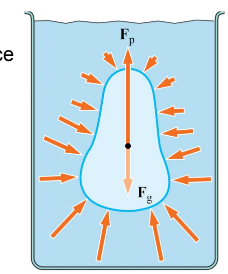
The buoyant force is F_p
the weight of the displaced fluid

Buoyant force on an object in a fluid

Now remove the fluid and place an object in the space of the displaced fluid.

The net force on the object is the difference between F_p and the weight F_g

Here the object floats because Fp is greater than the weight



(*b*)

Question

An ice cube floats in a glass of water. What fraction of the ice cube is under the surface of the water? (ρ_{ice}=0.92 kg/m^3, ρ_{water}=1.00 kg/m^3

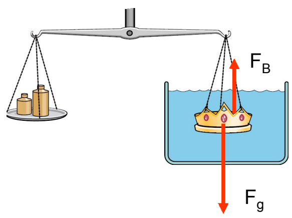
Question

A glass of water containing an ice cube is full to the brim. What will happen to the water level when the ice melts?

a) rise
b) lower
c) remain the same

Buoyant force

Archimedes was asked to determine if the King's crown was made of pure gold.



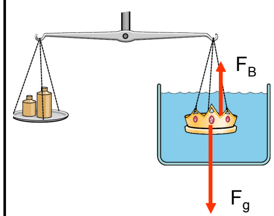
Archimedes of Syracuse
287 BC-212 BC

"Eureka, I have found it"

The density of the crown can be determined by weighing the crown in air and in water.

Buoyant force

Archimedes was asked to determine if the King's crown was made of pure gold. He used a test of weighing the crown in air and submerged in water. The weight in air was 25.0 N. What would the weight in water be if the crown was pure gold (ρ_{gold} =19.3g/cm^3, ρ_{water}=1.00g/cm^3)? What would it be if it were made of a silver alloy (ρ_{alloy} =17.1 g/cm^3)? What is the best way to perform the measurement accurately?



Question

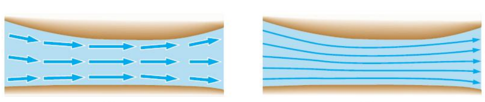
A spherical rubber balloon with mass 1.6 g and diameter of 20 cm is filled with helium (density 0.18 kg/m^3). How many 0.4 g paper clips can you hang from the balloon before it loses its buoyancy? (density of air is 1.2 kg/m^3)

Fluid Dynamics

- Fluid dynamics is the study of fluids in motion.
- The fluid is characterized by the velocity, and pressure in different parts of the fluid.
- Fluid dynamics has important applications such as aerodynamics of flight.

Fluid Dynamics

Fluid flow is described by velocity vectors or continuous streamlines



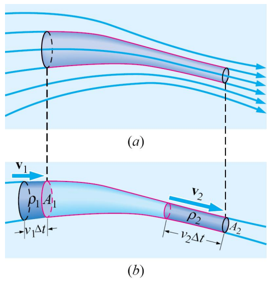
(a) Velocity vectors (b) Streamlines

Steady flow

- For steady flow the streamlines don't change with time
- For unsteady flow the streamlines change with time.

Continuity Equation
Conservation of mass in a flow tube

In a time Δt the same amount of mass must pass into the tube (at A_1) as leaves the tube (at A_2)



$$m = \rho_1 A_1 v_1 \Delta t$$

$$m = \rho_2 A_2 v_2 \Delta t$$

For a constant density, i.e. liquid

$$A_1 v_1 = A_2 v_2$$

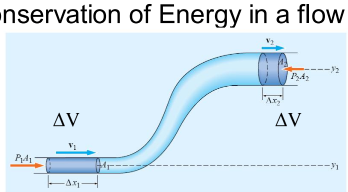
For a smaller area the flow velocity must be larger

Question

A 1.0 inch hose is constricted by a nozzle to a diameter 4 times smaller. By how much is the velocity of the water increased at the nozzle.

a) 1.5 times
b) 2 times
c) 4 times
d) 16 times.

Bernoulli's Equation
Conservation of Energy in a flow tube



The change in Energy of fluid volume ΔV in going from 1 to 2 is equal to the Work done on the mass

$$\Delta E = \Delta KE + \Delta PE = \frac{1}{2}m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

$$W = F_1 \Delta x_1 - F_2 \Delta x_2 = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = P_1 \Delta V - P_2 \Delta V$$

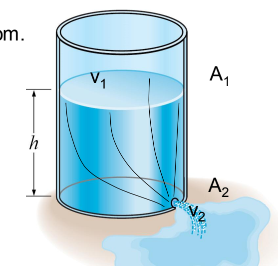
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \quad \Rightarrow \quad P + \frac{1}{2}\rho v^2 + \rho g y = \text{Constant}$$

Bernoulli's Equation

A tank of water with height h has a small hole at the bottom. Find the velocity of the water flowing from the hole.

Assume streamline flow

use the Bernoulli equation at two regions of the fluid



$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$A_1 >> A_2 \Rightarrow v_1 = 0$

set $y_2 = 0$, $y_1 = h$

$P_1 = P_2 = 1$ atm

$$\rho g h = \frac{1}{2}\rho v_2^2 \qquad v_2 = \sqrt{2gh}$$