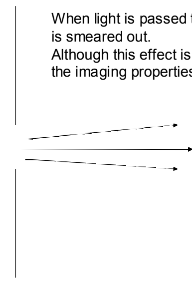
6.2 Interference and diffraction

Huygen's Principle
Single slit diffraction
Circular diffraction
The diffraction limit

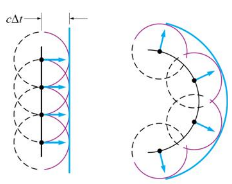
Diffraction effects of apertures

When light is passed though an aperture the light beam is smeared out.
Although this effect is small for large apertures. It limits the imaging properties of the light waves.



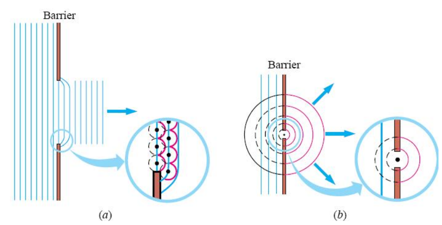
Hyugen's Principle

All points on a wavefront act as point sources of spherically propagating "wavelets" that travel at the speed of light appropriate to the medium. At a short time Δt later, the new wavefront is the unique surface tangent to all the forward-propagating wavelets.



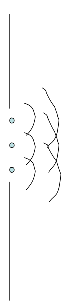
plane wave   spherical wave

Huygen's principle explains diffraction around a barrier.



large hole   small hole

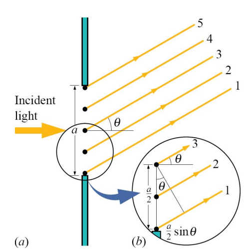
Single slit diffraction



From Hyugen's principle single slit diffraction results from wavelets from a large number of sources
The interference from these waves determine the intensity profile of the wave

Single slit diffraction

Waves from 5 sources propagating at angle θ
Find the condition for destructive interference.



Divide the slit into 2 halves
For each source in the bottom half there is a source in the top half a distance of a/2 away

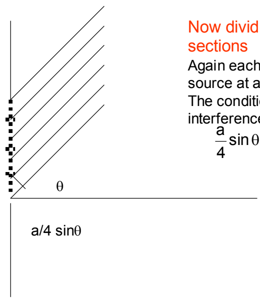
the two sources interfere destructively under the condition

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

The interference between the top half cancels the amplitude from the bottom half.

No light

Single slit diffraction



Now divide the slit up into 4 sections

Again each source will have another source at a distance of a/4 away.

The condition for destructive interference

$\frac{a}{4}\sin\theta = \frac{\lambda}{2}$ → $a\sin\theta = 2\lambda$

Single slit diffraction

The general condition for destructive interference for single slit diffraction

$$a\sin\theta = m\lambda \qquad \text{Minima}$$

m= ±1, ±2, ...

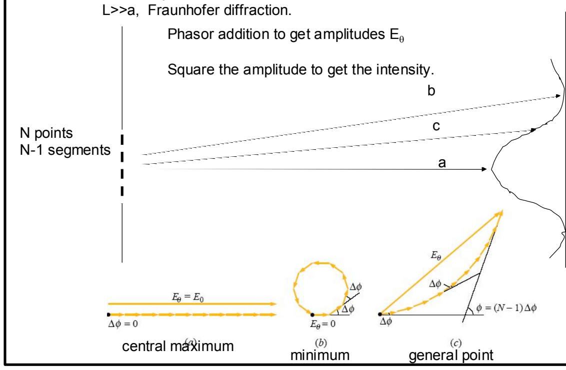
This relation is similar to the relation for two-slit interference but be careful to note the differences.

Intensities for single slit diffraction

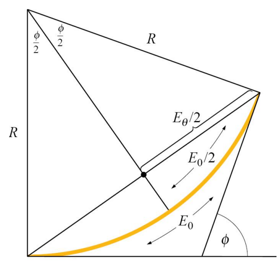
This analysis is for the case where the screen is far from the slit L>>a, Fraunhofer diffraction.

Phasor addition to get amplitudes E_θ

Square the amplitude to get the intensity.



Intensity



Electric field amplitude

$$\frac{E_\theta}{E_o} = \frac{\sin(\phi/2)}{\phi/2}$$

Intensity ∝ Amplitude²

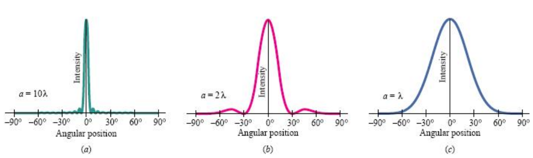
$$\overline{S}_\theta = \overline{S}_o\left[\frac{\sin(\phi/2)}{\phi/2}\right]^2$$

where

$$\phi = 2\pi\frac{a\sin\theta}{\lambda}$$

Single slit diffraction

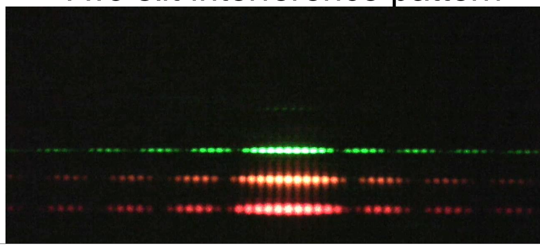
The width of the diffraction increases as the slit becomes smaller



Example

For single slit diffraction (a>>λ) find the amplitude of the first secondary maximum in terms of the intensity of the central maximum S_o. Assume that the peak lies between the first two minima.

Two slit interference pattern

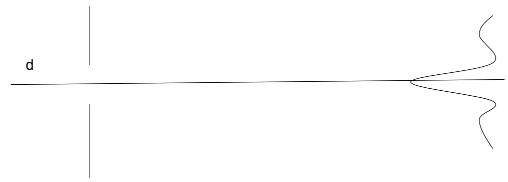


What is the ratio of d/a?

Diffraction Limit

Light is smeared out when passed through an aperture

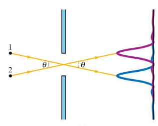
This smearing out limits the resolution for optical systems.



Diffraction Limit- single slit

Two objects well resolved

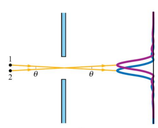
The separation between peaks is greater than the width of the peak



Two objects just resolved

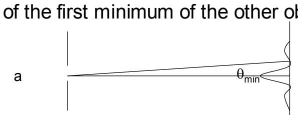
The separation between peaks is just equal to the width of the peak

Rayleigh criteria



Rayleigh Criteria- Single slit diffraction

The Rayleigh criterion is that the peak of one object is at the position of the first minimum of the other object.



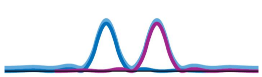
Position of the first minimum

$$\sin\theta_{min} = \frac{\lambda}{a}$$

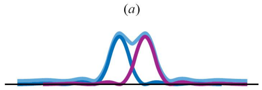
For small angles

$$\theta_{min} = \frac{\lambda}{a}$$

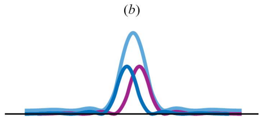
Resolution


(a)

Well resolved


(b)

Just resolved
Rayleigh criteria


(c)

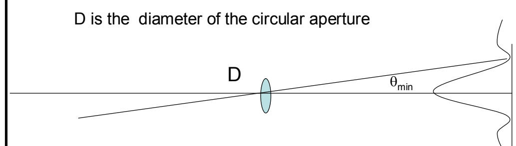
Not resolved
You cannot tell if there are two peaks present

Rayleigh Criterion- circular diffraction

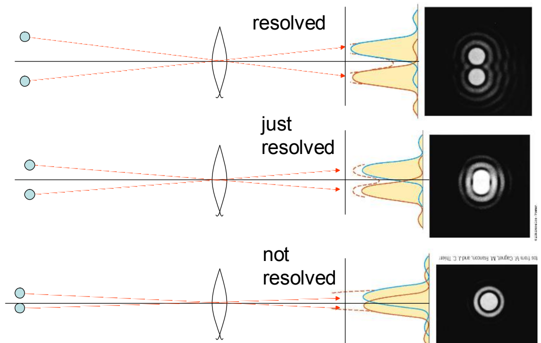
Most optical systems use a circular aperture e.g. camera lenses, telescopes, eye. The Rayleigh criterion for a circular aperture is

$$\theta_{min} = \frac{1.22\lambda}{D}$$

D is the diameter of the circular aperture



Resolution of two images by a lens



The resolution of images is limited by the diffraction pattern.

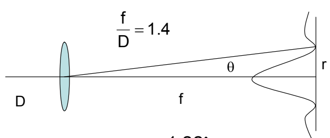
Diffraction Limit

- The diffraction limit determines the minimum spot size to which parallel light can be focused.
- The diffraction limit limits the smallest objects that can be resolved by an optical system.

Diffraction Limited spot size

A camera has an f/1.4 lens, meaning the ratio of focal length to lens diameter is 1.4. Find the radius to which the lens can focus parallel light with wavelength of 580 nm.

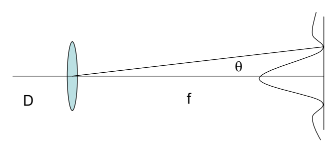
$$\frac{f}{D} = 1.4$$



$$\theta_{min} = \frac{1.22\lambda}{D} \approx \frac{r}{f}$$

$$r = \frac{(1.22)\lambda f}{D} = \frac{(1.22)(580)(1.4)}{1} = 1000\text{nm} = 1\mu\text{m}$$

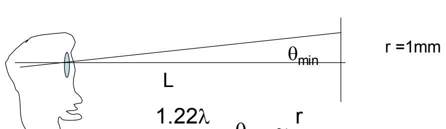
Microscope



If we use the same lens as in a microscope then the Rayleigh criterion limits the size of the smallest object that can be imaged by the microscope.

Diffraction limit of vision.

Light enters the human eye through an iris with an aperture with a diameter of about 4mm. In the diffraction limit how far away can you see two objects separated by 1mm (use λ =500nm)



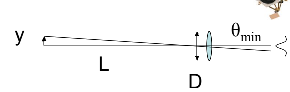
$$\frac{1.22\lambda}{D} = \theta_{min} \approx \frac{r}{L}$$

$$L = \frac{rD}{1.22\lambda} = \frac{10^{-3}(4\text{x}10^{-3})}{1.22(500\text{x}10^{-9})} = 6.5\text{m}$$

GeoEye 1 Imaging Satellite

1.1 m diameter mirror
Resolution 0.41 m
Altitude of 684 km

Diffraction limit (λ=550nm)

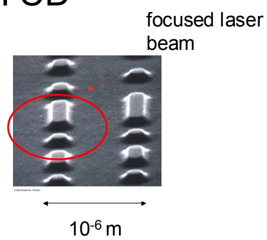


$$\theta_{min} = \frac{y}{L} = 1.22\frac{\lambda}{D}$$

$$y = \frac{1.22\lambda L}{D} = \frac{1.22(550\text{x}10^{-9}\text{m})(684\text{x}10^{3}\text{m})}{1.1\text{m}} = 0.41\text{m}$$

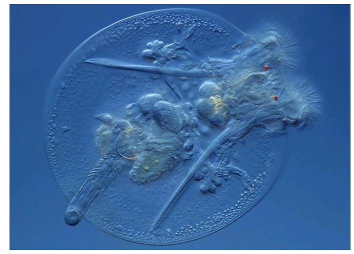
Satellite picture of Sea World

Optical CD



The amount of information that can be encoded is limited by the diameter of the diffraction-limited spot.

Diffraction limits the resolution of light microscopes



To get to smaller spot size use shorter wavelengths of light

- A DVD hold more information than a CD. DVDs need a shorter wavelength laser to focus on the smaller bumps.
- Light microscopes can't resolve molecular dimensions.
  - X-rays diffraction with shorter wavelengths can. (but there are no x-ray microscopes)
  - Electron microscopes can (electrons are waves too).