Electromagnetic Waves
3.1

Maxwell's equations

Electromagnetic waves

Speed of light

Properties of EM waves

James Clerk Maxwell

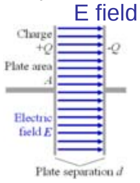
Electromagnetic waves

- EM waves carry energy in oscillating electric and magnetic fields that propagate through space with the speed of light.
- EM waves are governed by the laws of Electricity and Magnetism applied to a vacuum.
- Maxwell showed that these laws (called Maxwell's equations) give rise to EM waves and correctly predict the speed of light.

Electric and Magnetic Fields

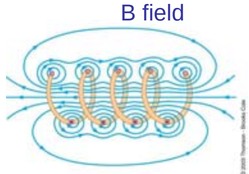
Capacitor

E field



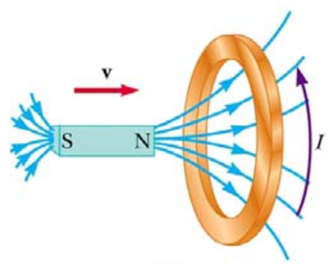
Inductor

B field



- Electric and magnetic fields exist in a vacuum.
- Electric and magnetic fields store energy

A time varying Magnetic field creates an Electric field

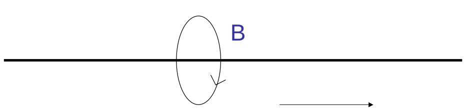


Induced Electric field

Faraday's Law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

Induced Magnetic field

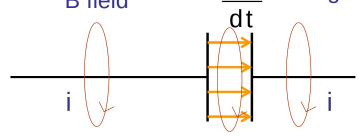


Ampere's Law
A Magnetic field is produced by a current

Maxwell: A magnetic field should be produced by a Changing electric field.

Problem with Ampere's law

B field $\frac{dE}{dt}$ changing E field



What about the B field around the capacitor?

Maxwell: a changing E field is equivalent to a current and also produces a B field in a surrounding loop

$$\oint \vec{B}\cdot d\vec{l} = \varepsilon_o \mu_o \frac{d\phi_E}{dt}$$ (in a vacuum)

Maxwell's Equations in a vacuum

Gauss' Law for E fields $\oint \vec{E}\cdot d\vec{a} = \frac{q}{\varepsilon_o} = 0$

Gauss' Law for B fields $\oint \vec{B}\cdot d\vec{a} = 0$

no electric or magnetic charges in vacuum. => E field and B field lines not terminated

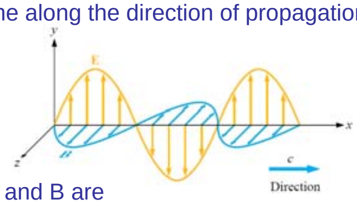
Faraday's Law $\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$

Ampere's Law $\oint \vec{B}\cdot d\vec{l} = \varepsilon_o\mu_o \frac{d\phi_E}{dt}$

time varying B and E fields through loops produce E and B fields around the loop. => E and B $\perp$

Electromagnetic Plane waves

Electric and magnetic field vectors in a line along the direction of propagation



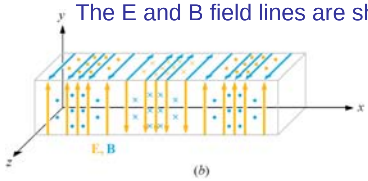
$\vec{E}\times\vec{B}$

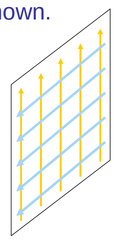
E and B are

- In phase
- Field directions are perpendicular to each other and to the direction of propagation

The E and B fields extend uniformly through space in the plane perpendicular to the direction of propagation

The E and B field lines are shown.





In a plane perpendicular to the direction of propagation the E and B fields are constant.

EM plane waves obey Maxwell's Equations

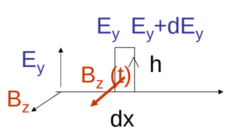
- EM plane waves follow Maxwell's equations
- Applying Faraday's Law and Ampere's Law to find the speed of light.

Faraday's Law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$$\oint \vec{E}_y \cdot d\vec{l} = hdE_y$$

$$\frac{d\phi_B}{dt} = hdx\frac{dB_z}{dt}$$



$$hdE_y = -hdx\frac{dB_z}{dt} \Rightarrow \frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}$$

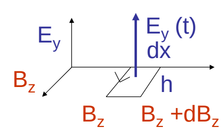
This is the expression of Faraday's Law for the plane wave geometry.

Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \varepsilon_o\mu_o\frac{d\phi_E}{dt}$$

$$\oint \vec{B}_z \cdot d\vec{l} = -hdB_z$$

$$\frac{d\phi_E}{dt} = hdx\frac{dE_y}{dt}$$



$$-hdB_z = \varepsilon_o\mu_o hdx\frac{dE_y}{dt} \Rightarrow \frac{\partial B_z(x,t)}{\partial x} = -\varepsilon_o\mu_o\frac{\partial E_y(x,t)}{\partial t}$$

This is an expression of Ampere's Law for the plane wave geometry.

Plane waves are solutions to Maxwell's Eq. Wave velocity is the speed of light

$$E_y = E_o \sin(kx - \omega t)$$
$$B_z = B_o \sin(kx - \omega t)$$

wave speed $c = \frac{\omega}{k}$

Faraday's Law

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}$$

$$kE_o \cos(kx - \omega t) = \omega B_o \cos(kx - \omega t) \quad \Rightarrow \quad \frac{E_o}{B_o} = c$$

Ampere's Law

$$\frac{\partial B_z(x,t)}{\partial x} = -\varepsilon_o \mu_o \frac{\partial E_y(x,t)}{\partial t}$$

$$kB_o \cos(kx - \omega t) = \varepsilon_o \mu_o \omega E_o \cos(kx - \omega t)$$

$$\Rightarrow \quad c = \sqrt{\frac{1}{\varepsilon_o \mu_o}}$$

The speed of light is determined by fundamental EM constants

Speed of Light

Measured values

permeability of free space $\mu_o = 4\pi x10^{-7}$ T·m/A

permittivity of free space $\varepsilon_o = 8.85419x10^{-12}$ $C^2/N \cdot m^2$

speed of light c= $2.99792458x10^8$ m/s

Calculated value

$$c = \sqrt{\frac{1}{\varepsilon_o \mu_o}} = 2.997924x10^8 \, m/s$$

Perfect agreement !!!!

The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.

James Clerk Maxwell, 1864

Properties of EM plane waves

- Have E field and B field components that are perpendicular to each other and the direction of propagation.
- The maximum E and B fields are related

$$E_o = cB_o$$

- The speed of light is

$$c = \sqrt{\frac{1}{\varepsilon_o \mu_o}} = 3.00x10^8 \, m/s$$

Question

You are calling your friend in London from San Diego, a distance of 9,000 km. If your voice travels at the speed of light. What is the time delay due to transmission.

a) 3 s
b) 0.3 s
c) 0.03 s
d) 0.003 s

Question

A fm radio station has a frequency of 100 MHz. What is the wavelength of the EM wave.

1) 3.0 mm
2) 3.0 cm
3) 3.0 m
4) 3.0 km

Example Radio waves

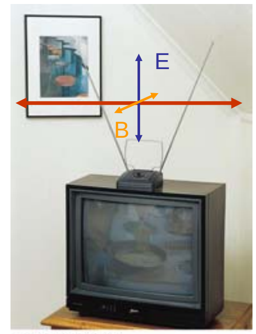
- Fm radio ~ 100 MHz - λ~ 3m
- Am radio 500 kHz - λ~ 600m

EM radiation can be produced by electric discharge. Hertz experiment

Radio frequency radiation is shielded by Faraday cage (Why not light?)

Radio frequency radiation signals can be detected by coupling E or B field of the radiation with a directional antenna.

What is the direction of propagation?



Speed of light in glass

When light travels through a transparent material such as glass the speed is reduced due to the dielectric constant κ of the material

$$\text{Speed of light in glass} = \frac{1}{\sqrt{\kappa_g \varepsilon_o \mu_o}}$$

If the speed of light in a glass is 2.00×10^8 m/s what is the dielectric constant of the glass at optical frequency?