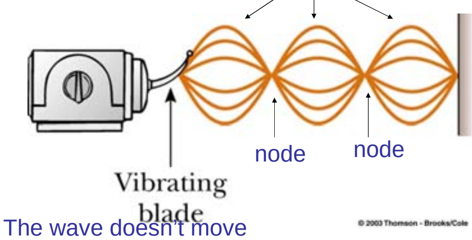
Sound
2.2

Standing Waves
- wave on a string
- wave in an air column

Resonance
Musical Instruments

Standing waves

Standing waves on a string with two ends fixed.

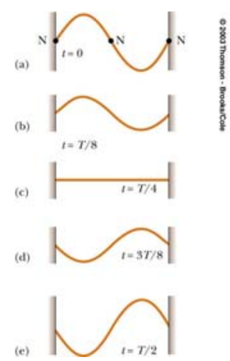


The wave doesn't move
Positions of nodes and antinodes are constant

Standing waves

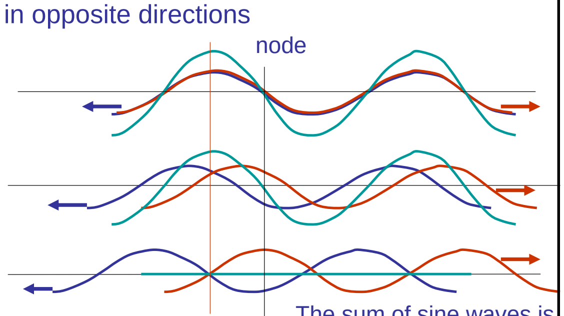
- Standing waves are harmonic waves that show oscillation but do not propagate in space.
- Standing waves are produced by two waves with the same wavelength moving in opposite directions.
- Opposite moving waves are produced by reflection at the end of the structure.
- Reflection at a fixed end produces a node
- Reflection at a free end produces an antinode.

Snapshots at different times.



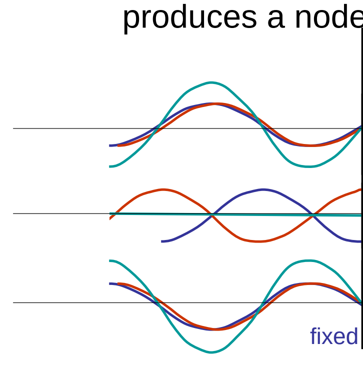
The points on the standing wave display harmonic oscillations with different amplitudes.

Standing waves are produced by superposition of two identical waves moving in opposite directions



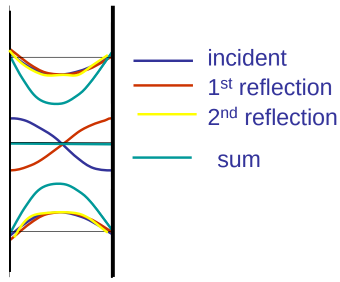
The sum of sine waves is a sine wave.

Reflection from a fixed end produces a node



The reflected wave is inverted.

Reflection at two ends



The wave reflected at two ends produces the standing wave pattern.

Standing Waves

- A standing wave is generated by superposition of two waves with the same frequency and wavelength traveling in opposite directions.

Simulation of a standing wave.

http://www.walter-fendt.de/ph14e/stwaverefl.htm

Mathematical description

Standing wave is the sum of waves with same wavelength and speed moving in opposite directions.

The minus sign accounts for the reflection at the fixed end.

$$y = A[\cos(kx - \omega t) - \cos(kx + \omega t)]$$

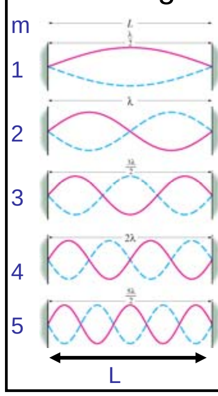
From trigonometry

$$\cos\alpha - \cos\beta = -2\sin(\frac{1}{2}(\alpha+\beta))\sin(\frac{1}{2}(\alpha-\beta))$$

$$y(x,t) = 2A\sin(kx)\cdot\sin(\omega t)$$

- For any time the y(x) is the same sine function
- The amplitude varies sinusoidally with time.
- Max and min displacement is $\pm 2A$.

standing wave with two fixed ends



Can accommodate standing waves with specific wavelengths.

node to node distance $=\lambda/2$

node to antinode distance $=\lambda/4$

For a length of L

$$\lambda = \frac{2L}{m}$$

m is an integer called Mode no.

$$f = \frac{v}{\lambda} = \frac{mv}{2L}$$

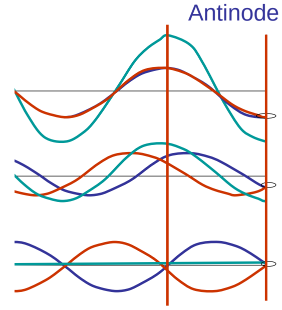
m=1 fundamental

m> 1 harmonics

Question

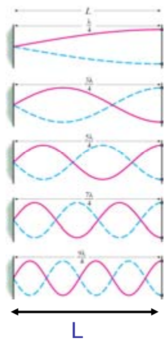
40 The A string in a piano (440Hz) is 38.9 cm long and is tightly clamped at both ends. If the string is under 667 N tension what is its mass?

Reflection from a fixed end produces an anti-node



reflection without inversion

Standing wave in an air column
One end closed



The structure contains and odd no. of quarter wavelengths

$$\lambda = \frac{4L}{m}$$

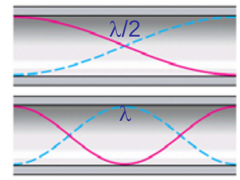
$$f = \frac{v}{\lambda} = \frac{mv}{4L}$$

m= 1, 3, 5, 7.....

odd integers.

Standing waves in an air column.

Two ends free



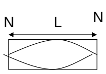
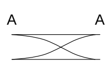
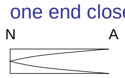
Results are similar to standing wave with 2 fixed ends.

$$\lambda = \frac{2L}{m}$$

$$f = \frac{v}{\lambda} = \frac{mv}{2L}$$

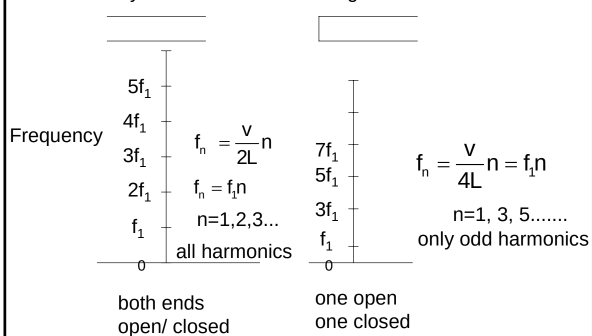
m= 1, 2, 3, 4........

Standing waves in air columns
Fundamental Frequency

2 ends closed	2 ends open	one end open one end closed
		
$L = \frac{\lambda_1}{2}$	$L = \frac{\lambda_1}{2}$	$L = \frac{\lambda_1}{4}$
$\lambda_1 = 2L$	$\lambda_1 = 2L$	$\lambda_1 = 4L$
$F_1 = \frac{v}{2L}$	$F_1 = \frac{v}{2L}$	$F_1 = \frac{v}{4L}$

F_1 lower by a factor of 2

Summary
For a cylinder with the same length



Question

Find the fundamental frequency of a tube 1.0 m long open at both ends. How does the frequency change when one end is closed?

Question

Find the fundamental frequency of standing wave in an aluminum rod 1.0 m long. The speed of sound in aluminum is 6400 m/s.

Find the frequency of the second harmonic.

Resonance

When the driving oscillations has a frequency that matches the oscillation frequency of the standing waves in the system then a large amount of energy can be put into the system.

Musical Instruments

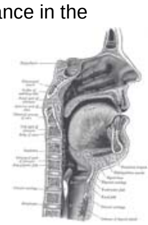
String Instruments

Frequency due to standing waves on the string.
The body of the instrument acts as a resonator to move air to amplify the sound.

Musical Instruments

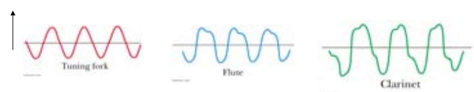
Wind instruments

The sound is produced by vibrating air and the frequency is enhanced by resonance in the air column



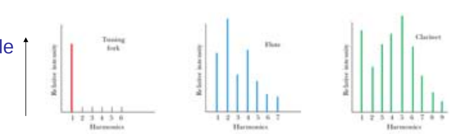
Complex waves consist of different frequency components , i.e. harmonics.

displacement



Time

relative amplitude



Frequency