Sound
2.1

Sound Waves
Speed of Sound
Intensity of Sound
Reflection of waves

Sound Waves

- Longitudinal waves – displacement in direction of propagation
- The displacement produces pressure differences due to differences in density

Pressure and Displacement out of phase by $\pi/2$ (90°)

Pressure is related to the slope of the displacement

An instructive simulation of sound wave

http://www.kettering.edu/~drussell/Demos/waves/wavemotion.html

Pressure

$P = \frac{F}{\text{Area}}$ Newton/m$^2$ Pascals

Compression increases pressure

$\Delta P = B \frac{\Delta V}{V} = B \frac{\Delta x}{x}$

Bulk modulus - $B$

Units of $B$ - Pascals
**Speed of sound**

\[ v = \sqrt{\frac{B}{\rho}} \]

for a solid and liquid

\[ \Delta P = B \frac{\Delta V}{V} \]

\( B \) is the bulk modulus

**F** compresses gas and changes speed.

\[ F = ma \]

\[ F = \Delta PA = \rho A \Delta V \left( \frac{\Delta V}{\Delta t} \right) \]

\[ v = \frac{\Delta P}{\rho \Delta V} \] but \[ \frac{\Delta V}{\Delta t} = \frac{\Delta V}{\Delta t} \]

\[ v^2 = \frac{\Delta PV}{\sigma \Delta V} = \frac{B}{\sigma} \]

**Speed of sound in a gas**

The bulk modulus for slow compression of a gas is equal to \( P \).

However, a sound wave compresses the gas quickly so that there is a temperature rise. (Adiabatic compression). For this case the bulk modulus is slightly larger by a factor of \( \gamma \) which varies according to the nature of the gas.

**Speed of sound in air**

- Air is a mixture of mostly diatomic gases. 80\% \( \text{N}_2 \), and 20\% \( \text{O}_2 \)

\[ \rho_{\text{air,20C}} = 1.20 \text{kg/m}^3 \]

\[ P_{\text{air,20C}} = 1.01 \times 10^5 \text{ Pascals} \]

\[ v = \sqrt{\frac{7 \times (1.0 \times 10^5)}{5(1.20)}} = 343 \text{ m/s} \]

- monatomic gas –He 5/3
- diatomic gas \( \text{O}_2 \), \( \text{N}_2 \) 7/5
- triatomic \( \text{CO}_2 \) 4/3
Speed of sound in water

\[ \rho_{\text{water}} = 1000 \text{kg/m}^3 \]
\[ B_{\text{water}} = 2 \times 10^9 \text{Pa} \]
\[ v = 1500 \text{m/s} \]

Speed of sound in water is about 5 times that in air. The higher bulk modulus compensates for the higher density.

Donald Duck Talk

\[ f = \frac{v}{\lambda} \]

Demonstrates the speed of sound in He is faster than in air. The wavelength of speech is governed by the length of the vocal cavity.

- He = 4 gm/mole (monatomic)
- Air ~ 30 gm/mole (diatomic)

\[ V_{\text{sound, He}} \approx 3 V_{\text{sound, air}} \]

Intensity of sound

average power is the energy in one wavelength / period

\[ P_w = \frac{\rho Av^2}{T} \]

For a harmonic oscillator the average KE is equal to the average PE so the total energy is

\[ 2xKE = mv^2 \]

\[ s = s_0 \cos(kx - \omega t) \]
\[ dm = \rho A dx \]
\[ v^2 = \omega^2 s_0^2 \sin^2(kx - \omega t) \]

Two expression for the intensity

Sound Intensity and Hearing

- The human ear can perceive changes in sound intensity over a wide range of intensities. (12 orders of magnitude)
- The perception of sound is not linear but logarithmic.
- The decibel scale is a logarithmic scale of intensities that is useful for characterizing sound.

\[ P_w = \int_0^T \frac{dmv^2}{T} = \rho A \omega^2 s_0^2 \int_0^T \sin^2(kx - \omega t) dx \]
\[ = \frac{\rho A \omega^2 s_0^2 \lambda}{2T} \]

divide by A to get intensity

\[ I_{av} = \frac{1}{2} \frac{\rho A \omega^2 s_0^2 \lambda}{2T} \]

also

\[ I_{av} = \frac{\Delta P_o^2}{2\rho v} \]

see text.

Problem

The threshold of hearing is a sound intensity of about \(1 \times 10^{-12}\) W/m². What is the maximum displacement of a sound wave in air at a frequency of 1000 Hz at this intensity? (\(\rho_{av} = 1.2 \text{ kg/m}^2\), speed of sound = 340 m/s)
Decibel
The unit of decibels $\beta$ is a logarithmic description of sound intensity.

$$\beta = 10 \log \frac{I}{I_0}$$

$I_0 = 10^{-12} \text{W/m}^2$

- $\beta$ is dimensionless, and increases as the log of the intensity
- $I_0$ is the threshold for hearing.

Hearing
Hearing covers a wide dynamic range of intensities. The maximum sensitivity is around 3 kHz. Low frequency vibrations require higher intensities.

Question
The ear perceives changes in loudness by a factor of 2 for a 10 dB change in intensity. What is the intensity that is 4 times louder than an intensity of $10^{-8} \text{W/m}^2$?

a) $10^{-7} \text{W/m}^2$

b) $10^{-6} \text{W/m}^2$

c) $10^{-5} \text{W/m}^2$

d) $10^{-4} \text{W/m}^2$

Reflection of waves
Waves are reflected at a boundary.

- The wave cannot propagate across the boundary.

Partial Reflection at the boundary

- Reflections are important to understand properties of waves in different media, i.e. partial reflection of light by glass.
- Standing waves that are the basis for musical instruments are formed by wave reflection.