week 4
**General Principles**

**Sources of V**

The electric potential, like the electric field, is created by charges.

Two major tools for calculating $V$ are

- The potential of a point charge $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$
- The principle of superposition

**Multiple point charges**

Use superposition: $V = V_1 + V_2 + V_3 + \cdots$

**Continuous distribution of charge**

- Divide the charge into point-like $\Delta Q$.
- Find the potential of each $\Delta Q$.
- Find $V$ by summing the potentials of all $\Delta Q$.

The summation usually becomes an integral. A critical step is replacing $\Delta Q$ with an expression involving a charge density and an integration coordinate. Calculating $V$ is usually easier than calculating $\bar{E}$ because the potential is a scalar.

**Consequences of V**

A charged particle has potential energy

$$U = qV$$

at a point where source charges have created an electric potential $V$.

The electric force is a conservative force, so the mechanical energy is conserved for a charged particle in an electric potential:

$$K_f + U_f = K_i + U_i$$

The potential energy of **two point charges** separated by distance $r$ is

$$U_{q_1+q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r}$$

The **zero point** of potential and potential energy is chosen to be convenient. For point charges, we let $U = 0$ when $r \to \infty$.

The potential energy in an electric field of an electric dipole with dipole moment $\vec{p}$ is

$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p} \cdot \vec{E}$$
Applications

Graphical representations of the potential:

- Potential graph
- Equipotential surfaces
- Contour map
- Elevation graph

Sphere of charge $Q$
Same as a point charge if $r \geq R$

Parallel-plate capacitor
$V = Es$, where $s$ is measured from the negative plate. The electric field inside is

$$E = \frac{\Delta V_c}{d}$$

Units
- Electric potential: $1 \text{ V} = 1 \text{ J/C}$
- Electric field: $1 \text{ V/m} = 1 \text{ N/C}$
## Terms and Notation

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Which requires the **most work**, to move a positive charge from \( P \) to points 1, 2, 3 or 4? All points are the same distance from \( P \).
Which requires the **most work**, to move a **positive** charge from $P$ to points 1, 2, 3 or 4? All points are the same distance from $P$.

1) $P \rightarrow 1$
2) $P \rightarrow 2$
3) $P \rightarrow 3$
4) $P \rightarrow 4$
5) all require the same amount of work

For **path #1**, you have to push the positive charge **against** the $E$ field, which is **hard to do**. By contrast, path #4 is the easiest, since the field does all the work.
Which requires zero work, to move a positive charge from point \( P \) to points 1, 2, 3 or 4? All points are the same distance from \( P \).

1) \( P \rightarrow 1 \)
2) \( P \rightarrow 2 \)
3) \( P \rightarrow 3 \)
4) \( P \rightarrow 4 \)
5) all require the same amount of work
For path #3, you are moving in a direction perpendicular to the field lines. This means you are moving along an equipotential, which requires no work (by definition).

Follow-up: Which path requires the least work?

Which requires zero work, to move a positive charge from P to points 1, 2, 3 or 4? All points are the same distance from P.

1) $P \rightarrow 1$
2) $P \rightarrow 2$
3) $P \rightarrow 3$
4) $P \rightarrow 4$
5) all require the same amount of work
The positive charge is the end view of a positively charged glass rod. A negatively charged particle moves in a circular arc around the glass rod. Is the work done on the charged particle by the rod’s electric field positive, negative or zero?

1. Positive
2. Negative
3. Zero
The positive charge is the end view of a positively charged glass rod. A negatively charged particle moves in a circular arc around the glass rod. Is the work done on the charged particle by the rod’s electric field positive, negative or zero?

1. Positive
2. Negative
3. Zero
When a positive charge moves in the direction of the electric field,

1. the field does positive work on it and the potential energy increases
2. the field does positive work on it and the potential energy decreases
3. the field does negative work on it and the potential energy increases
4. the field does negative work on it and the potential energy decreases
When a positive charge moves in the direction of the electric field,

1. the field does positive work on it and the potential energy increases
2. the field does positive work on it and the potential energy decreases
3. the field does negative work on it and the potential energy increases
4. the field does negative work on it and the potential energy decreases

The work done by the field is

\[ dW = qE \cdot dr \]

In this case the vectors \( E \) and \( dr \) are parallel and the charge is positive so

\[ dW > 0 \]

The change in potential energy is the negative of the work done by the field, so

\[ \Delta U = -W < 0 \]
When a negative charge moves in the direction of the electric field,

1. the field does positive work on it and the potential energy increases
2. the field does positive work on it and the potential energy decreases
3. the field does negative work on it and the potential energy increases
4. the field does negative work on it and the potential energy decreases
When a negative charge moves in the direction of the electric field,

1. the field does positive work on it and the potential energy increases
2. the field does positive work on it and the potential energy decreases
3. the field does negative work on it and the potential energy increases
4. the field does negative work on it and the potential energy decreases

The work done by the field is

\[ dW = qE \cdot dr \]

In this case the vectors \( E \) and \( dr \) are parallel and the charge is negative so \( dW < 0 \)

The change in potential energy is the negative of the work done by the field, so

\[ \Delta U = -W > 0 \]
A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from the positive side and the electron from the negative side. Which feels the larger electric force?

1) proton
2) electron
3) both feel the same force
4) neither – there is no force
5) they feel the same magnitude force but opposite direction
A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from the positive side and the electron from the negative side. Which feels the larger electric force?

Since $F = qE$ and the proton and electron have the same charge in magnitude, they both experience the same force. However, the forces point in opposite directions because the proton and electron are oppositely charged.

1) proton
2) electron
3) both feel the same force
4) neither – there is no force
5) they feel the same magnitude force but opposite direction
A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from the positive side and the electron from the negative side. Which has the larger acceleration?

1) proton
2) electron
3) both feel the same acceleration
4) neither – there is no acceleration
5) they feel the same magnitude acceleration but opposite direction
A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from the positive side and the electron from the negative side. Which has the larger acceleration?

1) proton
2) electron
3) both feel the same acceleration
4) neither – there is no acceleration
5) they feel the same magnitude acceleration but opposite direction

Since \( F = ma \) and the electron is much less massive than the proton, then the electron experiences the larger acceleration.
A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from the positive side and the electron from the negative side. When it strikes the opposite plate, which one has more KE?

1) proton
2) electron
3) both acquire the same KE
4) neither – there is no change of KE
5) they both acquire the same KE but with opposite signs
A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from the positive side and the electron from the negative side. When it strikes the opposite plate, which one has more KE?

1) proton
2) electron
3) both acquire the same KE
4) neither – there is no change of KE
5) they both acquire the same KE but with opposite signs

Since \( PE = qV \) and the proton and electron have the same charge in magnitude, they both have the same electric potential energy initially. Because energy is conserved, they both must have the same kinetic energy after they reach the opposite plate.
Here is a simple problem we already know how to solve using forces and $F=ma$. But let's solve it using conservation of energy:

The electric field strength is 50,000 N/C inside a parallel plate capacitor with a 2.0 mm spacing. A proton is released from rest at the positive plate. What is the proton's speed when it reaches the negative plate?

Ans: $1.38 \times 10^5$ m/s
Rank in order, from largest to smallest, the potential energies $U_a$ to $U_d$ of these four pairs of charges. Each + symbol represents the same amount of charge.

1. $U_a = U_b = U_c = U_d$

2. $U_d = U_b > U_c = U_a$

3. $U_d > U_b = U_c > U_a$

4. $U_b > U_a > U_d > U_c$

5. $U_d > U_b = U_c > U_a$
Rank in order, from largest to smallest, the potential energies $U_a$ to $U_d$ of these four pairs of charges. Each + symbol represents the same amount of charge.

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2. $U_d = U_b > U_c = U_a$

3. $U_d > U_b = U_c > U_a$

4. $U_b > U_a > U_d > U_c$

5. $U_d > U_b = U_c > U_a$

$U_a = k \frac{q^2}{r}$ $U_b = k \frac{2q^2}{r}$ $U_c = k \frac{(2q)q}{2r} = k \frac{q^2}{r}$ $U_d = k \frac{(2q)(2q)}{2r} = 2k \frac{q^2}{r}$
The electric potential energy of two point charges approaches zero as the two point charges move farther away from each other.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential energy of the system of three charges is

1. positive
2. negative
3. zero
4. not enough information given to decide
The electric potential energy of two point charges approaches zero as the two point charges move farther away from each other.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential energy of the system of three charges is

1. positive
2. negative
3. zero
4. not enough information given to decide

If the length of the side of the triangle is \( d \), then

\[
U = k \left( \frac{q^2}{d} + \frac{q(-q)}{d} + \frac{q(-q)}{d} \right) = - \frac{kq^2}{d} < 0
\]
Which group of charges took more work to bring together from a very large initial distance apart?

1. +2 \rightarrow +1

2. +1 \rightarrow +1 \rightarrow +1

Both took the same amount of work
Which group of charges took *more work* to bring together from a very large initial distance apart?

**Case 1:**
- **Pairs:** Only 1 pair
- **Potential Energy (PE):**
  \[ PE = k \frac{(+2)(+1)}{d} = k \frac{2}{d} \]

**Case 2:**
- **Pairs:** There are 3 pairs
- **Potential Energy (PE):**
  \[ PE = 3k \frac{(+1)(+1)}{d} = 3k \frac{1}{d} \]

The work needed to assemble a collection of charges is the same as the total PE of those charges:

\[ PE = k \frac{Q_1Q_2}{r} \]

Both took the same amount of work.
These four 1.0 g spheres are released simultaneously and allowed to move away from each other. What is the speed of each sphere when they are very far apart?

Ans (if needed, see next page): 0.49 m/s
These four 1.0 g spheres are released simultaneously and allowed to move away from each other. What is the speed of each sphere when they are very far apart?

Use conservation of energy: \[ U_f + K_f = U_i + K_i, \]

Initial energy is pure potential energy:
\[ U_i = (U_{12} + U_{23} + U_{34} + U_{41}) + (U_{13} + U_{24}) \]

\[ r_{12} = r_{23} = r_{34} = r_{41} = 1.0 \text{ cm}, \]
\[ r_{13} = r_{24} = \sqrt{2.0} \text{ cm}. \]

\[ U_i = 4 \left( \frac{1}{4\pi e_0} \frac{10 \times 10^{-9} \text{ C}}{0.010 \text{ m}} \right) (10 \times 10^{-9} \text{ C}) + 2 \left( \frac{1}{4\pi e_0} \frac{10 \times 10^{-9} \text{ C}}{0.01414 \text{ m}} \right) = 48.73 \times 10^{-5} \text{ J} \]

Final energy is pure kinetic energy:
\[ K_f = 4 \left( \frac{1}{2} m v_f^2 \right) = 2(1.0 \times 10^{-3} \text{ kg}) v_f^2 \]

\[ 2(1.0 \times 10^{-3} \text{ kg}) v_f^2 = 48.73 \times 10^{-5} \text{ J} \Rightarrow v_f = 0.49 \text{ m/s} \]
A proton is released from rest at point B, where the potential is 0 V. Afterward, the proton

1. moves toward A with an increasing speed.
2. moves toward A with a steady speed.
3. remains at rest at B.
4. moves toward C with a steady speed.
5. moves toward C with an increasing speed.
A proton is released from rest at point B, where the potential is 0 V. Afterward, the proton
1. moves toward A with an increasing speed.
2. moves toward A with a steady speed.
3. remains at rest at B.
4. moves toward C with a steady speed.
5. moves toward C with an increasing speed.
Rank in order, from largest to smallest, the potentials \( V_a \) to \( V_e \) at the points a to e.

1. \( V_a = V_b = V_c = V_d = V_e \)
2. \( V_a = V_b > V_c > V_d = V_e \)
3. \( V_d = V_e > V_c > V_a = V_b \)
4. \( V_b = V_c = V_e > V_a = V_d \)
5. \( V_a = V_b = V_d = V_e > V_c \)
Rank in order, from largest to smallest, the potentials $V_a$ to $V_e$ at the points a to e.

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2. $V_a = V_b > V_c > V_d = V_e$

3. $V_d = V_e > V_c > V_a = V_b$

4. $V_b = V_c = V_e > V_a = V_d$

5. $V_a = V_b = V_d = V_e > V_c$
An AA battery is connected to a parallel plate capacitor having 4.0-cm-diameter plates spaced 2.0 mm apart. How much charge does the battery supply to each plate?

Ans: $8.3 \times 10^{-12}$ C
What is the electric potential at point A?

1) $V > 0$
2) $V = 0$
3) $V < 0$
Since $Q_2$ (which is positive) is closer to point A than $Q_1$ (which is negative) and since the total potential is equal to $V_1 + V_2$, then the total potential is positive.

What is the electric potential at point A?

1) $V > 0$
2) $V = 0$
3) $V < 0$
What is the electric potential at point B?

1) $V > 0$
2) $V = 0$
3) $V < 0$
Since $Q_2$ and $Q_1$ are equidistant from point B, and since they have equal and opposite charges, then the total potential is zero.

Follow-up: What is the potential at the origin of the x-y axes?
Consider a point $P$ in space where the electric potential is zero. Which statement is correct?

1. a point charge placed at $P$ would feel no electric force
2. the electric field at points around $P$ is directed toward $P$
3. the electric field at points around $P$ is directed away from $P$
4. none of the above
5. not enough information given to decide
Consider a point \( P \) in space where the electric potential is zero. Which statement is correct?

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2. the electric field at points around \( P \) is directed toward \( P \)
3. the electric field at points around \( P \) is directed away from \( P \)
4. none of the above
5. not enough information given to decide

The value of the electric potential at any one point in space is arbitrary. Only differences in electric potential have physical significance.
The electric potential due to a point charge approaches zero as you move farther away from the charge.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential at the center of the triangle is

1. positive
2. negative
3. zero
4. not enough information given to decide
The electric potential due to a point charge approaches zero as you move farther away from the charge.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential at the center of the triangle is

1. positive
2. negative
3. zero
4. not enough information given to decide

If the distance from any charge to the center of the triangle is \( d \), then

\[
V = k \left( \frac{q}{d} + \frac{q}{d} - \frac{q}{d} \right) = \frac{kq}{d} > 0
\]
Four point charges are arranged at the corners of a square. Find the electric field $E$ and the potential $V$ at the center of the square.

1) $E = 0 \quad V = 0$
2) $E = 0 \quad V \neq 0$
3) $E \neq 0 \quad V \neq 0$
4) $E \neq 0 \quad V = 0$
5) $E = V$ regardless of the value $Q$.
Four point charges are arranged at the corners of a square. Find the electric field $E$ and the potential $V$ at the center of the square.

1. $E = 0 \quad V = 0$
2. $E = 0 \quad V \neq 0$
3. $E \neq 0 \quad V \neq 0$
4. $E \neq 0 \quad V = 0$

5. $E = V$ regardless of the value

The potential is zero: the scalar contributions from the two positive charges cancel the two minus charges. However, the contributions from the electric field add up as vectors, and they do not cancel (so it is non-zero).

Follow-up: What is the direction of the electric field at the center?
Example: electric potential of an electric dipole with dipole moment $p$ a distance $r$ much larger than the separation $2a$ between the charges, $r \gg 2a$.

$$ V(P) = \frac{kq}{r_1} + \frac{k(-q)}{r_2} $$

$$ = kq \left( \frac{1}{r_1} - \frac{1}{r_2} \right) $$

$$ = \frac{kq(r_2 - r_1)}{r_1 r_2} $$

or, for $a \ll r$

$$ V(r, \theta) \approx \frac{k(2qa) \cos \theta}{r^2} = kp \frac{\cos \theta}{r^2} $$
At which point does $V = 0$?

5) all of them
At which point does \( V = 0 \)?

\( \begin{align*}
1 & \quad 3 \\
2 & \\
3 & \\
4 & \quad \text{\textcolor{red}{+Q}} \\
& \quad \text{\textcolor{blue}{-Q}}
\end{align*} \)

\textcolor{green}{5) all of them}

All of the points are equidistant from both charges. Since the charges are equal and opposite, their contributions to the potential cancel out everywhere along the mid-plane between the charges.

Follow-up: What is the direction of the electric field at all 4 points?
Equipotential surfaces of a dipole

(show applet if possible)
Which of these configurations gives $V = 0$ at all points on the $x$-axis?

4) all of the above  
5) none of the above
Which of these configurations gives $V = 0$ at all points on the $x$-axis?

1) $+2\mu C$, $-2\mu C$, $+1\mu C$, $-1\mu C$
2) $+2\mu C$, $-1\mu C$, $+1\mu C$, $-2\mu C$
3) $+2\mu C$, $-1\mu C$, $-2\mu C$, $+1\mu C$
4) all of the above
5) none of the above

Only in case (1), where opposite charges lie directly across the $x$-axis from each other, do the potentials from the two charges above the $x$-axis cancel the ones below the $x$-axis.
Which of these configurations gives $V = 0$ at all points on the $y$-axis?

1) $+2\mu C -1\mu C -2\mu C$
2) $+2\mu C -1\mu C -2\mu C$
3) $+2\mu C -1\mu C +1\mu C$
4) all of the above
5) none of the above
Which of these configurations gives $V = 0$ at all points on the $y$-axis?

1) $+2\mu_C + 1\mu_C - 2\mu_C - 1\mu_C$

2) $+2\mu_C + 1\mu_C - 1\mu_C - 2\mu_C$

3) $+2\mu_C + 1\mu_C - 1\mu_C - 2\mu_C$

4) all of the above

5) none of the above

Only in case (3), where opposite charges lie directly across the $y$-axis from each other, do the potentials from the two charges above the $y$-axis cancel the ones below the $y$-axis.

Follow-up: Where is $V = 0$ for configuration #2?
Which two points have the same potential?

1) A and C
2) B and E
3) B and D
4) C and E
5) no pair
Which two points have the same potential?

1) A and C
2) B and E
3) B and D
4) C and E
5) no pair

Since the potential of a point charge is:

\[ V = k \frac{Q}{r} \]

only points that are at the same distance from charge Q are at the same potential. This is true for points C and E. They lie on an Equipotential Surface.

Follow-up: Which point has the smallest potential?
Rank in order, from largest to smallest, the potential differences $\Delta V_{12}, \Delta V_{13},$ and $\Delta V_{23}$ between points 1 and 2, points 1 and 3, and points 2 and 3. Here the symbol $\Delta V_{12}$ means $\Delta V_{12} = V_1 - V_2$, etc.

1. $\Delta V_{12} > \Delta V_{13} = \Delta V_{23}$
2. $\Delta V_{13} > \Delta V_{12} > \Delta V_{23}$
3. $\Delta V_{13} > \Delta V_{23} > \Delta V_{12}$
4. $\Delta V_{13} = \Delta V_{23} > \Delta V_{12}$
5. $\Delta V_{23} > \Delta V_{12} > \Delta V_{13}$
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1. $\Delta V_{12} > \Delta V_{13} = \Delta V_{23}$
2. $\Delta V_{13} > \Delta V_{12} > \Delta V_{23}$
3. $\Delta V_{13} > \Delta V_{23} > \Delta V_{12}$
4. $\Delta V_{13} = \Delta V_{23} > \Delta V_{12}$
5. $\Delta V_{23} > \Delta V_{12} > \Delta V_{13}$

1 and 2 the same distance from $q$: $V_1 = V_2$

3 is farther away since $V = kq/r$
potential at 3 is smaller than at 1&2
Example: ring of charge $Q$, radius $a$, with uniform charge density, on points along ring's axis.

Continuous charge distribution:

$$V = \int dV = \int \frac{k \, dq}{r}$$

$$V(x) = \frac{kQ}{\sqrt{a^2 + x^2}}$$
A thin rod with charge $Q$ has been bent into a semicircle of radius $R$. Find an expression for the electric potential at the center.

**Ans:** $V = k\frac{Q}{R}$
Example: disk of charge $Q$, radius $a$, with uniform charge density, on points along disk's axis.

Sum over concentric rings, using previous result for ring of:

- radius: $r$
- charge: $dq = (2\pi r \, dr) \sigma = (2\pi r \, dr) \frac{Q}{\pi a^2} = \frac{Q}{a^2} \, dr^2$

\[ V(x) = \frac{2kQ}{a^2} \left( \sqrt{x^2 + a^2} - |x| \right) \]
additional problems
At the back of a TV picture tube, a uniform electric field of 600 kN/C extends over region of 5.0 cm and points towards the back of the tube. (a) Find the potential difference between the back and the front of this field region. (b) How much work would it take to move an ion with charge $+2e$ from the back to the front of the field region? (c) What would happen to an electron released at the back of the field region?

Ans:
(a) 30 kV
(b) $9.6 \times 10^{-15}$ J (or $(2e)(30 \text{ kV}) = 60 \text{ keV}$, see next page)
An isolated, infinite charged sheet carries a uniform charge density $\sigma$.

(a) Find an expression for the potential difference from the sheet to a point a perpendicular distance $x$ from the sheet.

(b) What is the potential difference between two points the same distance from the sheet?

(COB)

ANS:

$$\Delta V_{0x} = -\frac{\sigma x}{2\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$