CMB Anisotropies

Recap: • Discussed development of density perturbations in an expanding Universe. Why are these interesting? Because present thinking is that ultimately these initially weak perturbations detatch from the background and form the non-linear density structures we observe today; i.e., galaxies, clusters, etc. In the end, these studies tell us about the origin of present-day structure

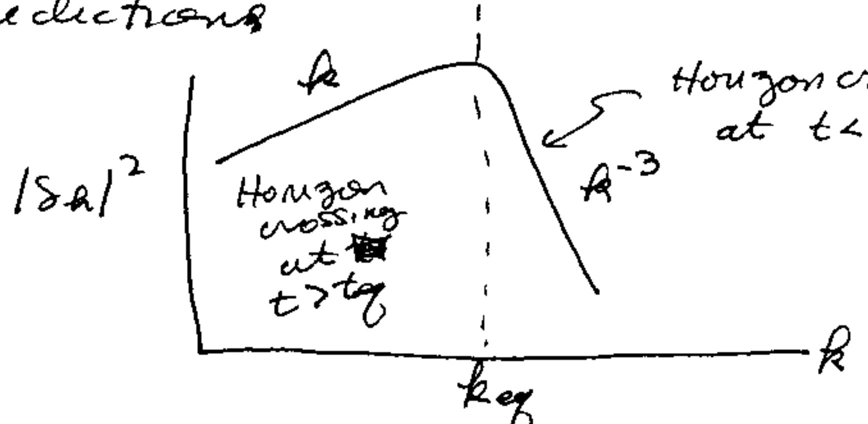
• Power-spectrum:

(1) I discussed how an "initial" scale-invariant power spectrum

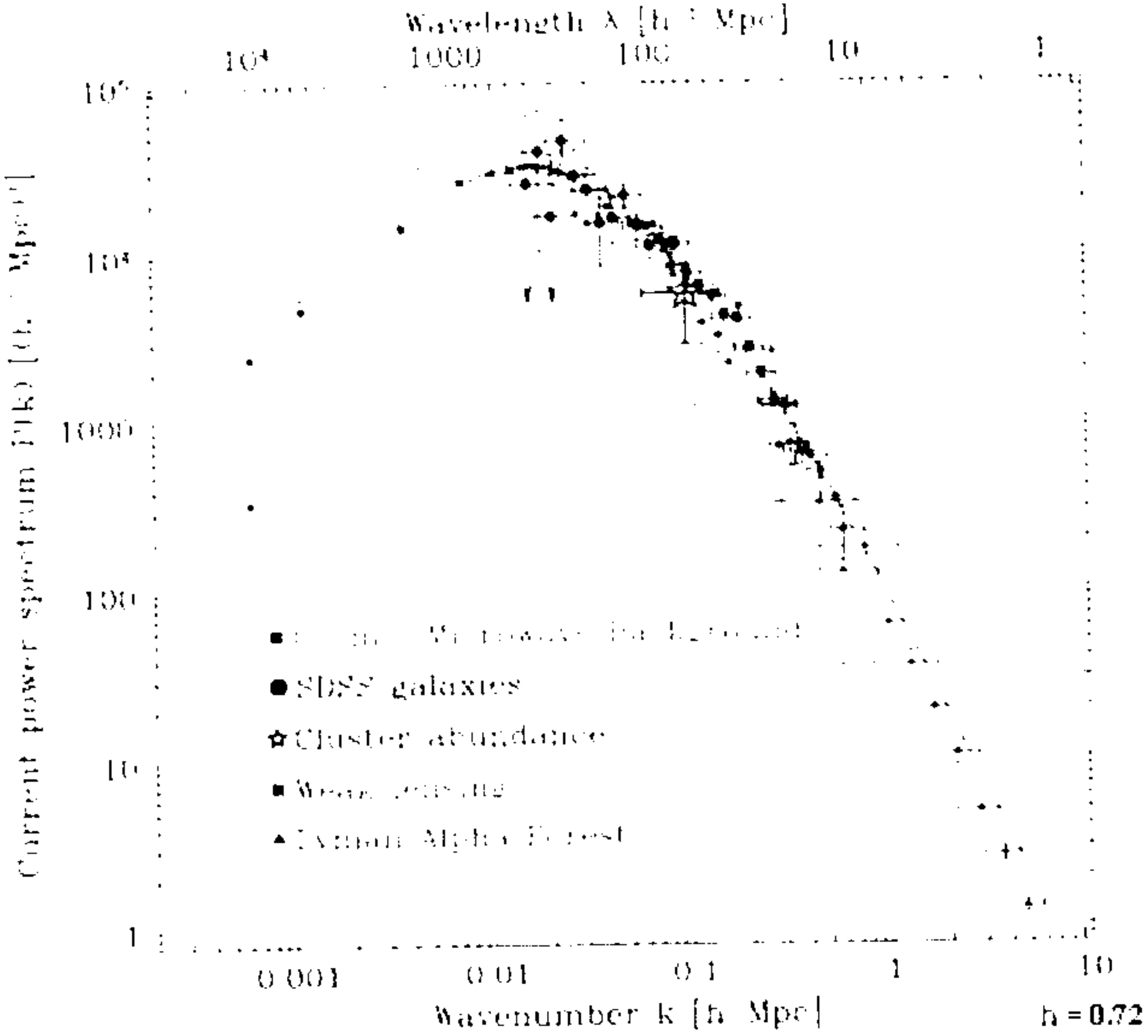
$|\delta_k|^2 \propto k$

gets modified when perturbation cross within horizon at $t < t_{eq}$.

Predictions



(2) • At $k > k_{eq}$: (i.e., $\lambda_{comov} < 300$ Mpc)
data is from galaxy surveys, etc.
• at $k < k_{eq}$ ($300 < \lambda_{comov} < 10000$ Mpc)
only CMB measurements are


Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
SDSS galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
Current power spectrum P(k) [(h⁻¹ Mpc)³]
10⁵
10⁴
1000
100
10
1
0.001
0.01
0.1
1
10
Wavenumber k [h Mpc⁻¹]
h = 0.72

relevant: ~~Any~~ in fact CMB data straddles both sides. Any successful theory for origin of structure <u>must</u> be consistent with observed power spectrum.

- But CMB "angular power" spectrum is equally, if not more important, for another reason: it tells us about fundamental parameters such as k, Ω_m, Ω_Λ, Ω_B, H_0, ... Especially in conjunction with galaxy data. moreover.
  - It has already provided evidence for re-ionization at $z \sim 8-10$
  - Potential from polarization data to tell us about inflation signatures.

S-W. Let's start with simplest case: Effect super-horizon density perturbation, SW effect. To do this you have to solve 3 problems in context of GR

(1) solutions to Einstein field equations perturbed about FRW metric

(2) Solve for lightlike geodesics in perturbed metric

(3) Solve GR Boltzmann eq. for

propagation of radiation in order to obtain ~~solutions~~ temperature variations to CMB.

I'll get to formalism shortly. But first I can present main result, which with a lot of hindsight can be explained by a simple physical picture.

<u>Super-horizon perturbations</u>: these are large-scale perturbations that did not enter horizon prior to decoupling. Today they have very large scales ($\gtrsim$ few hundred → thousands of Mpc). So they are

(1) Jeans unstable: pressure effects are irrelevant since sound waves don't have time to cross objects so large.

(2) Propagate in a pressure-free $P=0$ Universe.

<u>SW terms</u>:

Terms of interest are that in directions given by unit vector $\hat{e}$, {relative change in temperature

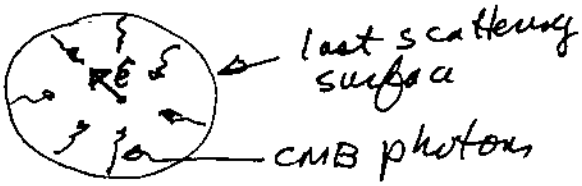
$$\frac{\delta T}{T} = \underbrace{\left(\hat{e}\cdot\frac{\delta \underline{v}}{c}\right)_0 - \left(\hat{e}\cdot\frac{\delta \underline{v}}{c}\right)_L}_{\text{Doppler}} + \frac{1}{3}\underbrace{\left\{\delta\phi(e\chi)_L - \delta\phi(0)\right\}}_{\text{Potential}}$$

where

- subscripts 0 and L denote us now and last-scattering surface respectively.

Doppler terms

(a) First terms: First harmonic due to our local peculiar motion wrst F.O. It has been measured



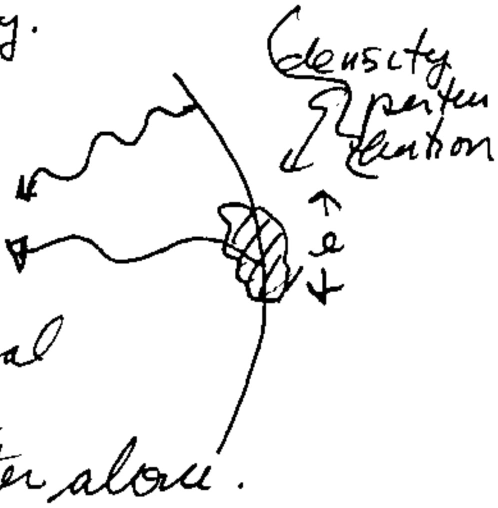
(b) Second term: Doppler terms due to peculiar motion wrst many FOs on last scattering surface. Gives rise to random changes in T as a function of $\hat{e}$.

Potential Terms

$$\frac{\delta T}{T} = \frac{1}{3}\left\{ \delta\phi(e^{\alpha}x)_L - \delta\phi(e^{\alpha}x)_0 \right\}$$

Second term is just a constant reference potential and has no intrinsic significance. But first term is just due to variation of gravitational potential as $\hat{e}$ sweeps across density inhomogeneity.

Physics: Photons propagating to us from direction of density lump suffer gravitational redshift in addition to cosmological redshift experienced by photons travelling through smooth matter alone.



Result: Photons propagating through lumps do extra work climbing out of grav. field due to lump.

- Toward lump: $\delta T/T < 0$
- Toward "void": $\delta T/T > 0$

Why does $\delta T/T = \Phi/3$ and not $\delta T/T = \Phi$?

Extra mass of lump slows down expansion rate in that direction causing a slightly lower value of the scale factor a. This alters T for following reasons:

$$T(a)\cdot a = \text{const.} \quad : \text{on last scattering surface}$$

So change in a causes change in T; i.e.,

$$\delta T \cdot a + T \cdot \delta a = 0 \Rightarrow \delta T/T = -\frac{\delta a}{a}$$

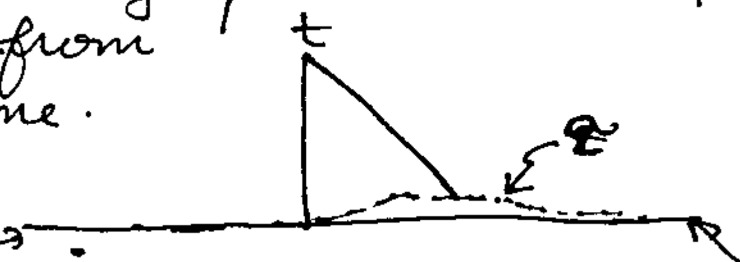
Thus if $\frac{\delta a}{a} < 0$, then $\frac{\delta T}{T} > 0$.

To evaluate $\delta a/a$, we note that

$$a \propto t^{2/3} \Rightarrow \ln a = \tfrac{2}{3}\ln t$$

$$\frac{\delta a}{a} = \frac{2}{3}\left(\frac{\delta t}{t}\right)$$

Thus density perturbation warps the t=const surface from cosmic time.



$$ds^2 = (1+2\phi)\,dt^2 + g_{\alpha\beta}\,dx^\alpha dx^\beta$$

$\therefore$ Proper time $d\tau = \sqrt{1+2\phi}\;dt$

or $\tau \simeq (1+\phi)\,t$ (weak ϕ)

Therefore: in regions of local density enhancement time intervals shift ~~as~~ wrst t:

$$\frac{\tau - t}{t} \simeq \phi$$

Consequently this effect causes ~~intrinsic shift~~

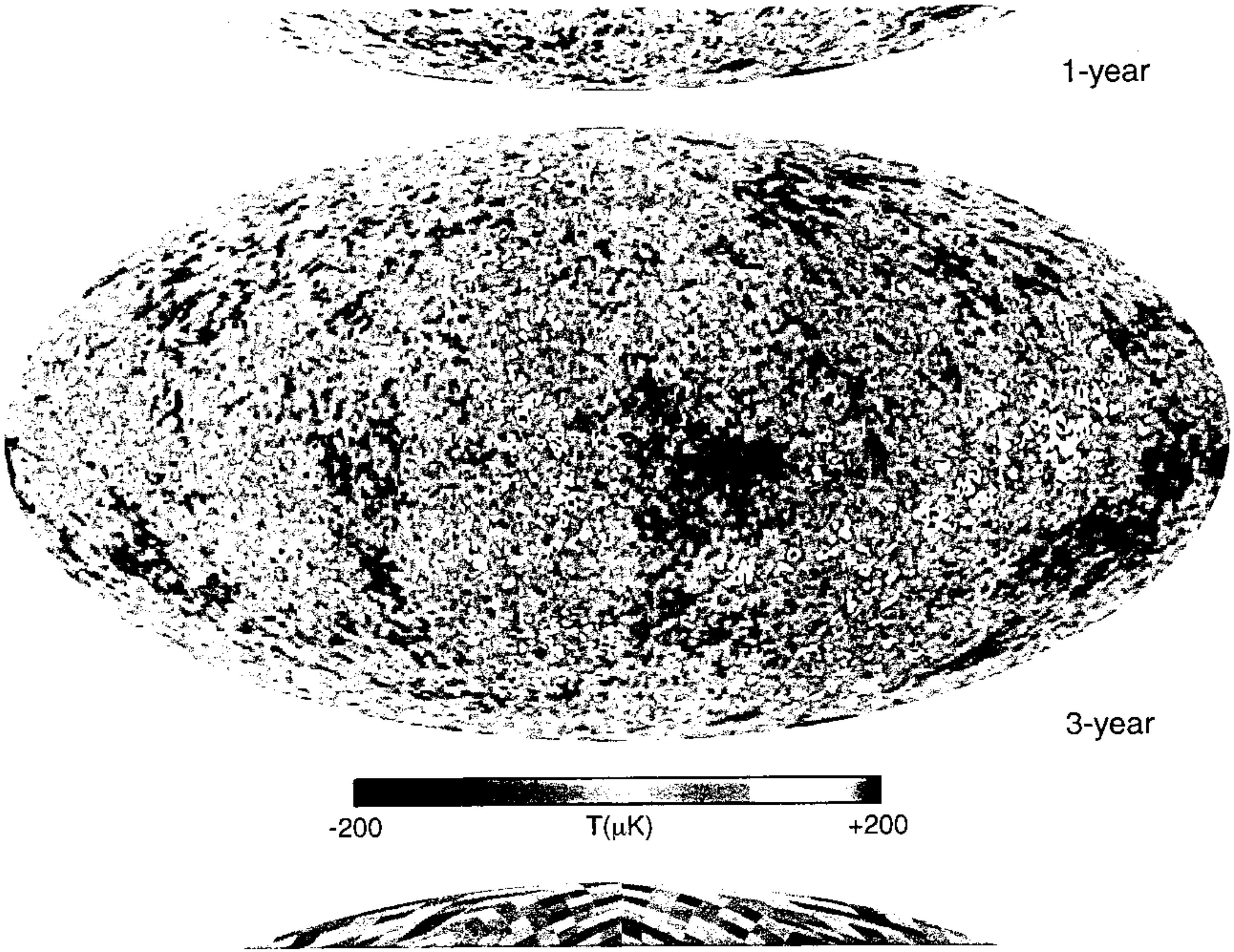
$$\frac{\delta a}{a} = \frac{2}{3}\phi$$

Thus full SW effect: $\frac{\delta T}{T} = \phi(x_L) + \left(\frac{\delta T}{T}\right)_L$

$$\boxed{\frac{\delta T}{T} = \phi - \frac{2}{3}\phi = \frac{1}{3}\phi}$$

{ effective dip since $\phi < 0$ }

CMB Normalization of Power Spectrum


1-year
3-year
-200
T(μK)
+200

Relationship between CMB and power Spectrum

(1) <u>Spherical Harmonics:</u>

First expand temperature variations in terms of spherical harmonics.

Toward direction $\hat{n} = (\theta, \phi)$ in sky define a fractional temperature fluctuation:

$$\Delta(\hat{n}) \equiv \frac{\delta T(\theta,\phi)}{T}$$

Expansion $\quad \Delta(\hat{n}) = \sum_{l,m}{}' a_{lm} Y_{lm}(\theta,\phi)$

But we also expand $\Delta(\hat{n})$ in terms of Fourier modes on last scattering surface (LSS)

$$\Delta(\hat{n}) = \int \frac{d^3k}{(2\pi)^3} \Delta(\underset{\sim}{k}) e^{i\underset{\sim}{k}\cdot\hat{n}L}$$

where $L = \chi = \eta_0 - \eta_{LSS}$ is comoving distance along past lightcone to the LSS.

But

$$\therefore \quad \int \frac{d^3k}{(2\pi)^3} \Delta(\underset{\sim}{k}) e^{i\underset{\sim}{k}\cdot\hat{n}L} = \sum_l{}' \sum_m{}' a_{lm} Y_{lm}$$

mult by $Y^*_{l'm'}$

$$\sum_l{}' \sum_m{}' a_{lm} Y_{lm} Y^*_{l'm'} = \int \frac{d^3k}{(2\pi)^3} \Delta(\underset{\sim}{k}) e^{i\underset{\sim}{k}\cdot\hat{n}L} Y^*_{l'm'}$$

Integrate latter equation over the sphere

$$\sum_{l}{}' \sum_{m}{}' a_{lm} \underbrace{\int_{4\pi} d\Omega\, Y_{lm} Y^*_{l'm'}}_{\delta_{ll'}\delta_{mm'}} = \int \frac{d^3k}{(2\pi)^3} \Delta(\underset{\sim}{k}) \int_{4\pi} d\Omega\, e^{i\underset{\sim}{k}\cdot\hat{n}L} Y^*_{l'm'}$$

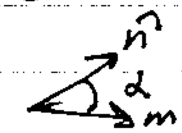
$$\text{i.e. } a_{l'm'} = \int \frac{d^3k}{(2\pi)^3} \Delta(\underset{\sim}{k}) \int_{4\pi} d\Omega\, e^{i\underset{\sim}{k}\cdot\hat{n}L} Y^*_{l'm'}(\hat{n})$$

$$\text{But } e^{i\underset{\sim}{k}\cdot\hat{n}L} = 4\pi \sum_{l}{}' \sum_{m}{}' i^l j_l(kL)\, Y^*_{lm}(\hat{k})\, Y_{lm}(\hat{n})$$

$$\text{Therefore: } a_{l'm'} = \int \frac{d^3k}{(2\pi)^3} \Delta(\underset{\sim}{k})\, 4\pi \sum_{l}{}' \sum_{m}{}' i^l j_l(kL)\, Y^*_{lm}(\hat{k}) \underbrace{\int d\Omega\, Y_{lm} Y^*_{l'm'}}_{\delta_{ll'}\delta_{mm'}}$$

$$\boxed{a_{l'm'} = \int \frac{d^3k}{(2\pi)^3} \Delta(\underset{\sim}{k})\, 4\pi i^{l'} j_{l'}(kL)\, Y_{l'm'}(\underset{\sim}{k})}$$

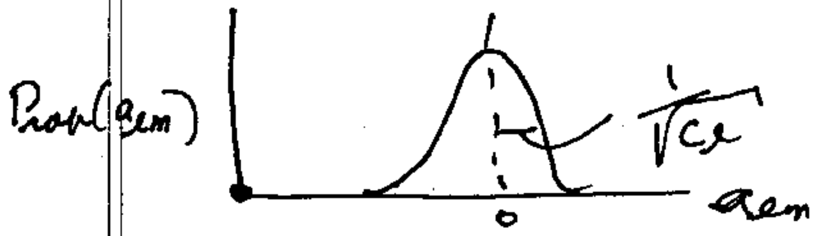
(2) Now consider angular correlation function



$$C(\alpha) = \langle \Delta(\hat{n})\, \Delta(\hat{m}) \rangle$$

$$= \sum{}' \sum{}' \langle a_{lm} a^*_{l'm'} \rangle Y_{lm}(\hat{n})\, Y^*_{l'm'}(\hat{m})$$

As with density perturbations we cannot make predictions about a particular a_{lm}, just about distribution from which they are drawn



Assuming $a_{\ell m}$ are Gaussian distributed with random mean: $\langle a_{\ell m}\rangle = 0$

variance C_ℓ ; $\langle a_{\ell m}\, a^*_{\ell' m'}\rangle = \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$

after squaring $a_{\ell m}\cdot a^*_{\ell' m'}$ one finds

$$\boxed{C_\ell = \frac{2}{\pi}\int dk\cdot k^2\,|\Delta(k)|^2\, j_\ell^2(kL)}$$

From definition of $C(\alpha)$ we have:

$$C(\alpha) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{+\ell} C_\ell\, Y_{\ell m}(\hat{n})\, Y^*_{\ell m}(\hat{m})$$

$$= \sum_{\ell=0}^{\infty} C_\ell \sum_{m=-\ell}^{+\ell} Y_{\ell m}(\hat{n})\, Y^*_{\ell m}(\hat{m})$$

But addition theorem $\Rightarrow \displaystyle\sum_{m=-\ell}^{+\ell} = \frac{P_\ell(\cos\alpha)(2\ell+1)}{4\pi}$

$$\therefore \boxed{C(\alpha) = \sum_{\ell=0}^{\infty} \frac{C_\ell\,(2\ell+1)}{4\pi}\, P_\ell(\cos\alpha)}$$

mean-Square fluctuation:

$$\left(\frac{\delta T}{T}\right)^2_{rms} = \boxed{C(0) = \sum_{\ell=0} \frac{C_\ell\,(2\ell+1)}{4\pi}\,(1)}$$

Quadrupole moment ($\ell=2$ term)

$$\boxed{\left(\frac{\Delta T}{T}\right)^2_Q = \frac{5}{4\pi}C_2}$$

To compute C_ℓ coefficients, let's go to large-scale where SW effect is relevant.

SW: $$\left(\frac{\delta T}{T}\right)_{obs} \equiv \Delta = \frac{1}{3}\delta\Phi(\eta_{LSS})$$

Fourier transform of this equation:

$$\boxed{\Delta(k) = (1/3)\,\delta\Phi(k, \eta_{LSS})}$$

Compute $\delta\Phi(k)$ from Poisson equation:
$\nabla^2(\delta\Phi) = 4\pi G\cdot\delta\rho = 4\pi G\rho\cdot\delta$

$$\left(\frac{a_0}{a}\right)^2\nabla^2(\delta\Phi) = 4\pi G\rho_M(0)\left(\frac{a_0}{a}\right)^3\left(\frac{a}{a_0}\right)\delta(\eta_0)$$

I Assume for simplicity that $\delta(\eta) = \frac{a}{a_0}\delta(\eta_0)$

In that case $\nabla^2\delta\Phi = 4\pi G\,\Omega_m\left(\frac{3H_0^2}{8\pi G}\right)\delta(\eta_0, x)$

$\therefore$ $\delta\Phi(\eta_{LSS}) \simeq \delta\Phi(\eta_0)$

Take the Fourier transform:

$$k^2\,\delta\Phi(k,\eta_0) = \frac{3}{2}\Omega_m H_0^2\,\delta(k,\eta_0)$$

$$\text{or}\quad \delta\Phi(k,\eta_0) = \frac{3\Omega_m H_0^2\,\delta(k,\eta_0)}{2k^2}$$

In that case temperature perturbation

$$\boxed{\Delta(k) = \frac{\Omega_m H_0^2}{2k^2}\,\delta(k,\eta_0)}$$

Since: $C_\ell = \frac{2}{\pi}\int dk\,k^2|\Delta(k)|^2 j_\ell^2(kL)$ we have

Since $\Delta^2(k) = \dfrac{\Omega_m^2 H_0^4}{4k^4}\,|\delta(k,\eta_0)|^2$,

we have

$$C_\ell = \frac{2}{\pi}\left(\frac{\Omega_m^2 H_0^4}{4}\right)\int_0^\infty \frac{dk}{k^2}\, j_\ell^2(kl)\, \overbrace{|\delta(k,\eta_0)|^2}^{P(k)}$$

So by measuring C_ℓ we normalize the power spectrum.

Conventional: Prediction from inflation:

$$P(k) = \frac{2\pi^2 \delta_H^2 k^n}{H_0^{n+3}} \qquad (\text{assuming } \delta \propto a)$$

where δ_H is scale-invariant amplitude at horizon crossing. In that case:

$$C_\ell = \left(\frac{\Omega_m^2 H_0^4}{2\pi}\right)\left(\frac{2\pi^2}{H_0^{n+3}}\right)(\delta_H^2)\int_0^\infty \frac{dk}{k^{2-n}}\, j_\ell^2(kl)$$

In special case $n=1$

$$\boxed{C_\ell = \frac{\pi}{2}\,\frac{\Omega_m^2 \delta_H^2}{\ell(\ell+1)}}$$

Observations. WMAP

Show ~~[illegible]~~ WMAP.

~~[illegible]~~ Quadrupole:

$$C_2 = \frac{4\pi}{5}\left(\frac{\Delta T}{T}\right)^2$$

$$C_2 = \frac{4\pi}{5}\left(\frac{(8.2\pm2)\times10^{-6}}{2.73}\right)^2$$

$$C_2 = 2.27\times10^{-11}$$

~~[illegible]~~ Since $C_2 = \dfrac{\pi}{2\cdot(2)(3)}\Omega_m^2\delta_H^2$

$$\delta_H^2 = \frac{12}{\pi}\,\frac{C_\ell}{\Omega_m^2} = 9.6\times10^{-10}$$

$$\boxed{\delta_H = 3.1\times10^{-5}}$$

Small amplitudes indeed!

Acoustic Oscillations

SW Last lecture I discussed super-horizon density perturbations. I showed how the gravitational potential generated by the density perturbations induced temperature variations in CMB; i.e., the Sachs-Wolfe effect.

AO This lecture I will speak about sub-horizon perturbations; i.e., perturbations with scales less than horizon length at decoupling. I will start out with very simplistic picture and go into details as I go along.

Boltzmann eq. for photons.

Distribution function $f = f(x, p, t)$ for photons

Boltzmann eq. $\frac{df}{dt} = C[f]$: $C[f]$ is collision-rate term.

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^a}\frac{dx^a}{dt} + \frac{\partial f}{\partial p}\frac{dp}{dt}$$

$$\text{Let } f = \exp\left[\frac{pc}{k_B T(t)[1 + \Theta(\underset{\sim}{x}, \hat{p}, t)]}\right]^{-1}$$

$\Theta(x, \hat{p}, t)$: perturbation of temperature depends on

(a) Location

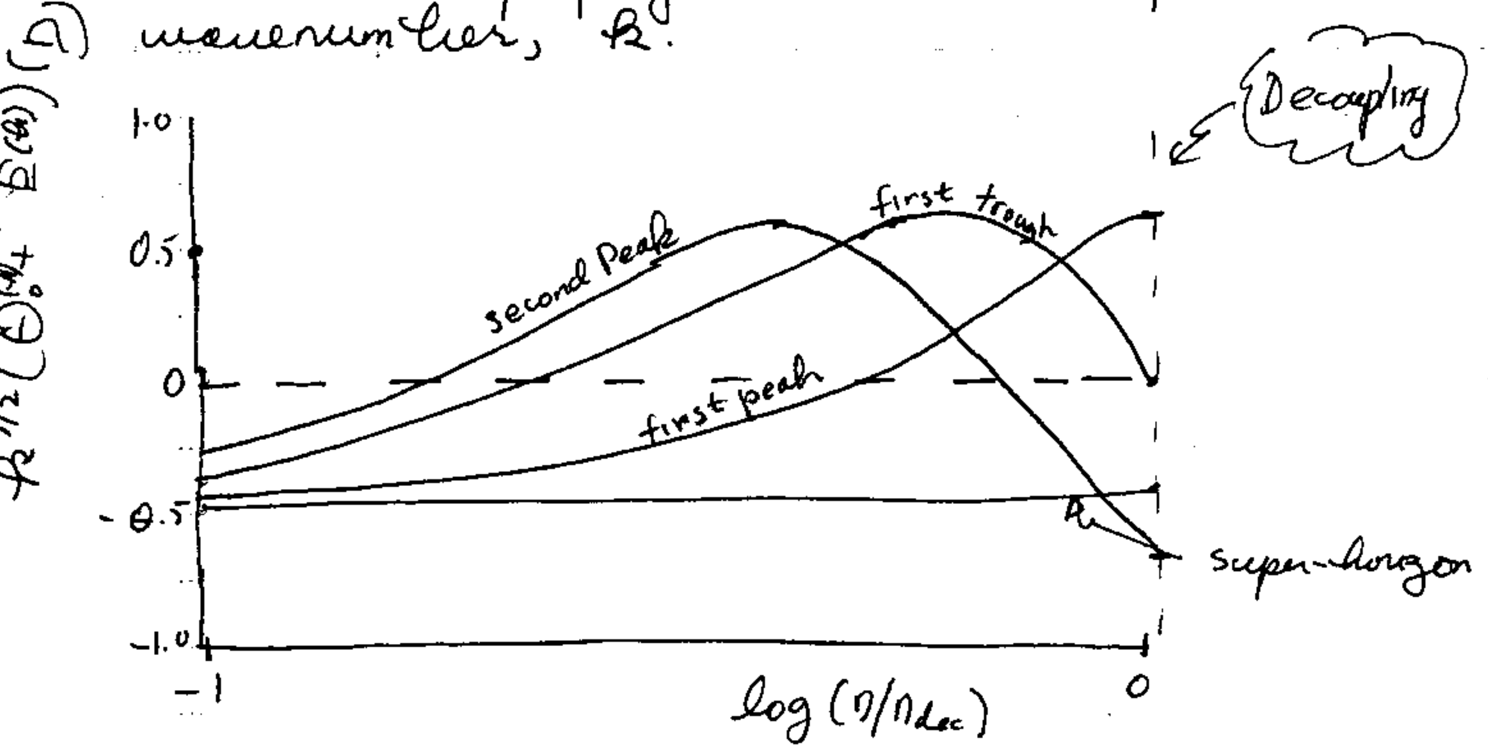
(b) direction in sky (Θ may change with angle)

Monopole: $\Theta_0(\underset{\sim}{x}, t) = \frac{1}{4\pi}\int_{4\pi} d\Omega' \, \Theta(\underset{\sim}{x}, t, \hat{p})$

Let's compare Fourier modes of $\Theta_0 + \delta\Phi$ as they propagate until recombination. We plot $\Theta_0 + \delta\Phi$, not just Θ_0, since $\left(\frac{\delta T}{T}\right)_{obs} = \Theta_0 + \delta\Phi$, as we saw

last lecture: photons undergo gravitational redshifts as they climb out of potential wells of density perturbations: recall $\Theta_0 = -\frac{2}{3}\delta\Phi_i$ for SW effect, where $\delta\Phi_i$ was initial potential fluctuation.

Fourier decompose Θ_0 ~~and~~ $\delta\Phi$ and see how modes propagate. Answer, depends on wavenumber, k.



(1) SH mode: low $k\eta_d \ll 1 \Rightarrow$ mode is outside horizon. We see these modes as they were set down initially

(2) Smaller scale modes: evolve in more complex fashion: they all oscillate after horizon crossing.

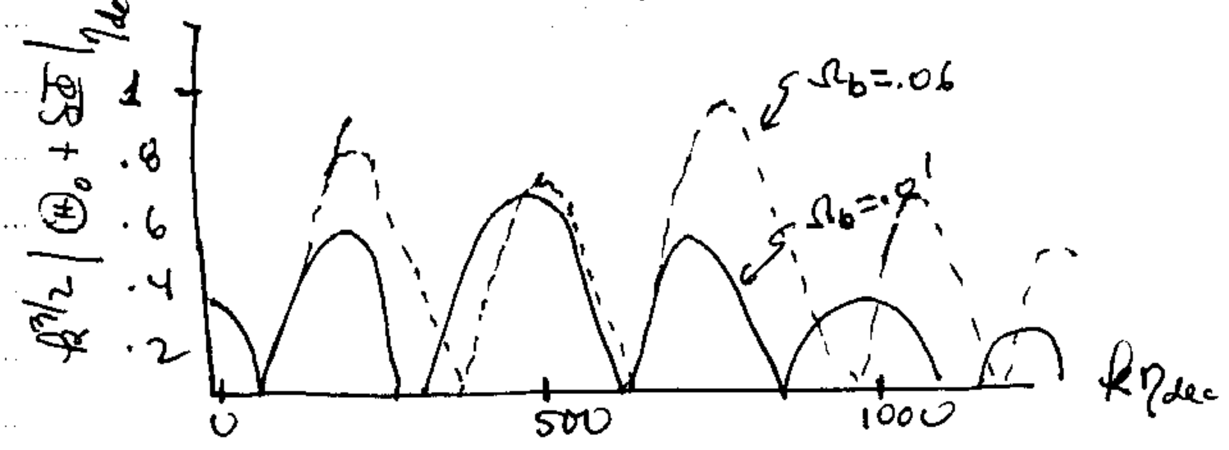
(a) "First Peak": Perturbation grows until it reaches max. at recombination epoch, η_{dec}

~~Thus~~ Anisotropies on scales corresponding to this mode would produce lg. fluctuations. Anisotropy $\delta T/T$ at scales corresponding to the mode which has peaked at recombination.

(b) "First trough": Enters horizon slightly earlier (because of larger k). This mode peaks earlier, then turns over so its amplitude at recombination is zero. By recombination it has undergone half an oscillation: this is first clear signal of acoustic oscillations due to photon pressure. Phase of this mode is such that its amplitude vanishes at recombination. Anisotropies on these scales will exhibit a trough

(c) "Second Peak": Entered horizon even earlier (owing to even larger value of k). Goes through full oscillation by recombination. $|\delta T|$ same as first peak at η_{dec}, so this will lead to a second peak in spectrum of C_ℓ (depend on $|\delta T/T|_k^2$).

(d) Series of peaks:
There should be a never-ending ~~...~~ series of peaks and troughs corresponding to modes entering horizon earlier & earlier.



~~Two features~~

Here we plot absolute value of $k^{3/2}|\delta T/T|$ versus $k \cdot \eta_{dec}$ for perturbations at $\eta = \eta_{rec} = \eta_{LSS}$.

- High Ω_B model: peaks alternate in height. Odd higher than even
- Low Ω_B model: Perturbations on small scales: $k > 500/\eta_{rec}$ are damped (This is due to larger Silk damping length $\lambda_d \propto 1/\Omega_b^{1/2}$

Qualitative Picture

Rough picture: Equations governing perturbation look like:

$$\ddot{\Theta}_0 + k^2 c_s^2 \Theta = F$$

sound speed in photon-Baryon fluid

c_s: sound speed in coupled photon/baryon fluid $\approx \sqrt{P/\rho} = \left(\frac{\partial P}{\partial \rho}\right)^{1/2}_{adiabatic}$

F: gravitational driving force

Description:

Consider simple harmonic oscillator with mass m at end of spring, spring constant k, external (constant) force F_0. Equation of motion is:

$$\ddot{x} + \frac{k}{m} x = \frac{F_0}{m}$$

- External force drives oscillator to lg. x
- Restoring force drives " " $x \to 0$.
- Solution: oscillations around zero pt. at lg. x

Solution to this equation: $x(t) = A\cos(\omega t) + \frac{F_0}{m\omega^2}$

- $\omega^2 = k/m$
- Boundary condition $\dot{x}(0) = 0$: particle initially at rest

Main features of solution shown in next figure

(1) Solid(red) line is unforced soln ($F_0 = 0$)

Natural • particle undergoes symmetric oscillations about $x = 0$

Fig1 (X)

Forced • Dashed curve show solns. for 2 high ~~w~~ ω (dotted curve) and low ω (dot-dashed curve)

• In both cases ~~[illegible]~~ oscillations are not about $x=0$, but about increasing values of x as $F_0/m\omega^2$ increases: Shift more dramatic as $\omega \to 0$

Fig.2 {X^2}

• All three oscillators experience peaks at $\omega t = n\pi$ ($n = 0, 1, 2, 3, \ldots$)

$F_0 = 0$ – Peak heights = for unforced oscillator

$F_0 \neq 0$ – Heights of odd peaks ($\omega t = (2n+1)\pi$) exceed heights of even peaks ($\omega t = 2n\pi$) (effect is more dramatic as $\omega \to 0$)

– Even peaks occur at $x < 0$, which are far from where F_0 "wants to go".

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(1) So simple picture explains odd-even effect seen in peak ~~[illegible]~~ heights

(2) what happens when Ω_B increases. In that case sound speed c_s decreases. Since $\omega^2 = k c_s^2$, ω goes down, which increases forcing term. Higher Ω_B causes peaks to increase.

Realistic Models

To compute evolution of temperature fluctuations, we solve the Boltzmann equation for the propagation of photon distribution through the coupled photon/baryon fluid.

Recall ~~that~~ ~~for~~ perturbations with wavelength within the horizon, pressure disterbances oppose gravity, which results oscillations both in density amplitude δ and temperature Θ_0.

~~For~~ Perturbations that came into the horizon at $t <$ $t_{decoupling}$, will generate osallations that will be present on the last scattering surface. ~~Neglecting the baryon inertia so that~~ $\boxed{c_s \neq c/\sqrt{3}}$ we have the solution for Fourier mode k.

$$\boxed{[\Theta_0 + \delta\Phi](\eta_{LSS}) = [\Theta_0 + \Phi](0)_{k\,\text{initial}} \cos(k s)}$$

(1) Sound horizon: Note s is distance that sound wave propagates through photon/baryon matter. These modes travel from initial time (end of inflation) to decoupling.

$$s = \int_0^{\eta_{LSS}} c_s \, d\eta$$

(2) Comments

(i) $\boxed{\text{Sound Speed}}$ $\quad c_s = c\sqrt{\dfrac{1}{3(1+R)}}$

$$\text{where } R = \cancel{\frac{\rho_b + \rho_\gamma}{\rho_\gamma + \rho_\gamma}} \approx \frac{3}{4}\left(\frac{\rho_b}{\rho_\gamma}\right)$$

Ignore: $R \ll 1$ for now

(b) Apparent Horizon: Unique prediction of inflation: perturbations in potential excited at time when they were outside apparent (FRW) horizon. But they exist in inflation because during or prior to inflation they were inside true horizon. Amplitude of these perturbations grows when they enter horizon

(c) Phases:

Inflation sets temporal phases of all wave modes by starting ~~them~~ them all in phase at $t_{inflation}$. As a result wave numbers which hit their extrema at recombination are given by $k_m s = m\pi$ $(m=0,1,2,\ldots)$. And these mark peaks of coherent oscillations in the power spectrum. This happens everywhere in the Universe at the same time.

(d) Illustration: Rewrite last equation as: ~~Peaks in T fluctuation~~

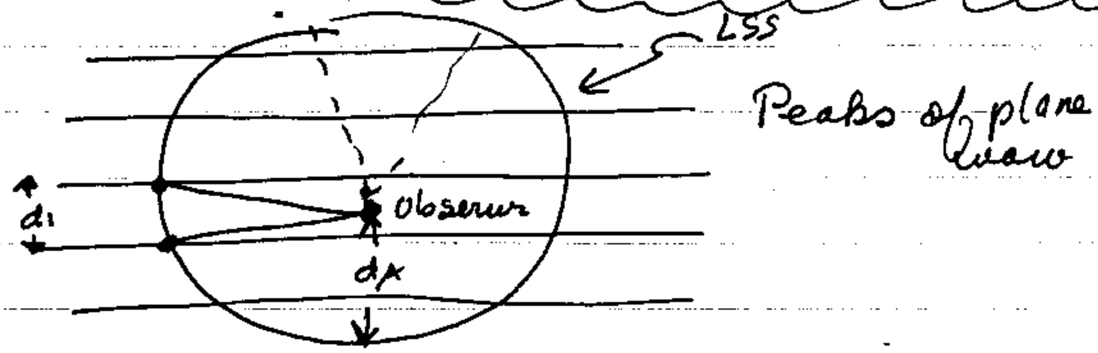
$$(\delta T/T)_{obs} = (\delta T/T)_{\eta_{LSS}} = \frac{\delta\Phi_{initial}}{3}\cos(k_m s)$$

- First peak: $k_1 = \frac{\pi}{s}$ will be a compression since $\delta\Phi_{init} < 0$
- Second peak $k_2 = \frac{2\pi}{s}$: will be a rarefaction and so on.

These peaks and troughs will exist in place everywhere on the last scattering surface and are the reason why sharp peaks in C_ℓ spectrum are present.

(e) Alternative Pictures: Fluctuations generated by causal mechanisms ⇒ random phases peaks get washed out

Crude Picture of inflation scenario



So a plane wave temperature perturbations with peaks separated by distance d_1 generates range of anisotropies corresponding to $\theta \simeq \frac{d_1}{d_A(L)}$ (solid lines) to $\theta \gg \frac{d_1}{d_A}$ (dashed lines).
Result is a sharp maximum around d_1/d_A

(C) ℓ of peaks:

Peaks separated $d_1 = \frac{1}{k_1} = \frac{S}{\pi}$ (linear scale)

$\Theta = d_1/(d_A)$ (angular scale)

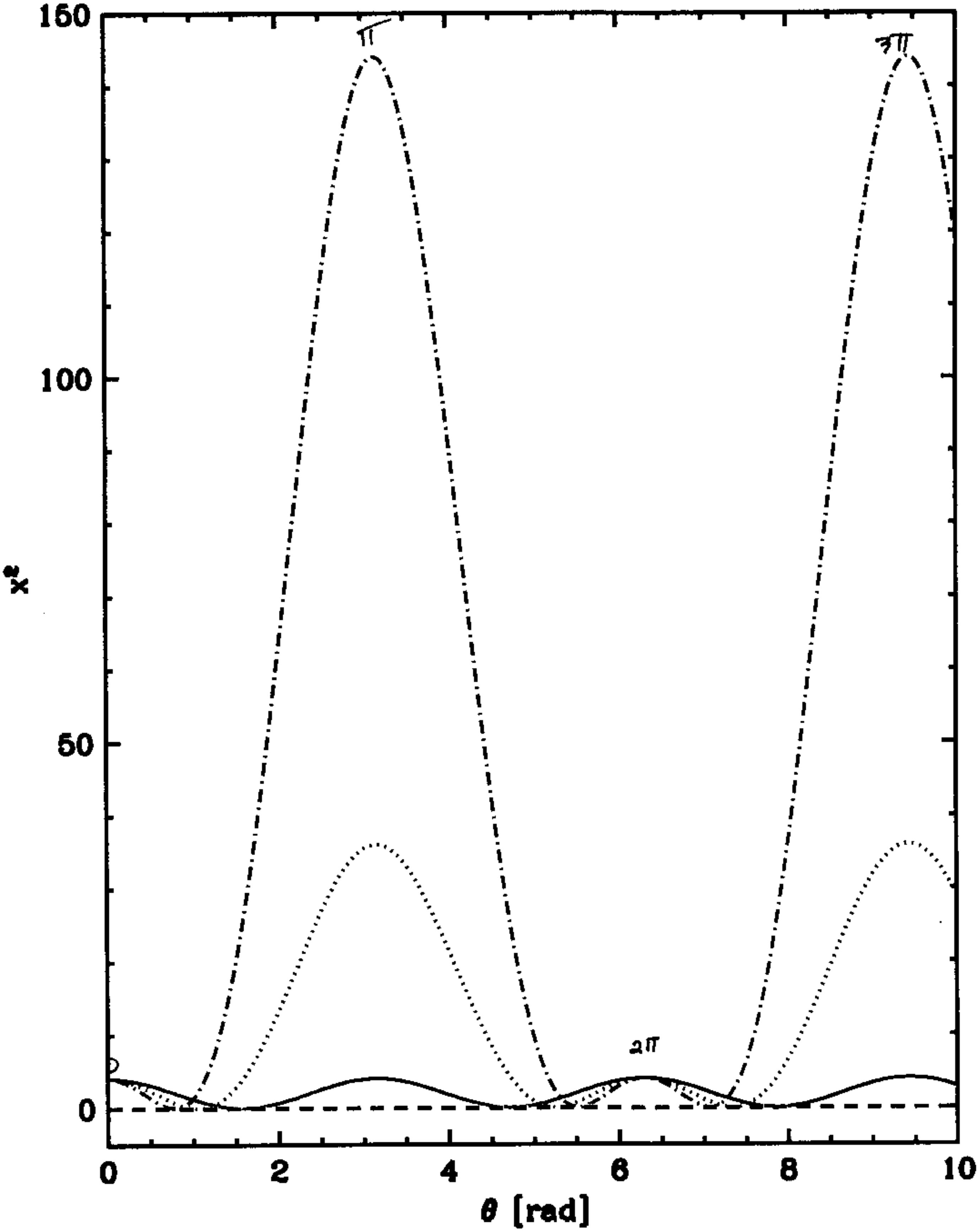
Since $\ell \simeq 1/\Theta \Rightarrow \ell_A = \frac{d_A}{d_1}$

If d_A is proper angular diameter distance, then

$$\ell \simeq \frac{\pi \cdot d_A}{S_{phys}}$$

$$S_{phys} = a(t_{dec}) \int_0^{\eta_{dec}} \frac{c\,dt}{a}$$

Fig. 2



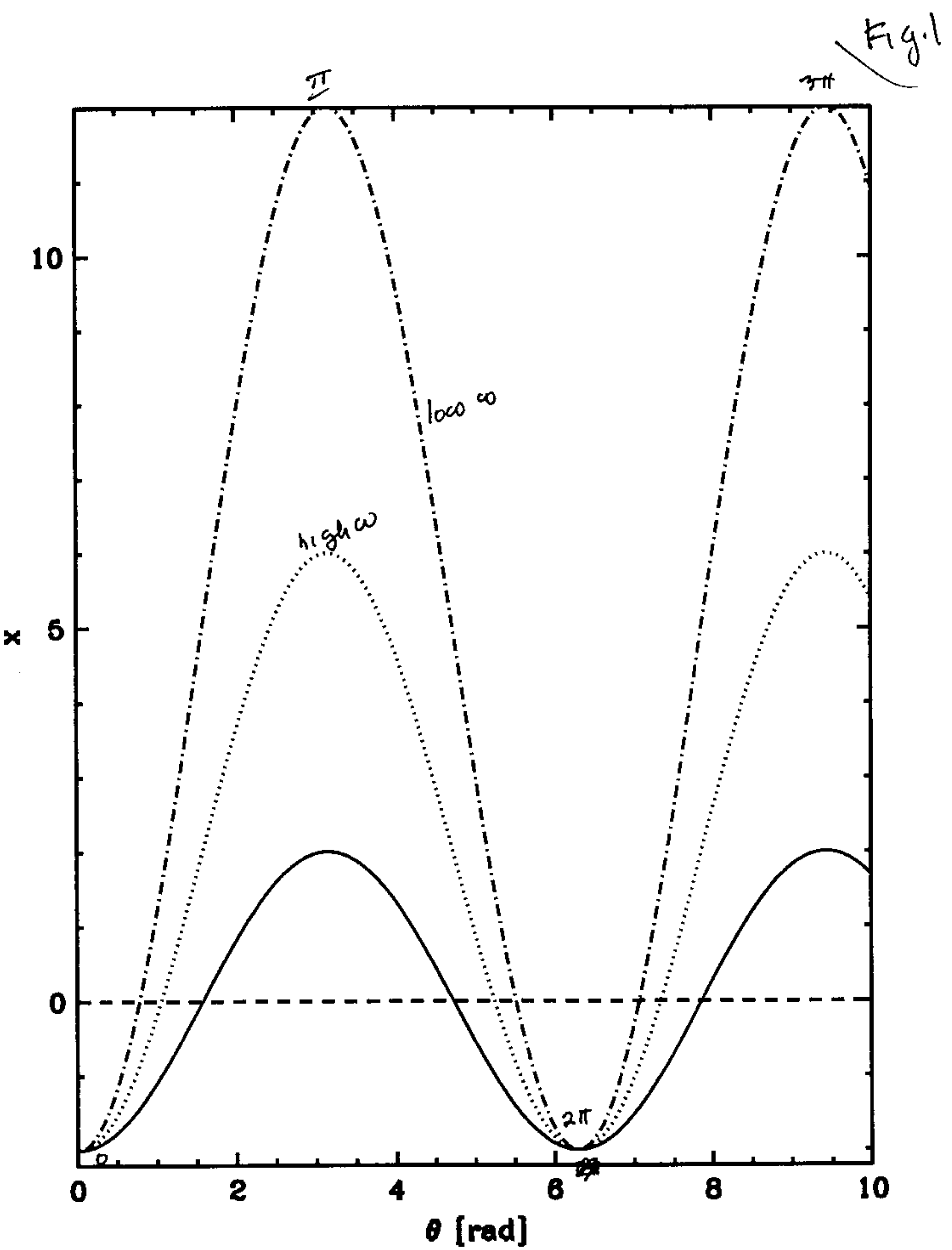


Fig.1
π
3π
10
5
0
x
low ω
high ω
2π
0
2
4
6
8
10
θ [rad]

Prediction:

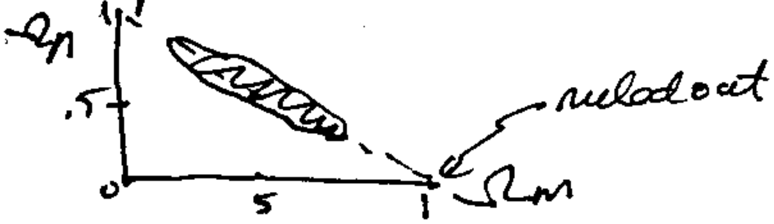
$\boxed{\cancel{\ell_A \approx \dots * \text{ correction terms}}}$

$$s_{phys} = \frac{2\,(c/H_0)}{\sqrt{3R_L\,\Omega_m}\,(1+z_L)^{3/2}} \ln\left(\frac{\sqrt{1+R_L}+\sqrt{R_{eq}+R_L}}{1+\sqrt{R_{eq}}}\right)$$

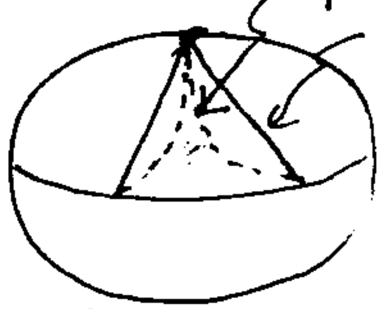
Putting in my expression for d_A I get

$$\ell_A \simeq 280$$

But $(\ell_A)_{obs} \simeq 220$ (other effects come in)
experts claim that $\ell_A \propto \frac{1}{\Omega_{TOT}}$.
this is principal reason
for
Ω_Λ vs Ω_m
degeneracy

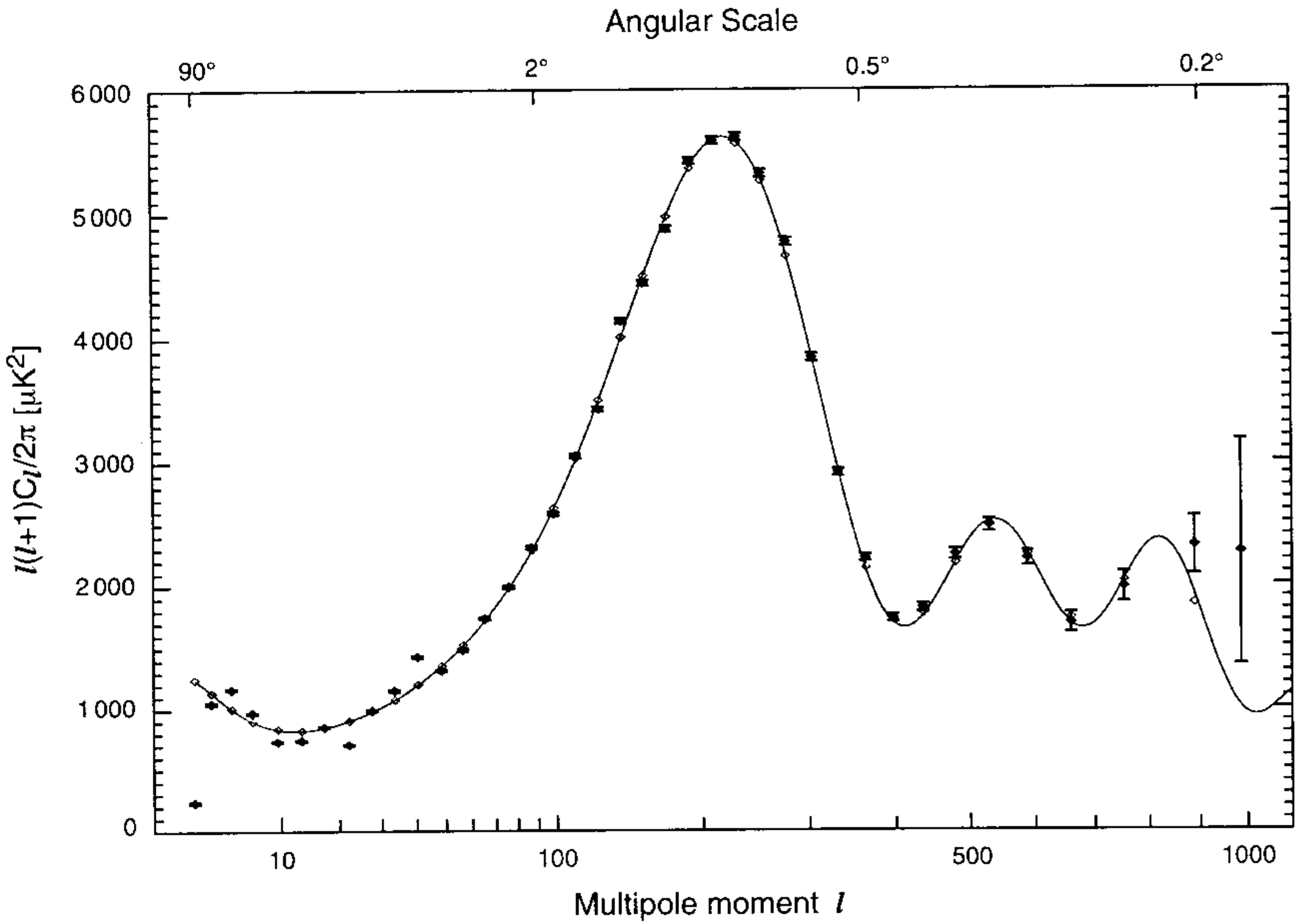


Geometry:



So open models give significantly smaller values for Θ, hence larger values for ℓ_A.
I compute $\ell_A \simeq 500$ for $\Omega_\Lambda = 0$, $\Omega_m = 0.3$
a model that used to be popular.

Peak locations tell us $\Omega_{TOT} = 1.02 \pm .02$

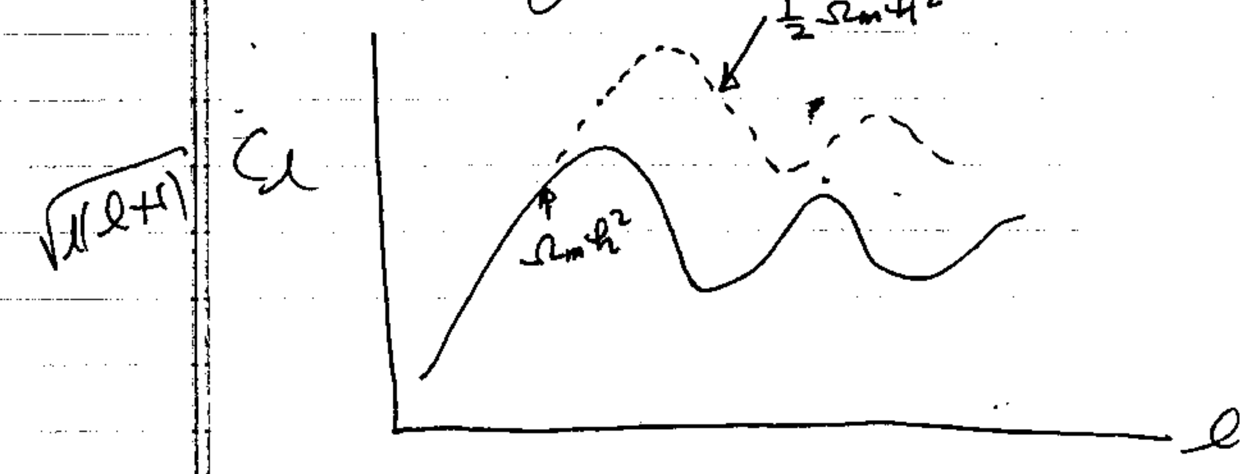

Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi\ [\mu K^2]$
10
100
500
1000
Multipole moment l

Baryons: Before we neglected Baryon inertia. By including baryons in the analysis, we slow down the sound speed since

$$c_s = \frac{c/\sqrt{3}}{\sqrt{1+R}}$$

(a) Baryons increase Peak amplitudes:
Resonant frequency decreases, which raises forcing term: baryons drag fluid deeper into potential well. Result is shift to larger amplitude. Results also in suppressed troughs: These effects result in $\Omega_B h^2 = 0.02$.

(b) Matter Density: Affects height and shape of acoustic peaks:



• $(1+z_{eq}) \propto \Omega_m/\Omega_R$. So decreasing Ω_m decreases redshift of matter=radiation nearer to $z_{decoupling}$. So radiation plays bigger role in computing growth of δ. Stronger driving force $\Rightarrow$ increases $\delta T/T$