PHYSICS 221A : NONLINEAR DYNAMICS
HW ASSIGNMENT #4

(1) Consider front propagation for the modified Fisher equation,

$$u_t = u_{xx} + u\,(1 - u^2) \ .$$

For what velocities is the front solution stable?

(2) Consider a predator-prey type model governed by the equations

$$\begin{aligned} u_t &= D\,u_{xx} - uv \\ v_t &= \lambda D\,v_{xx} + uv \ . \end{aligned}$$

Investigate the possibility of traveling wavefront solutions satisfying the boundary conditions

$$\begin{aligned} u(-\infty,t) &= v(+\infty,t) = 0 \\ v(-\infty,t) &= u(+\infty,t) = K \ , \end{aligned}$$

where K is a positive constant. Note any special cases.

(3) Compute the growth rate η for the Brusselator amplitude within a purely linearized treatment of the problem, in terms of the parameters a, c, and $\varepsilon \equiv b - b_{\mathrm{T}}$. Compare your answer with the result in eqn. 9.67. Do they agree? Why or why not?

There is somewhat of an ambiguity in what we might mean by the growth rate. We could (i) compute η at fixed $Q^2(\varepsilon = 0) = a/\sqrt{D_u D_v}$, which is the critical wavevector at $\varepsilon = 0$, or (ii) allow Q to vary with ε, *i.e.* compute the growth of the maximally unstable wavevector at each value of b. Show that these to calculations yield the same growth rate, to order ε^2.

(4) Consider the real Ginsburg-Landau equation,

$$\psi_t = \mu\psi + \psi_{xx} - |\psi|^2\,\psi \ ,$$

where ψ is a complex field. Static solutions of the form $\psi(x) = \sqrt{\mu - Q^2}\,e^{iQx}$ exist, provided $\mu > 0$ and $Q^2 < \mu$. Investigate the stability of these solutions by writing

$$\psi(x,t) = \sqrt{\mu - Q^2}\ e^{iQx} + \eta(x)\,e^{\lambda t}$$

and solving the resulting eigenvalue equation. The resulting instability is the Eckhaus instability.