## PHYSICS 210A, HOMEWORK ASSIGNMENT #4

May 21, 2009

1. Solve Problem 8.2 of the text; here, you are supposed to show that the ratio  $T_0 / T_F$  is about 0.989.

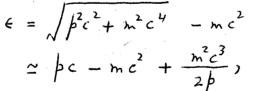
**2.** Solve Problem 8.13 of the text.

[Hint:- Since the answers to this problem are in terms of the single-particle density of states  $g(\varepsilon)$ , keep your integrals for N and U over  $\varepsilon$  --- and <u>do not</u> employ the phase-space approach we normally resort to.

Secondly, to evaluate these integrals, you need to use *only* the first two terms of the formula (E.16); however, in doing so, you'll have to further approximate the first term of this formula (for, in it, the upper limit on the integral is  $\mu / kT$ , not  $\varepsilon_F / kT$ .]

3. In the class, I presented a simplified treatment for deriving a formula for the magnetic susceptibility of a gas of spin-half fermions valid at *all* temperatures --- the formula that appears in Problem 8.15 as well. Generalize that treatment to a gas of fermions carrying a magnetic moment  $\mu^*$ , with spin  $\mathcal{F}_n$  (where J = 1/2, 3/2, 5/2, ----).

**4.** Re-visit the problem of the "statistical equilibrium of white dwarf stars", for which it is essential that the motion of the electrons be treated as *relativistic*. Using the approximation



determine the pressure of the electron gas and thereby the mass-radius relationship for such stars --- thus establishing the existence of a limiting mass  $M_0$  that signals the gravitational collapse of the star.

In determining  $M_0$ , do not worry about *exact* numerical factors --- just focus on the main characters of the play!

5. In the class, I discussed an Ising-like model in which the spin quantum number of atoms on the various lattice sites is s (instead of  $\frac{1}{2}$ ), with g = 2s+1 (instead of 2). Using mean-field approach, I showed that this system undergoes a phase transition at a critical temperature  $T_c$ , given by

$$T_c = \{(s+1)/3s\} qJ/k$$
.

Now, you are supposed to show that, for  $T \leq T_c$ , the order parameter  $m_0$  is given by

$$m_0 \simeq \frac{\sqrt{10} (s+1)}{\sqrt{3(2s^2+2s+1)}} \left(1 - \frac{T}{T_c}\right)^{1/2}$$

6. A collection of spin- $\frac{1}{2}$  paricles is adsorbed on a surface that has N adsorption sites. For each site, let  $\sigma$  be +1 if the site is occupied by a particle with spin up, -1 if it is occupied by a particle with spin down, 0 if it is unoccupied. The Hamiltonian of the system, in obvious notation, is

$$\mathcal{H} = -W \sum_{i} \sigma_{i}^{2} - \mu^{*} H \sum_{i} \sigma_{i} - J \sum_{n.n.} \sigma_{i} \sigma_{j},$$

where W is the binding energy of an adsorbed atom --- regardless of whether its spin is up or down.

(a) Let  $p_+$  be the probability that a site is occupied by an atom with spin up,  $p_-$  the probability that it is occupied by a particle with spin down and  $p_0$  the probability that it is vacant. Let f be the "adsorbate density" and m the "magnetisation density" of the lattice, i.e.,  $f = p_+ + p_-$  (so f lies between 0 and 1) and  $m = p_+ - p_-$  (so m lies between -f and +f). Using these probabilities, evaluate the entropy S of the system in terms of the parameters f and m.

Next, using mean-field approximation, write down an expression for the energy U of the system and then for the Helmholtz free energy A --- again as a function of f and m.

(b) Now, with *m* fixed, minimize *A* with respect to *f* to determine the optimal value of *f*. [which you may call  $f^*(m)$ ]. What are the special values of  $f^*(m)$  when m = 0 and when m = 1? And what are its limiting values as *T* goes to zero and as *T* goes to infinity?

(c) Finally, with f fixed, minimize A with respect to m to determine the optimal value of m [which you may call  $m^*(f)$ ]. Based on this result, establish a criterion that determines the critical temperature  $T_c$  at which a phase transition occurs in this system.