PHYSICS 4C: QUIZ 1 SOLUTIONS

Problem 1

Since the distance $r = \frac{0.5*\sqrt{2}}{2}$ meters from the center charge to each of the corner charges is the same,

$$\begin{aligned}
\mathbf{F} &= \frac{kQ\Sigma q_i \hat{r}_i}{r^2} \\
&= \frac{kQ}{r^2}(q_1\frac{\hat{x}-\hat{y}}{\sqrt{2}} + q_2\frac{-\hat{x}-\hat{y}}{\sqrt{2}} + q_3\frac{-\hat{x}+\hat{y}}{\sqrt{2}} + q_4\frac{\hat{x}+\hat{y}}{\sqrt{2}}) \\
&= \frac{(8.99\times 10^9 \text{N m}^2\text{C}^{-2})(3.2\times 10^{-6}\text{C})}{(\frac{0.5\text{m}\times\sqrt{2}}{2})^2} \\
&\quad \times(2.3\times 10^{-6}\text{C}\frac{\hat{x}-\hat{y}}{\sqrt{2}} - 7.1\times 10^{-6}\text{C}\frac{-\hat{x}-\hat{y}}{\sqrt{2}} \\
&\quad -0.7\times 10^{-6}\text{C}\frac{-\hat{x}+\hat{y}}{\sqrt{2}} - 5.6\times 10^{-6}\text{C}\frac{\hat{x}+\hat{y}}{\sqrt{2}}) \\
&= 2.30\times 10^5\text{N C}^{-1}\frac{1\times 10^{-6}\text{C}}{\sqrt{2}}(4.5\hat{x} - 1.5\hat{y}) \\
&= 0.244(3\hat{x}-\hat{y})\text{N},
\end{aligned}$$

the magnitude of which is

$$F = 0.244\sqrt{3^2 + (-1)^2} = 0.77\text{N},$$

and the direction is $3\hat{x} - \hat{y}$, which corresponds to 18.4° below the positive x-axis.

Problem 2

The ring can be represented as a superposition of two rings, one (a) with uniform charge density λ and another (b) with charge density 2λ on the top half and -2λ on the bottom half. Consider the electric field on the x-axis. By symmetry, ring (a) will not cause an electric field in the $\hat{y}$ or $\hat{z}$ directions. We are left with

$$E_{x,a} = \frac{kQx}{(x^2+a^2)^{3/2}} = \frac{2\pi a\lambda kx}{(x^2+a^2)^{3/2}}.$$

The second ring (b) will produce no E_x or E_z along x-axis by symmetry. E_y will be downward, pointing from the positive charge on the top to the negative charge on the bottom of the ring. Adding the two fields, we get

$$E_x = \frac{2\pi a\lambda kx}{(x^2+a^2)^{3/2}}$$

along x-axis; E_y is negative and E_z is 0 everywhere on the x-axis.

Alternative solution. Since a ring of uniform charge is symmetric around x-axis, every bit of the ring contributes the same amount to E_x along the x-axis. This means E_x of a half-ring is the same as E_x of a ring as long as they have the same total charge. Thus, we can calculate the two fields for the two half-rings of charges $Q_1 = 3\pi a\lambda$ and $Q_2 = -\pi a\lambda$, and add them. This is *not* true for E_y or E_z along the x-axis (both are 0 for a uniformly charged ring, but not necessarily in this case) or any electric field off x-axis. In short, be careful when using a formula derived for a symmetric object by dividing the object into asymmetric components.

$$\begin{aligned} E_x &= \frac{kQ_1x}{(x^2+a^2)^{3/2}} + \frac{kQ_2x}{(x^2+a^2)^{3/2}} \\ &= \frac{3\pi a\lambda kx}{(x^2+a^2)^{3/2}} + \frac{-\pi a\lambda kx}{(x^2+a^2)^{3/2}} \\ &= \frac{2\pi a\lambda kx}{(x^2+a^2)^{3/2}}. \end{aligned}$$

Field lines start at the positively charged half of the ring and end on the negatively charged half, so E_y is down, and $E_z = 0$ by symmetry.

Problem 3

Since electrostatic force on the particle is only in the y-direction, the particle's velocity in the x-direction is constant, $v_x(t) = v_0$. This means the particle takes the time

$$t = x/v_0 = (0.04\text{m})/(1500\text{m/s}) = 2.67 \times 10^{-5}\text{s}$$

to clear the capacitor. Since the particle enters halfway between the plates, its maximum vertical displacement is $\Delta y = 2\text{mm}$ during that time. The y-force on the particle is

$$F_y = qE = ma_y,$$

so its y-acceleration is

$$a_y = \frac{qE}{m}.$$

From classical mechanics,

$$\Delta y = \frac{a_y t^2}{2} = \frac{qEt^2}{2m}.$$

From this, maximum value of the electric field is

$$E = \frac{2m\Delta y}{qt^2} = \frac{2(6.6\times10^{-27}\text{kg})(0.002\text{m})}{(3.2\times10^{-19}\text{C})(2.67\times10^{-5}\text{s})^2} = 0.12\text{N/C}.$$