Ch. 22: 4, 9, 10, 19, 20, 21, 24, 29, 31, 34, 38

4. (*a*) From the diagram in the textbook, we see that the flux outward through the hemispherical surface is the same as the flux inward through the circular surface base of the hemisphere. On that surface all of the flux is perpendicular to the surface. Or, we say that on the circular base, $\vec{\mathbf{E}} \parallel \vec{\mathbf{A}}$. Thus $\Phi_{\mathrm{E}} = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = \boxed{\pi r^2 E}$.

   (b) $\vec{\mathbf{E}}$ is perpendicular to the axis, then every field line would both enter through the hemispherical surface and leave through the hemispherical surface, and so $\Phi_{\mathrm{E}} = \boxed{0}$.

9. The only contributions to the flux are from the faces perpendicular to the electric field. Over each of these two surfaces, the magnitude of the field is constant, so the flux is just $\vec{\mathbf{E}} \cdot \vec{\mathbf{A}}$ on each of these two surfaces.

$$\Phi_{\mathrm{E}} = \left(\vec{\mathbf{E}} \cdot \vec{\mathbf{A}}\right)_{\text{right}} + \left(\vec{\mathbf{E}} \cdot \vec{\mathbf{A}}\right)_{\text{left}} = E_{\text{right}} l^{2} - E_{\text{left}} l^{2} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$Q_{\text{encl}} = \left(E_{\text{right}} - E_{\text{left}}\right) l^{2} \varepsilon_0 = \left(410\,\mathrm{N/C} - 560\,\mathrm{N/C}\right)\left(25\,\mathrm{m}\right)^2 \left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right) = \boxed{-8.3\times10^{-7}\,\mathrm{C}}$$

10. Because of the symmetry of the problem one sixth of the total flux will pass through each face.

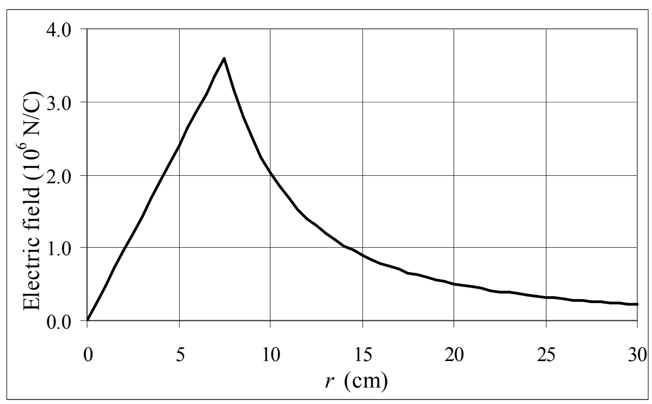
$$\Phi_{\text{face}} = \tfrac{1}{6}\Phi_{\text{total}} = \tfrac{1}{6}\frac{Q_{\text{encl}}}{\varepsilon_0} = \boxed{\frac{Q_{\text{encl}}}{6\varepsilon_0}}$$

$\boxed{19.}$ For points inside the nonconducting spheres, the electric field will be determined by the charge inside the spherical surface of radius *r*.

$$Q_{\text{encl}} = Q\left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3}\right) = Q\left(\frac{r}{r_0}\right)^3$$

The electric field for $r \le r_0$ can be calculated from Gauss's law.

$$E(r \le r_0) = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = Q\left(\frac{r}{r_0}\right)^3 \frac{1}{4\pi\varepsilon_0 r^2} = \left(\frac{Q}{4\pi\varepsilon_0 r_0^3}\right) r$$



The electric field outside the sphere is calculated from Gauss's law with $Q_{\text{encl}} = Q$.

$$E\left(r \ge r_0\right) = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.19."

20. (*a*) When close to the sheet, we approximate it as an infinite sheet, and use the result of Example
22-7. We assume the charge is over both surfaces of the aluminum.

$$E=\frac{\sigma}{2\varepsilon_o}=\frac{\dfrac{275\times10^{-9}\,\mathrm{C}}{(0.25\,\mathrm{m})^2}}{2\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}=\boxed{2.5\times10^5\,\mathrm{N/C},\text{ away from the sheet}}$$

(*b*) When far from the sheet, we approximate it as a point charge.

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}=\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{275\times10^{-9}\,\mathrm{C}}{(15\,\mathrm{m})^2}=\boxed{11\,\mathrm{N/C},\text{ away from the sheet}}$$

21. (*a*) Consider a spherical gaussian surface at a radius of 3.00 cm. It encloses all of the charge.

$$\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=E\left(4\pi r^2\right)=\frac{Q}{\varepsilon_0}\quad\rightarrow$$

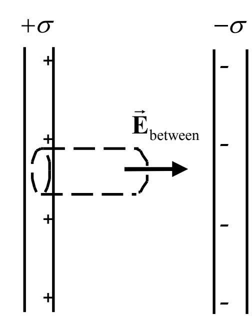
$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}=\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{5.50\times10^{-6}\,\mathrm{C}}{\left(3.00\times10^{-2}\,\mathrm{m}\right)^2}=\boxed{5.49\times10^7\,\mathrm{N/C},\text{ radially outward}}$$

(*b*) A radius of 6.00 cm is inside the conducting material, and so the field must be $\boxed{0}$. Note that
there must be an induced charge of $-5.50\times10^{-6}\,\mathrm{C}$ on the surface at $r = 4.50$ cm, and then an induced charge of $5.50\times10^{-6}\,\mathrm{C}$ on the outer surface of the sphere.

(*c*) Consider a spherical gaussian surface at a radius of 3.00 cm. It encloses all of the charge.

24. Since the charges are of opposite sign, and since the charges are free to move since they are on conductors, the charges will attract each other and move to the inside or facing edges of the plates. There will be no charge on the outside edges of the plates. And there cannot be charge in the plates themselves, since they are conductors. All of the charge must reside on surfaces. Due to the symmetry of the problem, all field lines must be perpendicular to the plates, as discussed in Example 22-7.

(*a*) To find the field between the plates, we choose a gaussian cylinder, perpendicular to the plates, with area A for the ends of the cylinder. We place one end inside the left plate (where the field must be zero), and the other end between the plates. No flux passes through the curved surface of the cylinder.

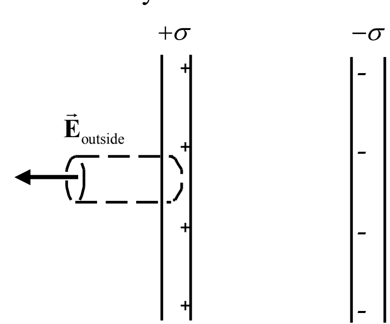


$$\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=\int_{\text{ends}}\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}+\int_{\text{side}}\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=\int_{\substack{\text{right}\\\text{end}}}\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=\frac{Q_{\text{encl}}}{\varepsilon_0}\quad\rightarrow$$

$$E_{\text{between}}A=\frac{\sigma A}{\varepsilon_0}\quad\rightarrow\quad\boxed{E_{\text{between}}=\frac{\sigma}{\varepsilon_0}}$$

The field lines between the plates leave the inside surface of the left plate, and terminate on the inside surface of the right plate. A similar derivation could have been done with the right end of the cylinder inside of the right plate, and the left end of the cylinder in the space between the plates.

(*b*) If we now put the cylinder from above so that the right end is



inside the conducting material, and the left end is to the left of the left plate, the only possible location for flux is through the left end of the cylinder. Note that there is NO charge enclosed by the Gaussian cylinder.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\ \text{end}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$E_{\text{outside}} A = \frac{0}{\varepsilon_0} \rightarrow \boxed{E_{\text{outside}} = \frac{0}{\varepsilon_0}}$$

(c) If the two plates were nonconductors, the results would not change. The charge would be distributed over the two plates in a different fashion, and the field inside of the plates would not be zero, but the charge in the empty regions of space would be the same as when the plates are conductors.

29. Due to the spherical symmetry of the problem, Gauss's law using a sphere of radius r leads to the following.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$$

(a) For the region $0 < r < r_1$, the enclosed charge is 0.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \boxed{0}$$

(b) For the region $r_1 < r < r_0$, the enclosed charge is the product of the volume charge density times the volume of charged material enclosed. The charge density is given by

$$\rho = \frac{Q}{\frac{4}{3}\pi r_0^3 - \frac{4}{3}\pi r_1^3} = \frac{3Q}{4\pi\left(r_0^3 - r_1^3\right)}.$$

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{\rho V_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{\rho\left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3\right]}{4\pi\varepsilon_0 r^2} = \frac{\frac{3Q}{4\pi\left(r_0^3 - r_1^3\right)}\left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3\right]}{4\pi\varepsilon_0 r^2} = \boxed{\frac{Q}{4\pi\varepsilon_0 r^2}\frac{\left(r^3 - r_1^3\right)}{\left(r_0^3 - r_1^3\right)}}$$

(c) For the region $r > r_0$, the enclosed charge is the total charge, Q.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$\boxed{31.}$ (a) Create a gaussian surface that just encloses the inner surface of the spherical shell. Since the

electric field inside a conductor must be zero, Gauss's law requires that the enclosed charge be zero. The enclosed charge is the sum of the charge at the center and charge on the inner surface of the conductor.

$$Q_{\text{enc}} = q + Q_{\text{inner}} = 0$$

Therefore $Q_{\text{inner}} = \boxed{-q}$.

(b) The total charge on the conductor is the sum of the charges on the inner and outer surfaces.

$$Q = Q_{\text{outer}} + Q_{\text{inner}} \quad \to \quad Q_{\text{outer}} = Q - Q_{\text{inner}} = \boxed{Q + q}$$

(c) A gaussian surface of radius $r < r_1$ only encloses the center charge, q. The electric field will

therefore be the field of the single charge.

$$\boxed{E(r < r_1) = \frac{q}{4\pi\varepsilon_0 r^2}}$$

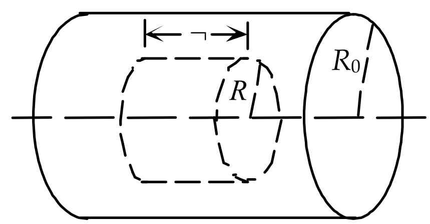
(d) A gaussian surface of radius $r_1 < r < r_0$ is inside the conductor so $\boxed{E = 0}$.

(e) A gaussian surface of radius $r > r_0$ encloses the total charge $q + Q$. The electric field will then

be the field from the sum of the two charges.

$$\boxed{E(r > r_0) = \frac{q + Q}{4\pi\varepsilon_0 r^2}}$$

34. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho_E V_{\text{encl}}}{\varepsilon_0} \quad \to \quad E = \frac{\rho_E V_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(a) For $R > R_0$, the enclosed volume of the shell is

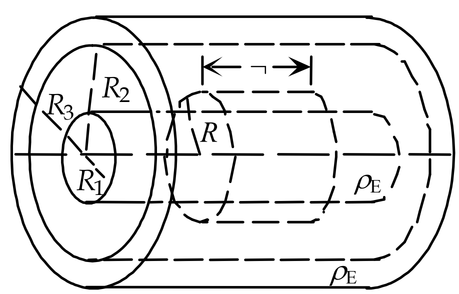
$V_{\text{encl}} = \pi R_0^2 \ell$.

$$E = \frac{\rho_E V_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R_0^2}{2\varepsilon_0 R}, \text{ radially outward}}$$

(b) For $R < R_0$, the enclosed volume of the shell is $V_{\text{encl}} = \pi R^2 \ell$.

$$E = \frac{\rho_E V_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R}{2\varepsilon_0}, \text{ radially outward}}$$

38. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \to \quad E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(a) For $0 < R < R_1$, the enclosed charge is the volume of

charge enclosed, times the charge density.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R^2 \ell}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R}{2\varepsilon_0}}$$

(b) For $R_1 < R < R_2$, the enclosed charge is all of the charge on the inner cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R \ell} = \frac{\rho_{\text{E}}\pi R_1^2 \ell}{2\pi\varepsilon_0 R \ell} = \boxed{\frac{\rho_{\text{E}} R_1^2}{2\varepsilon_0 R}}$$

(c) For $R_2 < R < R_3$, the enclosed charge is all of the charge on the inner cylinder, and the part of
the charge on the shell that is enclosed by the gaussian cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R \ell} = \frac{\rho_{\text{E}}\pi R_1^2 \ell + \rho_{\text{E}}\left(\pi R^2 \ell - \pi R_2^2 \ell\right)}{2\pi\varepsilon_0 R \ell} = \boxed{\frac{\rho_{\text{E}}\left(R_1^2 + R^2 - R_2^2\right)}{2\varepsilon_0 R}}$$

(d) For $R > R_3$, the enclosed charge is all of the charge on both the inner cylinder and the shell.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R \ell} = \frac{\rho_{\text{E}}\pi R_1^2 \ell + \rho_{\text{E}}\left(\pi R_3^2 \ell - \pi R_2^2 \ell\right)}{2\pi\varepsilon_0 R \ell} = \boxed{\frac{\rho_{\text{E}}\left(R_1^2 + R_3^2 - R_2^2\right)}{2\varepsilon_0 R}}$$

(e) See the graph. The spreadsheet used for
this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.38e."