Homework for the week of October 27. 5th week of classes.
Ch. 25: 12, 23, 32, 36, 39, 45, 49, 57
Ch. 26: 12, 17, 19, 34, 38, 40, 46, 50

Ch. 25

12. Use Eq. 25-3 to calculate the resistance, with the area as $A = \pi r^2 = \pi d^2/4$.

$$R = \rho \frac{l}{A} = \rho \frac{4l}{\pi d^2} = \left(1.68\times10^{-8}\,\Omega\cdot\text{m}\right)\frac{4(4.5\,\text{m})}{\pi\left(1.5\times10^{-3}\,\text{m}\right)^2} = \boxed{4.3\times10^{-2}\,\Omega}$$

23. The original resistance is $R_0 = V/I_0$, and the high temperature resistance is $R = V/I$, where the two voltages are the same. The two resistances are related by Eq. 25-5, multiplied by l/A so that it expresses resistance instead of resistivity.

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right] \rightarrow T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0}-1\right) = T_0 + \frac{1}{\alpha}\left(\frac{V/I}{V/I_0}-1\right) = T_0 + \frac{1}{\alpha}\left(\frac{I_0}{I}-1\right)$$

$$= 20.0^\circ\text{C} + \frac{1}{0.00429\left(\text{C}^\circ\right)^{-1}}\left(\frac{0.4212\,\text{A}}{0.3818\,\text{A}}-1\right) = \boxed{44.1^\circ\text{C}}$$

32. Use Eq. 25-7b to find the resistance from the voltage and the power.

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(240\,\text{V})^2}{3300\,\text{W}} = \boxed{17\,\Omega}$$

36. (*a*) Since $P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P}$ says that the resistance is inversely proportional to the power for a constant voltage, we predict that the 850 W setting has the higher resistance.

(*b*) $$R = \frac{V^2}{P} = \frac{(120\,\text{V})^2}{850\,\text{W}} = \boxed{17\,\Omega}$$

(*c*) $$R = \frac{V^2}{P} = \frac{(120\,\text{V})^2}{1250\,\text{W}} = \boxed{12\,\Omega}$$

39. To find the kWh of energy, multiply the kilowatts of power consumption by the number of hours in operation.

$$\text{Energy} = P(\text{in kW})\,t(\text{in h}) = (550\,\text{W})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(6.0\,\text{min})\left(\frac{1\,\text{h}}{60\,\text{min}}\right) = \boxed{0.055\,\text{kWh}}$$

To find the cost of the energy used in a month, multiply times 4 days per week of usage, times 4 weeks per month, times the cost per kWh.

$$\text{Cost} = \left(0.055\frac{\text{kWh}}{\text{d}}\right)\left(\frac{4\,\text{d}}{1\,\text{week}}\right)\left(\frac{4\,\text{week}}{1\,\text{month}}\right)\left(\frac{9.0\,\text{cents}}{\text{kWh}}\right) = \boxed{7.9\,\text{cents/month}}$$

45. Find the current used to deliver the power in each case, and then find the power dissipated in the
resistance at the given current.

$$P = IV \quad \rightarrow \quad I = \frac{P}{V} \qquad P_{\text{dissipated}} = I^2 R = \frac{P^2}{V^2} R$$

$$P_{\substack{\text{dissipated} \\ 12{,}000\,\text{V}}} = \frac{\left(7.5\times10^5\,\text{W}\right)^2}{\left(1.2\times10^4\,\text{V}\right)^2}\left(3.0\,\Omega\right) = 11719\,\text{W}$$

$$P_{\substack{\text{dissipated} \\ 50{,}000\,\text{V}}} = \frac{\left(7.5\times10^5\,\text{W}\right)^2}{\left(5\times10^4\,\text{V}\right)^2}\left(3.0\,\Omega\right) = 675\,\text{W} \qquad \text{difference} = 11719\,\text{W} - 675\,\text{W} = \boxed{1.1\times10^4\,\text{W}}$$

49. Use Ohm's law and the relationship between peak and rms values.

$$I_{\text{peak}} = \sqrt{2} I_{\text{rms}} = \sqrt{2}\frac{V_{\text{rms}}}{R} = \sqrt{2}\frac{220\,\text{V}}{2700\,\Omega} = \boxed{0.12\,\text{A}}$$

57. (*a*) We follow the derivation in Example 25-14. Start with Eq. 25-14, in absolute value.

$$j = nev_{\text{d}} \quad \rightarrow \quad v_{\text{d}} = \frac{j}{ne} = \frac{I}{neA} = \frac{I}{\left(\frac{N\left(1\text{ mole}\right)}{m\left(1\text{ mole}\right)}\rho_{\text{D}}\right)e\left[\pi\left(\tfrac{1}{2}d\right)^2\right]} = \frac{4Im}{N\rho_{\text{D}}e\pi d^2}$$

$$v_{\text{d}} = \frac{4\left(2.3\times10^{-6}\,\text{A}\right)\left(63.5\times10^{-3}\,\text{kg}\right)}{\left(6.02\times10^{23}\right)\left(8.9\times10^3\,\text{kg/m}^3\right)\left(1.60\times10^{-19}\,\text{C}\right)\pi\left(0.65\times10^{-3}\,\text{m}\right)^2} = \boxed{5.1\times10^{-10}\,\text{m/s}}$$

(*b*) Calculate the current density from Eq. 25-11.

$$j = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{4I}{\pi d^2} = \frac{4\left(2.3\times10^{-6}\,\text{A}\right)}{\pi\left(6.5\times10^{-4}\,\text{m}\right)^2} = 6.931\,\text{A/m}^2 \approx \boxed{6.9\,\text{A/m}^2}$$

(*c*) The electric field is calculated from Eq. 25-17.

Ch. 26

12. (*a*) Each bulb should get one-eighth of the total voltage, but let us prove that instead of assuming it.

Since the bulbs are identical, the net resistance is $R_{eq} = 8R$. The current flowing through the

bulbs is then $V_{tot} = IR_{eq} \rightarrow I = \frac{V_{tot}}{R_{eq}} = \frac{V_{tot}}{8R}$. The voltage across one bulb is found from

Ohm's law.

$$V = IR = \frac{V_{tot}}{8R}R = \frac{V_{tot}}{8} = \frac{110\,\mathrm{V}}{8} = 13.75\,\mathrm{V} \approx \boxed{14\,\mathrm{V}}$$

(*b*) $$I = \frac{V_{tot}}{8R} \rightarrow R = \frac{V_{tot}}{8I} = \frac{110\,\mathrm{V}}{8(0.42\,\mathrm{A})} = 32.74\,\Omega \approx \boxed{33\,\Omega}$$

$$P = I^2R = (0.42\,\mathrm{A})^2(32.74\,\Omega) = 5.775\,\mathrm{W} \approx \boxed{5.8\,\mathrm{W}}$$

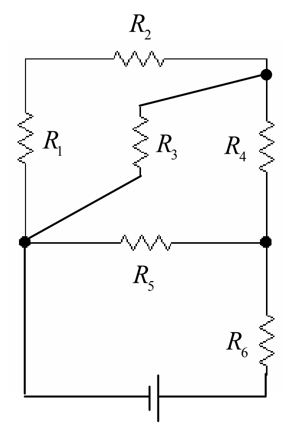
17. The resistance of each bulb can be found by using Eq. 25-7b, $P = V^2/R$. The two individual

resistances are combined in parallel. We label the bulbs by their wattage.

$$P = V^2/R \rightarrow \frac{1}{R} = \frac{P}{V^2}$$

$$R_{eq} = \left(\frac{1}{R_{75}} + \frac{1}{R_{40}}\right)^{-1} = \left(\frac{75\,\mathrm{W}}{(110\,\mathrm{V})^2} + \frac{25\,\mathrm{W}}{(110\,\mathrm{V})^2}\right)^{-1} = 121\,\Omega \approx \boxed{120\,\Omega}$$

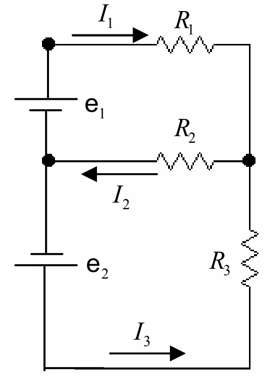
$\boxed{19.}$ The resistors have been numbered in the accompanying diagram to help in



the analysis. R_1 and R_2 are in series with an equivalent resistance of $R_{12} = R + R = 2R$. This combination is in parallel with R_3, with an equivalent resistance of $R_{123} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \frac{2}{3}R$. This combination is in series with R_4, with an equivalent resistance of $R_{1234} = \frac{2}{3}R + R = \frac{5}{3}R$. This combination is in parallel with R_5, with an equivalent resistance of $R_{12345} = \left(\frac{1}{R} + \frac{3}{5R}\right)^{-1} = \frac{5}{8}R$. Finally, this combination is in series with R_6, and we calculate the final equivalent resistance.

$$R_{eq} = \frac{5}{8}R + R = \boxed{\frac{13}{8}R}$$

34. (a) There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.



$$I_2 = I_1 + I_3 \rightarrow I_1 = I_2 - I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$\varepsilon_1 - I_1R_1 - I_2R_2 = 0 \;\rightarrow\; 9 = 25I_1 + 48I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$\varepsilon_2 - I_3R_3 - I_2R_2 = 0 \;\rightarrow\; 12 = 35I_3 + 48I_2$$

Substitute $I_1 = I_2 - I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9 = 25I_1 + 48I_2 = 25(I_2 - I_3) + 48I_2 = 73I_2 - 25I_3 \;\;;\;\; 12 = 35I_3 + 48I_2$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$12 = 35I_3 + 48I_2 \;\rightarrow\; I_2 = \frac{12 - 35I_3}{48}$$

$$9 = 73I_2 - 25I_3 = 73\left(\frac{12 - 35I_3}{48}\right) - 25I_3 \;\rightarrow\; 432 = 876 - 2555I_3 - 1200I_3 \;\rightarrow$$

$$I_3 = \frac{444}{3755} = 0.1182\text{ A} \approx \boxed{0.12\text{ A , up}} \;\;;\;\; I_2 = \frac{12 - 35I_3}{48} = 0.1638\text{ A} \approx \boxed{0.16\text{ A , left}}$$

$$I_1 = I_2 - I_3 = 0.0456\text{ A} \approx \boxed{0.046\text{ A , right}}$$

(*b*) We can include the internal resistances simply by adding 1.0Ω to R_1 and R_3. So let $R_1 = 26\Omega$ and let $R_3 = 36\Omega$. Now re-work the problem exactly as in part (*a*).

$$I_2 = I_1 + I_3 \;\rightarrow\; I_1 = I_2 - I_3$$

$$\varepsilon_1 - I_1R_1 - I_2R_2 = 0 \;\rightarrow\; 9 = 26I_1 + 48I_2$$

$$\varepsilon_2 - I_3R_3 - I_2R_2 = 0 \;\rightarrow\; 12 = 36I_3 + 48I_2$$

$$9 = 26I_1 + 48I_2 = 26(I_2 - I_3) + 48I_2 = 74I_2 - 26I_3 \;\;;\;\; 12 = 36I_3 + 48I_2$$

$$12 = 36I_3 + 48I_2 \;\rightarrow\; I_2 = \frac{12 - 36I_3}{48} = \frac{1 - 3I_3}{4}$$

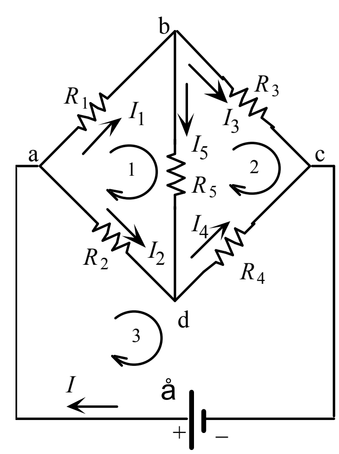
$$9 = 74I_2 - 26I_3 = 74\left(\frac{1 - 3I_3}{4}\right) - 26I_3 \;\rightarrow\; 36 = 74 - 222I_3 - 104I_3 \;\rightarrow$$

$$I_3 = \frac{38}{326} = 0.1166\text{ A} \approx \boxed{0.12\text{ A , up}} \;\;;\;\; I_2 = \frac{1 - 3I_3}{4} = 0.1626\text{ A} \approx \boxed{0.16\text{ A, left}}$$

$$I_1 = I_2 - I_3 = \boxed{0.046\text{ A , right}}$$

The currents are unchanged to 2 significant figures by the inclusion of the internal resistances.

38. The circuit diagram has been labeled with six different currents. We apply the junction rule to junctions a, b, and c. We apply the loop rule to the three loops labeled in the diagram.



$$1)\;\; I = I_1 + I_2 \;\;;\;\; 2)\;\; I_1 = I_3 + I_5 \;\;;\;\; 3)\;\; I_3 + I_4 = I$$

$$4)\;\; -I_1R_1 - I_5R_5 + I_2R_2 = 0 \;\;;\;\; 5)\;\; -I_3R_3 + I_4R_4 + I_5R_5 = 0$$

$$6)\;\; \varepsilon - I_2R_2 - I_4R_4 = 0$$

Eliminate *I* using equations 1) and 3).

$$1)\ I_3+I_4=I_1+I_2\ ;\ 2)\ I_1=I_3+I_5$$
$$4)\ -I_1R_1-I_5R_5+I_2R_2=0\ ;\ 5)\ -I_3R_3+I_4R_4+I_5R_5=0$$
$$6)\ \mathsf{e}-I_2R_2-I_4R_4=0$$

Eliminate I_1 using equation 2.

$$1)\ I_3+I_4=I_3+I_5+I_2 \quad \rightarrow \quad I_4=I_5+I_2$$
$$4)\ -(I_3+I_5)R_1-I_5R_5+I_2R_2=0 \quad \rightarrow \quad -I_3R_1-I_5(R_1+R_5)+I_2R_2=0$$
$$5)\ -I_3R_3+I_4R_4+I_5R_5=0$$
$$6)\ \mathsf{e}-I_2R_2-I_4R_4=0$$

Eliminate I_4 using equation 1.

$$4)\ -I_3R_1-I_5(R_1+R_5)+I_2R_2=0$$
$$5)\ -I_3R_3+(I_5+I_2)R_4+I_5R_5=0 \quad \rightarrow \quad -I_3R_3+I_5(R_4+R_5)+I_2R_4=0$$
$$6)\ \mathsf{e}-I_2R_2-(I_5+I_2)R_4=0 \quad \rightarrow \quad \mathsf{e}-I_2(R_2+R_4)-I_5R_4=0$$

Eliminate I_2 using equation 4: $I_2=\frac{1}{R_2}\left[I_3R_1+I_5(R_1+R_5)\right]$.

$$5)\ -I_3R_3+I_5(R_4+R_5)+\frac{1}{R_2}\left[I_3R_1+I_5(R_1+R_5)\right]R_4=0 \quad \rightarrow$$
$$I_3(R_1R_4-R_2R_3)+I_5(R_2R_4+R_2R_5+R_1R_4+R_5R_4)=0$$
$$6)\ \mathsf{e}-\frac{1}{R_2}\left[I_3R_1+I_5(R_1+R_5)\right](R_2+R_4)-I_5R_4=0 \quad \rightarrow$$
$$\mathsf{e}R_2-I_3R_1(R_2+R_4)-I_5(R_1R_2+R_1R_4+R_5R_2+R_5R_4+R_2R_4)=0$$

Eliminate I_3 using equation 5: $I_3=-I_5\frac{(R_2R_4+R_2R_5+R_1R_4+R_5R_4)}{(R_1R_4-R_2R_3)}$

$$\mathsf{e}R_2+\left[I_5\frac{(R_2R_4+R_2R_5+R_1R_4+R_5R_4)}{(R_1R_4-R_2R_3)}\right]R_1(R_2+R_4)-I_5(R_1R_2+R_1R_4+R_5R_2+R_5R_4+R_2R_4)=0$$
$$\mathsf{e}=-\frac{I_5}{R_2}\left\{\left[\frac{(R_2R_4+R_2R_5+R_1R_4+R_5R_4)}{(R_1R_4-R_2R_3)}\right]R_1(R_2+R_4)-(R_1R_2+R_1R_4+R_5R_2+R_5R_4+R_2R_4)\right\}$$
$$=-\frac{I_5}{25\Omega}\left\{\begin{array}{l}\left[\frac{(25\Omega)(14\Omega)+(25\Omega)(15\Omega)+(22\Omega)(14\Omega)+(15\Omega)(14\Omega)}{(22\Omega)(14\Omega)-(25\Omega)(12\Omega)}\right](22\Omega)(25\Omega+14\Omega)\\ -\left[(22\Omega)(25\Omega)+(22)(14)+(15\Omega)(25\Omega)+(15\Omega)(14\Omega)+(25\Omega)(14\Omega)\right]\end{array}\right\}$$

$$=-I_5(5261\Omega) \quad \rightarrow \quad I_5=-\frac{6.0\,\text{V}}{5261\Omega}=-1.140\,\text{mA}\ (\text{upwards})$$

$$I_3 = -I_5 \frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}$$

$$= -(-1.140\,\text{mA})\frac{(25\,\Omega)(14\,\Omega)+(25\,\Omega)(15\,\Omega)+(22\,\Omega)(14\,\Omega)+(15\,\Omega)(14\,\Omega)}{(22\,\Omega)(14\,\Omega)-(25\,\Omega)(12\,\Omega)} = 0.1771\,\text{A}$$

$$I_2 = \frac{1}{R_2}\left[I_3R_1 + I_5(R_1+R_5)\right] = \frac{1}{25\,\Omega}\left[(0.1771\,\text{A})(22\,\Omega) + (-0.00114\,\text{A})(37\,\Omega)\right] = 0.1542\,\text{A}$$

$$I_4 = I_5 + I_2 = -0.00114\,\text{A} + 0.1542\,\text{A} = 0.1531\,\text{A}$$

$$I_1 = I_3 + I_5 = 0.1771\,\text{A} - 0.00114\,\text{A} = 0.1760\,\text{A}$$

We keep an extra significant figure to show the slight difference in the currents.

$$\boxed{I_{22\Omega} = 0.176\text{A} \quad I_{25\Omega} = 0.154\,\text{A} \quad I_{12\Omega} = 0.177\,\text{A} \quad I_{14\Omega} = 0.153\,\text{A} \quad I_{15\Omega} = 0.001\,\text{A, upwards}}$$

40. (*a*) As shown in the diagram, we use symmetry to reduce the number of independent currents to six. Using Kirchhoff's junction rule, we write equations for junctions a, c, and d. We then use Kirchhoff's loop rule to write the loop equations for loops afgba, hedch, and aba (through the voltage source).

$$I = 2I_1 + I_2 \;\; [1] \;\; ; \;\; I_3 + I_4 = I_1 \;\; [2] \;\; ; \;\; I_5 = 2I_4 \;\; [3]$$

$$0 = -2I_1R - I_3R + I_2R \;\; [4] \;\; ; \;\; 0 = -2I_4R - I_5R + I_3R \;\; [5]$$

$$0 = \mathcal{E} - I_2R \;\; [6]$$

We have six equations with six unknown currents. We use the method of substitution to reduce the equations to a single equation relating the emf from the power source to the current through the power source. This resulting ratio is the effective resistance between points a and b. We insert Eqs. [2], [3], and [6] into the other three equations to eliminate I_1, I_2, and I_5.

$$I = 2(I_3 + I_4) + \frac{\mathcal{E}}{R} = 2I_3 + 2I_4 + \frac{\mathcal{E}}{R} \qquad [1^*]$$

$$0 = -2(I_3 + I_4)R - I_3R + \frac{\mathcal{E}}{R}R = -2I_4R - 3I_3R + \mathcal{E} \qquad [4^*]$$

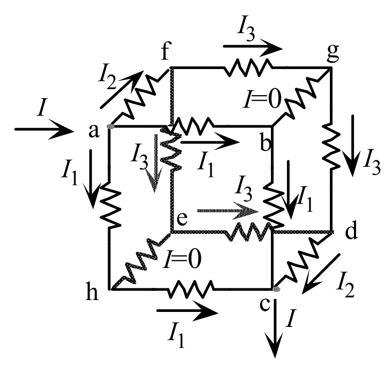
$$0 = -2I_4R - 2I_4R + I_3R = -4I_4R + I_3R \qquad [5^*]$$

We solve Eq. [5*] for I_3 and insert that into Eq. [4*]. We then insert the two results into Eq. [1*] and solve for the effective resistance.

$$I_3 = 4I_4 \;\; ; \;\; 0 = -2I_4R - 3(4I_4)R + \mathcal{E} \rightarrow I_4 = \frac{\mathcal{E}}{14R}$$

$$I = 2(4I_4) + 2I_4 + \frac{\mathcal{E}}{R} = 10I_4 + \frac{\mathcal{E}}{R} = \frac{10\mathcal{E}}{14R} + \frac{\mathcal{E}}{R} = \frac{24\mathcal{E}}{14R} = \frac{12\mathcal{E}}{7R} \;\; \rightarrow \;\; R_{\text{eq}} = \frac{\mathcal{E}}{I} = \boxed{\tfrac{7}{12}R}$$

(*b*) As shown in the diagram, we use symmetry to reduce the number of currents to four. We use Kirchhoff's junction rule at junctions a and d and the loop rule around loops abca (through the voltage source) and afgdcha. This results in four equations with four unknowns. We solve



these equations for the ratio of the voltage source to current I, to obtain the effective resistance.

$$I = 2I_1 + I_2 \ [1] \quad ; \quad 2I_3 = I_2 \ [2]$$

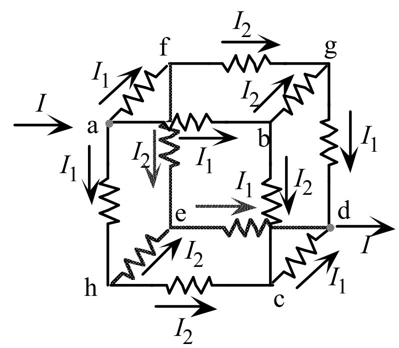
$$0 = -2I_2R + \mathcal{E} \ [3] \ ; \ 0 = -2I_2R - 2I_3R + 2I_1R \ [4]$$

We solve Eq. [3] for I_2 and Eq. [2] for I_3. These results are inserted into Eq. [4] to determine I_1. Using these results and Eq. [1] we solve for the effective resistance.

$$I_2 = \frac{\mathcal{E}}{2R} \ ; \ I_3 = \frac{I_2}{2} = \frac{\mathcal{E}}{4R} \ ; \ I_1 = I_2 + I_3 = \frac{\mathcal{E}}{2R} + \frac{\mathcal{E}}{4R} = \frac{3\mathcal{E}}{4R}$$

$$I = 2I_1 + I_2 = 2\left(\frac{3\mathcal{E}}{4R}\right) + \frac{\mathcal{E}}{2R} = \frac{2\mathcal{E}}{R} \ ; \ R_{eq} = \frac{\mathcal{E}}{I} = \boxed{\tfrac{1}{2}R}$$

(*c*) As shown in the diagram, we again use symmetry to reduce the number of currents to three. We use Kirchhoff's junction rule at points a and b and the loop rule around the loop abgda (through the power source) to write three equations for the three unknown currents. We solve these equations for the ratio of the emf to the current through the emf (I) to calculate the effective resistance.



$$I = 3I_1 \ [1] \ ; \ I_1 = 2I_2 \ [2]$$

$$0 = -2I_1R - I_2R + \mathcal{E} \ [3]$$

We insert Eq. [2] into Eq. [3] and solve for I_1. Inserting I_1 into Eq. [1] enables us to solve for the effective resistance.

$$0 = -2I_1R - \tfrac{1}{2}I_1R + \mathcal{E} \to I_1 = \frac{2\mathcal{E}}{5R} \ ; \ I = 3I_1 = \frac{6\mathcal{E}}{5R} \to R_{eq} = \frac{\mathcal{E}}{I} = \boxed{\tfrac{5}{6}R}$$

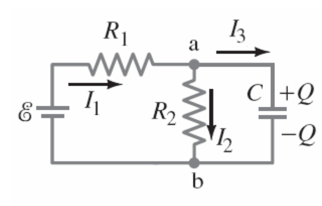
46. Express the stored energy in terms of the charge on the capacitor, using Eq. 24-5. The charge on the capacitor is given by Eq. 26-6a.

$$U = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{\left[C\mathcal{E}\left(1-e^{-t/\tau}\right)\right]^2}{C} = \tfrac{1}{2}C\mathcal{E}^2\left(1-e^{-t/\tau}\right)^2 = U_{max}\left(1-e^{-t/\tau}\right)^2 \ ;$$

$$U = 0.75U_{max} \ \to \ U_{max}\left(1-e^{-t/\tau}\right)^2 = 0.75U_{max} \ \to \ \left(1-e^{-t/\tau}\right)^2 = 0.75 \ \to$$

$$t = -\tau \ln\left(1-\sqrt{0.75}\right) = \boxed{2.01\tau}$$

50. (*a*) With the currents and junctions labeled as in the diagram, we use point a for the junction rule and the right and left loops for the loop rule. We set current I_3 equal to the derivative of the charge on the capacitor and combine the equations to obtain a single differential equation in terms of the capacitor charge. Solving this equation yields the charging time constant.



$$I_1 = I_2 + I_3 \ [1] \ ; \ \mathcal{E} - I_1R_1 - I_2R_2 = 0 \ [2] \ ; \ -\frac{Q}{C} + I_2R_2 = 0 \ [3]$$

We use Eq. [1] to eliminate I_1 in Eq. [2]. Then we use Eq. [3] to eliminate I_2 from Eq. [2].

$$0 = \varepsilon - (I_2 + I_3)R_1 - I_2R_2 \;\; ; \;\; 0 = \varepsilon - I_2(R_1 + R_2) - I_3R_1 \;\; ; \;\; 0 = \varepsilon - \left(\frac{Q}{R_2C}\right)(R_1 + R_2) - I_3R_1$$

We set I_3 as the derivative of the charge on the capacitor and solve the differential equation by separation of variables.

$$0 = \varepsilon - \left(\frac{Q}{R_2C}\right)(R_1 + R_2) - \frac{dQ}{dt}R_1 \;\rightarrow\; \int_0^Q \frac{dQ'}{Q' - \left(\frac{R_2C\varepsilon}{R_1 + R_2}\right)} = \int_0^t \frac{-(R_1 + R_2)}{R_1R_2C}dt' \;\rightarrow$$

$$\ln\left[Q' - \left(\frac{R_2C\varepsilon}{R_1 + R_2}\right)\right]_0^Q = -\frac{(R_1 + R_2)}{R_1R_2C}t'\Big|_0^t \;\rightarrow\; \ln\left[\frac{Q - \left(\frac{R_2C\varepsilon}{R_1 + R_2}\right)}{\left(\frac{R_2C\varepsilon}{R_1 + R_2}\right)}\right] = -\frac{(R_1 + R_2)}{R_1R_2C}t \;\rightarrow$$

$$Q = \frac{R_2C\varepsilon}{R_1 + R_2}\left(1 - e^{-\frac{(R_1 + R_2)}{R_1R_2C}t}\right)$$

From the exponential term we obtain the time constant, $\tau = \boxed{\dfrac{R_1R_2C}{R_1 + R_2}}$.

(*b*) We obtain the maximum charge on the capacitor by taking the limit as time goes to infinity.

$$Q_{\max} = \lim_{t \to \infty} \frac{R_2C\varepsilon}{R_1 + R_2}\left(1 - e^{-\frac{(R_1 + R_2)}{R_1R_2C}t}\right) = \boxed{\frac{R_2C\varepsilon}{R_1 + R_2}}$$