

Ch. 21: 7, 11, 13, 16, 32, 34, 40, 47, 48, 49, 58, 59, 62

7. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the force is tripled, the distance has been reduced by a factor of $\sqrt{3}$.

$$r = \frac{r_0}{\sqrt{3}} = \frac{8.45 \text{ cm}}{\sqrt{3}} = \boxed{4.88 \text{ cm}}$$

11. (a) Let one of the charges be q , and then the other charge is $Q_T - q$. The force between the

charges is $F_E = k \frac{q(Q_T - q)}{r^2} = \frac{k}{r^2}(qQ_T - q^2)$. To find the maximum and minimum force, set the first derivative equal to 0. Use the second derivative test as well.

$$F_E = \frac{k}{r^2}(qQ_T - q^2) ; \quad \frac{dF_E}{dq} = \frac{k}{r^2}(Q_T - 2q) = 0 \rightarrow q = \frac{1}{2}Q_T$$

$$\frac{d^2F_E}{dq^2} = -\frac{2k}{r^2} < 0 \rightarrow q = \frac{1}{2}Q_T \text{ gives } (F_E)_{\max}$$

So $\boxed{q_1 = q_2 = \frac{1}{2}Q_T}$ gives the maximum force.

- (b) If one of the charges has all of the charge, and the other has no charge, then the force between

them will be 0, which is the minimum possible force. So $\boxed{q_1 = 0, q_2 = Q_T}$ gives the minimum force.

13. The forces on each charge lie along a line connecting the charges. Let the variable d represent the length of a side of the triangle. Since the triangle is equilateral, each angle is 60° . First calculate the magnitude of each individual force.

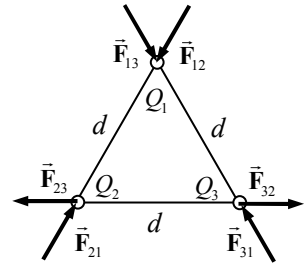
$$F_{12} = k \frac{|Q_1 Q_2|}{d^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(7.0 \times 10^{-6} \text{ C})(8.0 \times 10^{-6} \text{ C})}{(1.20 \text{ m})^2}$$

$$= 0.3495 \text{ N}$$

$$F_{13} = k \frac{|Q_1 Q_3|}{d^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(7.0 \times 10^{-6} \text{ C})(6.0 \times 10^{-6} \text{ C})}{(1.20 \text{ m})^2}$$

$$= 0.2622 \text{ N}$$

$$F_{23} = k \frac{|Q_2 Q_3|}{d^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(8.0 \times 10^{-6} \text{ C})(6.0 \times 10^{-6} \text{ C})}{(1.20 \text{ m})^2} = 0.2996 \text{ N} = F_{32}$$



Now calculate the net force on each charge and the direction of that net force, using components.

$$F_{1x} = F_{12x} + F_{13x} = -(0.3495 \text{ N}) \cos 60^\circ + (0.2622 \text{ N}) \cos 60^\circ = -4.365 \times 10^{-2} \text{ N}$$

$$F_{1y} = F_{12y} + F_{13y} = -(0.3495 \text{ N}) \sin 60^\circ - (0.2622 \text{ N}) \sin 60^\circ = -5.297 \times 10^{-1} \text{ N}$$

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \boxed{0.53 \text{ N}} \quad \theta_1 = \tan^{-1} \frac{F_{1y}}{F_{1x}} = \tan^{-1} \frac{-5.297 \times 10^{-1} \text{ N}}{-4.365 \times 10^{-2} \text{ N}} = \boxed{265^\circ}$$

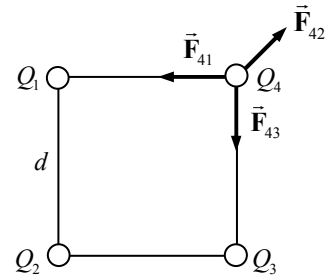
$$F_{2x} = F_{21x} + F_{23x} = (0.3495 \text{ N}) \cos 60^\circ - (0.2996 \text{ N}) = -1.249 \times 10^{-1} \text{ N}$$

$$F_{2y} = F_{21y} + F_{23y} = (0.3495 \text{ N}) \sin 60^\circ + 0 = 3.027 \times 10^{-1} \text{ N}$$

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \boxed{0.33 \text{ N}} \quad \theta_2 = \tan^{-1} \frac{F_{2y}}{F_{2x}} = \tan^{-1} \frac{3.027 \times 10^{-1} \text{ N}}{-1.249 \times 10^{-1} \text{ N}} = \boxed{112^\circ}$$

16. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other charges.

The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 4.15 mC charge at each corner.



$$F_{41} = k \frac{Q^2}{d^2} \rightarrow F_{41x} = -k \frac{Q^2}{d^2}, F_{41y} = 0$$

$$F_{42} = k \frac{Q^2}{2d^2} \rightarrow F_{42x} = k \frac{Q^2}{2d^2} \cos 45^\circ = k \frac{\sqrt{2}Q^2}{4d^2}, F_{42y} = k \frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k \frac{Q^2}{d^2} \rightarrow F_{43x} = 0, F_{43y} = -k \frac{Q^2}{d^2}$$

Add the x and y components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = -k \frac{Q^2}{d^2} + k \frac{\sqrt{2}Q^2}{4d^2} + 0 = k \frac{Q^2}{d^2} \left(-1 + \frac{\sqrt{2}}{4} \right) = -0.64645k \frac{Q^2}{d^2} = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k \frac{Q^2}{d^2} (0.64645) \sqrt{2} = k \frac{Q^2}{d^2} (0.9142)$$

$$= (8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(4.15 \times 10^{-3} \text{ C})^2}{(0.100 \text{ m})^2} (0.9142) = \boxed{1.42 \times 10^7 \text{ N}}$$

$$\theta = \tan^{-1} \frac{F_{4y}}{F_{4x}} = \boxed{225^\circ} \text{ from the } x\text{-direction, or exactly towards the center of the square.}$$

For each charge, there are two forces that point towards the adjacent corners, and one force that points away from the center of the square. Thus for each charge, the net force will be the magnitude of $\boxed{1.42 \times 10^7 \text{ N}}$ and will lie along the line from the charge inwards towards the center of the square.

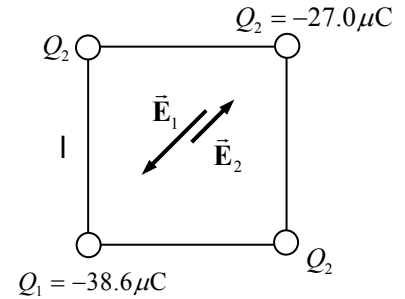
32. The field due to the negative charge will point towards

the negative charge, and the field due to the positive charge will point towards the negative charge. Thus the magnitudes of the two fields can be added together to find the charges.

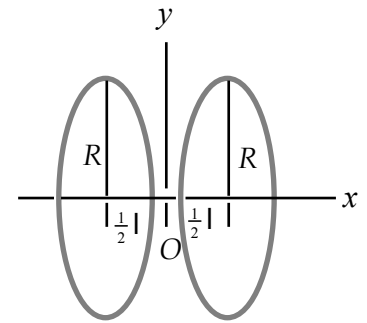
$$E_{\text{net}} = 2E_Q = 2k \frac{Q}{(l/2)^2} = \frac{8kQ}{l^2} \rightarrow Q = \frac{El^2}{8k} = \frac{(586 \text{ N/C})(0.160 \text{ m})^2}{8(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)} = \boxed{2.09 \times 10^{-10} \text{ C}}$$

34. The field at the center due to the two $-27.0 \mu\text{C}$ negative charges on opposite corners (lower right and upper left in the diagram) will cancel each other, and so only the other two charges need to be considered. The field due to each of the other charges will point directly toward the charge. Accordingly, the two fields are in opposite directions and can be combined algebraically.

$$\begin{aligned} E &= E_1 - E_2 = k \frac{|Q_1|}{l^2/2} - k \frac{|Q_2|}{l^2/2} = k \frac{|Q_1| - |Q_2|}{l^2/2} \\ &= (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(38.6 - 27.0) \times 10^{-6} \text{ C}}{(0.525 \text{ m})^2/2} \\ &= \boxed{7.57 \times 10^6 \text{ N/C, towards the } -38.6 \mu\text{C charge}} \end{aligned}$$



40. Consider Example 21-9. We use the result from this example, but shift the center of the ring to be at $x = \frac{1}{2}l$ for the ring on the right, and at $x = -\frac{1}{2}l$ for the ring on the left. The fact that the original expression has a factor of x results in the interpretation that the sign of the field expression will give the direction of the field. No special consideration needs to be given to the location of the point at which the field is to be calculated.

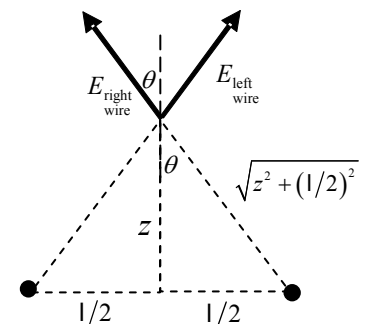


$$\begin{aligned} \vec{E} &= \vec{E}_{\text{right}} + \vec{E}_{\text{left}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q(x - \frac{1}{2}l)}{[(x - \frac{1}{2}l)^2 + R^2]^{3/2}} \hat{i} + \frac{1}{4\pi\epsilon_0} \frac{Q(x + \frac{1}{2}l)}{[(x + \frac{1}{2}l)^2 + R^2]^{3/2}} \hat{i} \\ &= \hat{i} \frac{Q}{4\pi\epsilon_0} \left\{ \frac{(x - \frac{1}{2}l)}{[(x - \frac{1}{2}l)^2 + R^2]^{3/2}} + \frac{(x + \frac{1}{2}l)}{[(x + \frac{1}{2}l)^2 + R^2]^{3/2}} \right\} \end{aligned}$$

47. If we consider just one wire, then from the answer to problem 46, we would have the following. Note that the distance from the wire to the point in question is $x = \sqrt{z^2 + (l/2)^2}$.

$$E_{\text{wire}} = \frac{\lambda}{2\pi\epsilon_0} \frac{l}{\sqrt{z^2 + (l/2)^2} (4[z^2 + (l/2)^2 + l^2]^{1/2})}$$

But the total field is not simply four times the above expression, because the fields due to the four wires are not parallel to each other. Consider a side view of the problem. The two dots represent two



parallel wires, on opposite sides of the square. Note that only the vertical component of the field due to each wire will actually contribute to the total field. The horizontal components will cancel.

$$E_{\text{wire}} = 4(E_{\text{wire}}) \cos \theta = 4(E_{\text{wire}}) \frac{z}{\sqrt{z^2 + (1/2)^2}}$$

$$E_{\text{wire}} = 4 \left[\frac{\lambda}{2\pi\epsilon_0} \frac{1}{\sqrt{z^2 + (1/2)^2} \left(4 \left[z^2 + (1/2)^2 \right] + 1^2 \right)^{1/2}} \right] \frac{z}{\sqrt{z^2 + (1/2)^2}}$$

$$= \frac{8\lambda z}{\pi\epsilon_0 (4z^2 + 1^2)(4z^2 + 2^2)^{1/2}}$$

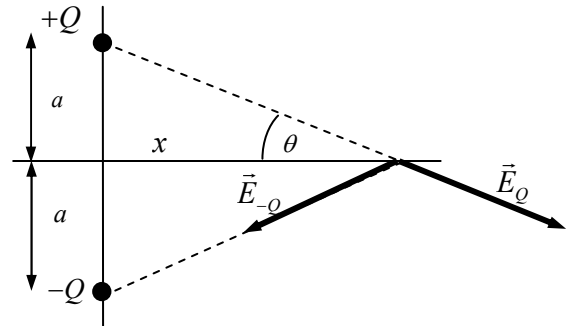
The direction is vertical, perpendicular to the loop.

48. From the diagram, we see that the x components of the two fields will cancel each other at the point P. Thus the net electric field will be in the negative y -direction, and will be twice the y -component of either electric field vector.

$$E_{\text{net}} = 2E \sin \theta = 2 \frac{kQ}{x^2 + a^2} \sin \theta$$

$$= \frac{2kQ}{x^2 + a^2} \frac{a}{(x^2 + a^2)^{1/2}}$$

$$= \frac{2kQa}{(x^2 + a^2)^{3/2}} \text{ in the negative } y \text{ direction}$$



49. Select a differential element of the arc which makes an angle of θ with the x axis. The length of this element is $Rd\theta$, and the charge on that element is $dq = \lambda Rd\theta$.

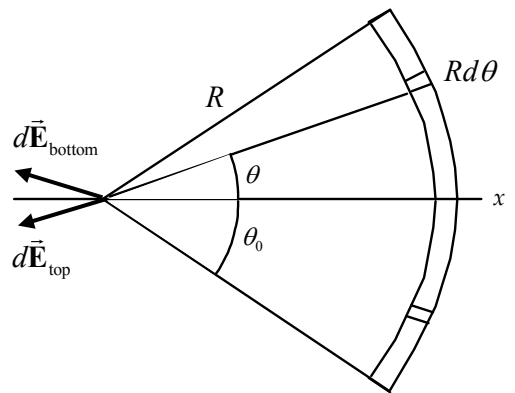
The magnitude of the field produced by that element is $dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda Rd\theta}{R^2}$. From the diagram, considering

pieces of the arc that are symmetric with respect to the x axis, we see that the total field will only have an x component. The vertical components of the field due to symmetric portions of the arc will cancel each other.

So we have the following.

$$dE_{\text{horizontal}} = \frac{1}{4\pi\epsilon_0} \frac{\lambda Rd\theta}{R^2} \cos \theta$$

$$E_{\text{horizontal}} = \int_{-\theta_0}^{\theta_0} \frac{1}{4\pi\epsilon_0} \cos \theta \frac{\lambda Rd\theta}{R^2} = \frac{\lambda}{4\pi\epsilon_0 R} \int_{-\theta_0}^{\theta_0} \cos \theta d\theta = \frac{\lambda}{4\pi\epsilon_0 R} [\sin \theta_0 - \sin(-\theta_0)] = \frac{2\lambda \sin \theta_0}{4\pi\epsilon_0 R}$$



The field points in the negative x direction, so $E = \boxed{-\frac{2\lambda \sin \theta_0}{4\pi\epsilon_0 R} \hat{\mathbf{i}}}$

58. (a) The electron will experience a force in the opposite direction to the electric field. Since the

electron is to be brought to rest, the electric field must be in the same direction as the initial velocity of the electron, and so is to the **right**.

(b) Since the field is uniform, the electron will experience a constant force, and therefore have a constant acceleration. Use constant acceleration relationships to find the field strength.

$$F = qE = ma \rightarrow a = \frac{qE}{m} \quad v^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{qE}{m}\Delta x \rightarrow$$

$$E = \frac{m(v^2 - v_0^2)}{2q\Delta x} = \frac{-mv_0^2}{2q\Delta x} = -\frac{(9.109 \times 10^{-31} \text{ kg})(7.5 \times 10^5 \text{ m/s})^2}{2(-1.602 \times 10^{-19} \text{ C})(0.040 \text{ m})} = \boxed{40 \text{ N/C}} \quad (2 \text{ sig. fig.})$$

59. The angle is determined by the velocity. The x component of the velocity is constant. The time to pass through the plates can be found from the x motion. Then the y velocity can be found using constant acceleration relationships.

$$x = v_0 t \rightarrow t = \frac{x}{v_0} ; v_y = v_{y0} + a_y t = -\frac{eE}{m} \frac{x}{v_0}$$

$$\tan \theta = \frac{v_y}{v_x} = \frac{-\frac{eE}{m} \frac{x}{v_0}}{v_0} = -\frac{eEx}{mv_0^2} = -\frac{(1.60 \times 10^{-19} \text{ C})(5.0 \times 10^3 \text{ N/C})(0.049 \text{ m})}{(9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^7 \text{ m/s})^2} = -.4303 \rightarrow$$

$$\theta = \tan^{-1}(-0.4303) = \boxed{-23^\circ}$$

62. (a) The dipole moment is given by the product of the positive charge and the separation distance.

$$p = ql = (1.60 \times 10^{-19} \text{ C})(0.68 \times 10^{-9} \text{ m}) = 1.088 \times 10^{-28} \text{ C}\cdot\text{m} \approx \boxed{1.1 \times 10^{-28} \text{ C}\cdot\text{m}}$$

(b) The torque on the dipole is given by Eq. 21-9a.

$$\tau = pE \sin \theta = (1.088 \times 10^{-28} \text{ C}\cdot\text{m})(2.2 \times 10^4 \text{ N/C})(\sin 90^\circ) = \boxed{2.4 \times 10^{-24} \text{ C}\cdot\text{m}}$$

(c) $\tau = pE \sin \theta = (1.088 \times 10^{-28} \text{ C}\cdot\text{m})(2.2 \times 10^4 \text{ N/C})(\sin 45^\circ) = \boxed{1.7 \times 10^{-24} \text{ N}\cdot\text{m}}$

(d) The work done by an external force is the change in potential energy. Use Eq. 21-10.

$$W = \Delta U = (-pE \cos \theta_{\text{final}}) - (-pE \cos \theta_{\text{initial}}) = pE (\cos \theta_{\text{initial}} - \cos \theta_{\text{final}}) \\ = (1.088 \times 10^{-28} \text{ C}\cdot\text{m})(2.2 \times 10^4 \text{ N/C})[1 - (-1)] = \boxed{4.8 \times 10^{-24} \text{ J}}$$