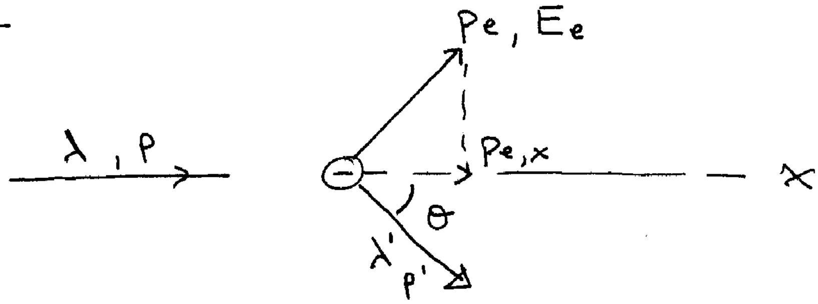
Problem 1



(a) Conservation of energy: K_e = electron kinetic energy:

$$K_e = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 12{,}400\,eV\left(1 - \frac{1}{1.01}\right) = 122.8\,eV$$

$$\boxed{K_e = 122.8\,eV} \text{ (a)}$$

(b) $$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \Rightarrow \cos\theta = 1 - \frac{\lambda' - \lambda}{h/m_e c} = 1 - \frac{0.01}{0.0243} = 0.588$$

$$\cos\theta = 0.588 \Rightarrow \boxed{\theta = 53.95°}$$

(c) Conservation of momentum along the x direction:

$$p = p'\cos\theta + p_{e,x} \Rightarrow p_{e,x} = p - p'\cos\theta$$

For photon, $E = hf = \frac{hc}{\lambda} = pc \Rightarrow p = \frac{h}{\lambda} \Rightarrow$

$$p_{e,x} = \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta \Rightarrow$$

$$p_{e,x} \cdot c = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\theta\right) = 5175\,eV$$

$$\Rightarrow \boxed{p_{e,x} = 5175\,eV/c}$$

Problem 2

Rutherford's formula says: $\Delta n = \dfrac{c}{\sin^4 \phi/2}$

$\phi = 180° \Rightarrow \phi/2 = 90° \Rightarrow \sin^4\phi/2 = 1$

$\phi = 90° \Rightarrow \phi/2 = 45° \Rightarrow \sin\phi/2 = \sqrt{2}/2 \Rightarrow \sin^4\phi/2 = 1/4$

Therefore, Rutherford's formula predicts

$$\Delta n(\phi = 180°) = \Delta n(\phi = 90°)/4$$

For $K_\alpha = 6$ MeV, $\Delta n(90°) = 600$, $\Delta n(180°) = 150 = 600/4 \Rightarrow$

$\Rightarrow$ Rutherford's formula works.

For $K_\alpha = 7$ MeV, $\Delta n(90°) = 700$, $\Delta n(180°) = 150 \neq 700/4 \Rightarrow$

Rutherford's formula doesn't work $\Rightarrow$ α-particles penetrate the nucleus.

(a) If Rutherford's formula works for $K_\alpha = 6$ MeV it will work for smaller $K_\alpha = 5$ MeV $\Rightarrow$

$$\boxed{\Delta n(180°) = \Delta n(90°)/4 = 500/4 = 125}$$

(b) Distance of closest approach: from $K_\alpha = kZe \times (2e)/d_{min}$

$$d_{min} = \frac{2kZe^2}{K_\alpha} = \frac{2 \times 14.4\, eV\,Å \times 12}{K_\alpha} = \frac{345.6\, eV\,Å}{K_\alpha}$$

For $K_\alpha = 7$ MeV, $d_{min} = 4.94 \times 10^{-5}$ Å, because Rutherford's fla doesn't work $\Rightarrow$ $d_{min} <$ radius of nucleus.

For $K_\alpha = 6$ MeV, $d_{min} = 5.76 \times 10^{-5}$ Å, because Rutherford's fla works $\Rightarrow$ $d_{min} >$ radius of nucleus

$$\Rightarrow \boxed{4.94 \times 10^{-5}\,Å < \text{radius of Mg nucleus} < 5.76 \times 10^{-5}\,Å}$$

Problem 3

$$E_n = -\frac{Z^2}{n^2} E_0 \quad . \; Z = 1 \text{ for } H, \; Z = 3 \text{ for } Li^{++}.$$

For H in ground state, $n=1$, $E_1 = -E_0 = -13.6\,eV$

For Li^{++}, if electron has energy $-E_0 \Rightarrow \boxed{n=3}$ (a)

(b) The kinetic energy, potential energy and total energy satisfy

$$E = K + U, \quad K = -\frac{U}{2} \Rightarrow U = -2K \Rightarrow E = K - 2K = -K$$

Since $K = \frac{1}{2} m_e v^2$, if the energy is the same for both electrons

$\Rightarrow$ $\boxed{\text{the speed is the same}}$

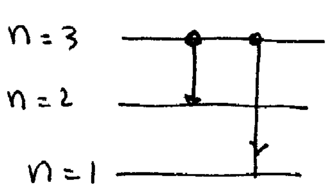
Alternative solution:

$$H: \quad L_1 = \hbar = m_e v_1 r_1 = m_e v_1 a_0 \Rightarrow v_1 = \frac{\hbar}{m_e a_0}$$

$$Li^{++}, n=3: \quad L_3 = 3\hbar = m_e v_3 r_3 = m_e v_3 r_0 \cdot 3^2 = m_e v_3 \frac{a_0}{Z} \cdot 3^2 = m_e v_3 a_0 \cdot 3$$

$$\Rightarrow \quad v_3 = \frac{3\hbar}{m_e a_0 \cdot 3} = \frac{\hbar}{m_e a_0} = v_1 \text{ (for } H, n=1)$$

(c)



$$\frac{hc}{\lambda_{nm}} = E_0 Z^2 \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

possible transitions: $3 \to 1$, $3 \to 2$

$$\lambda_{13} = \frac{hc}{E_0 Z^2 \left(1 - \frac{1}{3^2}\right)} = \frac{hc}{E_0 (9-1)} = \frac{911.76\,\mathring{A}}{8} = 114.0\,\mathring{A}$$

$$\lambda_{23} = \frac{hc}{E_0 Z^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right)} = \frac{hc}{E_0 \left(\frac{9}{4} - 1 \right)} = 911.76\,\mathring{A} \times \frac{4}{5} = 729.4\,\mathring{A}$$

So possible wavelengths are $\boxed{\lambda = 114.0\,\mathring{A}, \; \lambda = 729.4\,\mathring{A}}$