Formulas:
Time dilation; Length contraction: $\Delta t = \frac{\gamma \Delta t'}{\gamma} = \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{m/s}$
Lorentz transformation: $x' = \gamma (x - vt)$; $y' = y$; $z' = z$; $t' = \gamma (t - vx/c^2)$; inverse: $v \rightarrow -v$
Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$
Velocity transformation: $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \frac{u_y}{\gamma (1 - u_x v/c^2)}$; inverse: $v \rightarrow -v$
Relativistic Doppler shift: $f_{\text{obs}} = f_{\text{source}} \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}}$ (approaching)
Momentum: $\tilde{p} = \gamma m \tilde{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$
Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$
Electron: $m_e = 0.511 \text{MeV/c}^2$ Proton: $m_p = 938.26 \text{MeV/c}^2$ Neutron: $m_n = 939.55 \text{MeV/c}^2$
Atomic mass unit: $1 u = 931.5 \text{MeV/c}^2$; electron volt: $1 \text{eV} = 1.6 \times 10^{-19} \text{J}$
Stefan's law: $e_{\text{tot}} = \sigma T^4$, $e_{\text{tot}}$ = power/unit area; $\sigma = 5.67 \times 10^{-8} \text{W/m}^2\text{K}^4$
$e_{\text{tot}} = cU/4$, $U = \text{energy density} = \int_0^\infty u(\lambda,T) d\lambda$; Wien's law: $\lambda_m T = \frac{hc}{4.96 k_B}$
Boltzmann distribution: $P(E) = Ce^{E/(k_B T)}$
Planck's law: $u_\lambda(\lambda,T) = N_\lambda(\lambda,T) \times \bar{E}(\lambda,T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$
Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12,400 \text{ eV A}$; $k_B = (1/11,600) \text{eV K}$
Photoelectric effect: $eV_s = K_{\text{max}} = hf - \phi$, $\phi$ = work function; Bragg equation: $n\lambda = 2d \sin \theta$
Compton scattering: $\lambda' - \lambda = \frac{h}{m_c c}(1 - \cos \theta)$; $h/m_c c = 0.0243\text{A}$; Coulomb constant: $ke^2 = 14.4 \text{ eV A}$
Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$; Drag force: $D = 6\pi a \eta v$
Rutherford scattering: $\Delta n = \frac{C}{\sin^4(\phi/2)}$; $hc = 1,973 \text{ eV A}$
Hydrogen spectrum: $\frac{1}{\lambda_{mn}} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right)$; $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3A}$
Electrostatic force, energy: $F = \frac{kq_1 q_2}{r^2}$; $U = \frac{kq_1 q_2}{r}$. Centripetal force: $F_c = \frac{mv^2}{r}$
Bohr atom: $E_n = -\frac{ke^2 Z^2}{2r_n}$; $E_0 = \frac{ke^2}{2a_0} = 13.6 \text{eV}$; $K = \frac{m_a \gamma^2}{2}$; $U = -\frac{ke^2 Z^2}{r}$
hf = $E_1 - E_f$; $r_n = r_n h^2$; $r_0 = \frac{a_0}{Z}$, $a_0 = \frac{h^2}{m_e ke^2} = 0.529 \text{A}$; $L = m_a \gamma r = nh$ angular momentum
de Broglie: $\frac{\hbar}{p} = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar \omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$

Justify all your answers to all problems
Problem 1 (10 points)
In a Compton scattering experiment, X-rays incident along the x direction of wavelength \( \lambda = 1 \text{A} \) are scattered by free electrons and the outcoming X-rays have wavelength 1.01A.

(a) Find the kinetic energy of the scattered electrons, in eV.
(b) Find the angle of the scattered X-rays relative to the incident direction (x-axis), in degrees.
(c) Find the x-component of the momentum of the scattered electrons. Give your answer in units eV/c.

Problem 2 (10 points)
In a Rutherford scattering experiment with a Magnesium (Mg) foil (Z=12) and \( \alpha \)-particle energy 6 MeV, it is found that for every 600 particles scattered at angle 90° there are 150 \( \alpha \)-particles scattered at angle 180°. When the \( \alpha \)-particle energy is 7 MeV it is found that for every 700 \( \alpha \)-particles scattered at angle 90° there are 150 \( \alpha \)-particles scattered at angle 180°.

(a) For \( \alpha \)-particle energy 5 MeV, for every 500 \( \alpha \)-particles scattered at angle 90°, how many \( \alpha \)-particles are scattered at angle 180°?
(b) Find minimum and maximum sizes of the nucleus of Mg from this information (i.e. give the radii, in Angstroms).

Problem 3 (10 points)
A hydrogen atom and a Li\(^{++}\) ion (Z=3) have electrons in states that have the same total energy. The electron in hydrogen is in the lowest energy state (ground state).

(a) What state is the electron in the Li\(^{++}\) ion in? Give its quantum number.
(b) Compare the speed of these electrons. I.e. give the ratio of the speed of the electron in this state of Li\(^{++}\) to the speed of the electron in the ground state of H.
(c) What are the possible wavelengths of the photons emitted when this electron in Li\(^{++}\) makes a transition from this state to a lower energy state? Give your answer in Angstroms.

Justify all your answers to all problems