Physics 2D | Quiz 2 Solutions | Fall 2009

Problem 1:

(a) By conservation of momentum, $p' = p = 300\ \text{MeV}/c$, since the momentum of the particle with mass $m$ is zero.

(b) Conservation of energy: $E_{initial} = E_{final}$

$$E_{initial} = \sqrt{M^2c^4 + p^2c^2} = \sqrt{100^2 + 300^2}\ \text{MeV} = 316.23\ \text{MeV}$$

$$E_{final} = mc^2 + \sqrt{\left(\frac{M}{2}c^2\right)^2 + p^2c^2} = mc^2 + \sqrt{50^2 + 300^2}\ \text{MeV} = mc^2 + 304.14\ \text{MeV}$$

So, $\boxed{m = 12.09\ \text{MeV}/c^2} = (316.23 - 304.13)\ \text{MeV}/c^2$

(c) Initial kinetic energy:

$$K_{in} = E_{initial} - Mc^2 = 216.23\ \text{MeV}$$

Final kinetic energy:

$$K_{final} = \sqrt{\left(\frac{M}{2}c^2\right)^2 + p^2c^2} - \frac{M}{2}c^2 = 304.14\ \text{MeV} - 50\ \text{MeV} = 254.14\ \text{MeV}$$

So kinetic energy increased by $\boxed{K_{final} - K_{initial} = 37.91\ \text{MeV}}$

That means, $\boxed{\text{37.91 MeV}/c^2 \text{ of mass was converted into kinetic energy.}}$

Initial mass: $100\ \text{MeV}/c^2$

Final mass: $50\ \text{MeV}/c^2 + 12.09\ \text{MeV}/c^2 = 62.09\ \text{MeV}/c^2$

Missing mass: $(100 - 62.09)\ \frac{\text{MeV}}{c^2} = \boxed{37.91\ \frac{\text{MeV}}{c^2}}$

Alternative solution prob. 1:

$$p = \gamma M u = 300\,MeV/c = \frac{u/c \times 100}{\sqrt{1-u^2/c^2}}\,MeV/c^2 \quad \text{solve for} \quad u/c = 3/\sqrt{10} = 0.9487$$

$$\gamma = 3/(u/c) = 3.1623$$

$$E_{in} = \gamma M c^2 = 316.23\,MeV$$

$$E_{fin} = E_{in} = \gamma'(M/2)c^2 + mc^2$$

Find γ' from p'=(M/2)u' γ' ==> γ' =6.0828

$$mc^2 = E_{in} - \gamma'(M/2)c^2 = 12.09\,MeV$$

etc

Problem 2

$$\lambda_m T = \frac{hc}{4.96\,k_B} \Rightarrow \lambda_m = \frac{12{,}400}{4.96 \times \frac{1}{11{,}600} \cdot 3000} \text{ Å} \Rightarrow$$

(a)

$$\Rightarrow \boxed{\lambda_m = 9666.67 \text{ Å}}$$

(b) Stefan law: $e_{tot} = \sigma T^4$

If $T$ goes from 3000K to 6000K $\Rightarrow$ factor of 2 $\Rightarrow$

$$e_{tot} \longrightarrow 2^4 \times 100\,W = \boxed{1600\,W}$$

(c)

$$u_\lambda(\lambda, T) = \frac{8\pi}{\lambda^4} \frac{hc/\lambda}{e^{hc/\lambda k T} - 1}$$

For $\lambda = 100{,}000$ Å : $\frac{hc}{\lambda k T} \cong 0.1$. For larger $\lambda$ and for larger $T$,
$T = 3000$ K

$\frac{hc}{\lambda k T}$ is even smaller. So we can use $e^x \approx 1 + x$, with $x = \frac{hc}{\lambda k T}$. So

$$u_\lambda(\lambda, T) \cong \frac{8\pi}{\lambda^4} \frac{hc/\lambda}{1 + \frac{hc}{\lambda k T} - 1} = \frac{8\pi}{\lambda^4} k T$$

So when $T$ goes from 3000K to 6000 K, in the range of $\lambda > 100{,}000$ Å the power emitted is approximately $\boxed{\text{double}}$

## Problem 3

$$eV_s = \frac{hc}{\lambda_1} - \phi \qquad \lambda_1 = 3000\,\text{Å}$$

$$\frac{eV_s}{2} = \frac{hc}{\lambda_2} - \phi \qquad \lambda_2 = 3500\,\text{Å}$$

Subtract: $\frac{eV_s}{2} = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \Rightarrow$

$$eV_s = 2hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = 2 \times 12,400 \times \left(\frac{1}{3000} - \frac{1}{3500}\right) eV \Rightarrow$$

$$\Rightarrow \boxed{eV_s = 1.18\,eV}$$

$$\phi = \frac{hc}{\lambda_1} - eV_s = \frac{12,400}{3000}\,eV - 1.18\,eV \Rightarrow \boxed{\phi = 2.95\,eV}$$

(b) $\frac{hc}{\lambda_t} = \phi \Rightarrow \lambda_t = \frac{hc}{\phi} = \frac{12,400}{2.95}\,\text{Å}$

$$\Rightarrow \boxed{\lambda_t = 4203\,\text{Å}}$$